

Open system approach to neutrinos propagating in an ultralight scalar background

Gustavo F. S. Alves

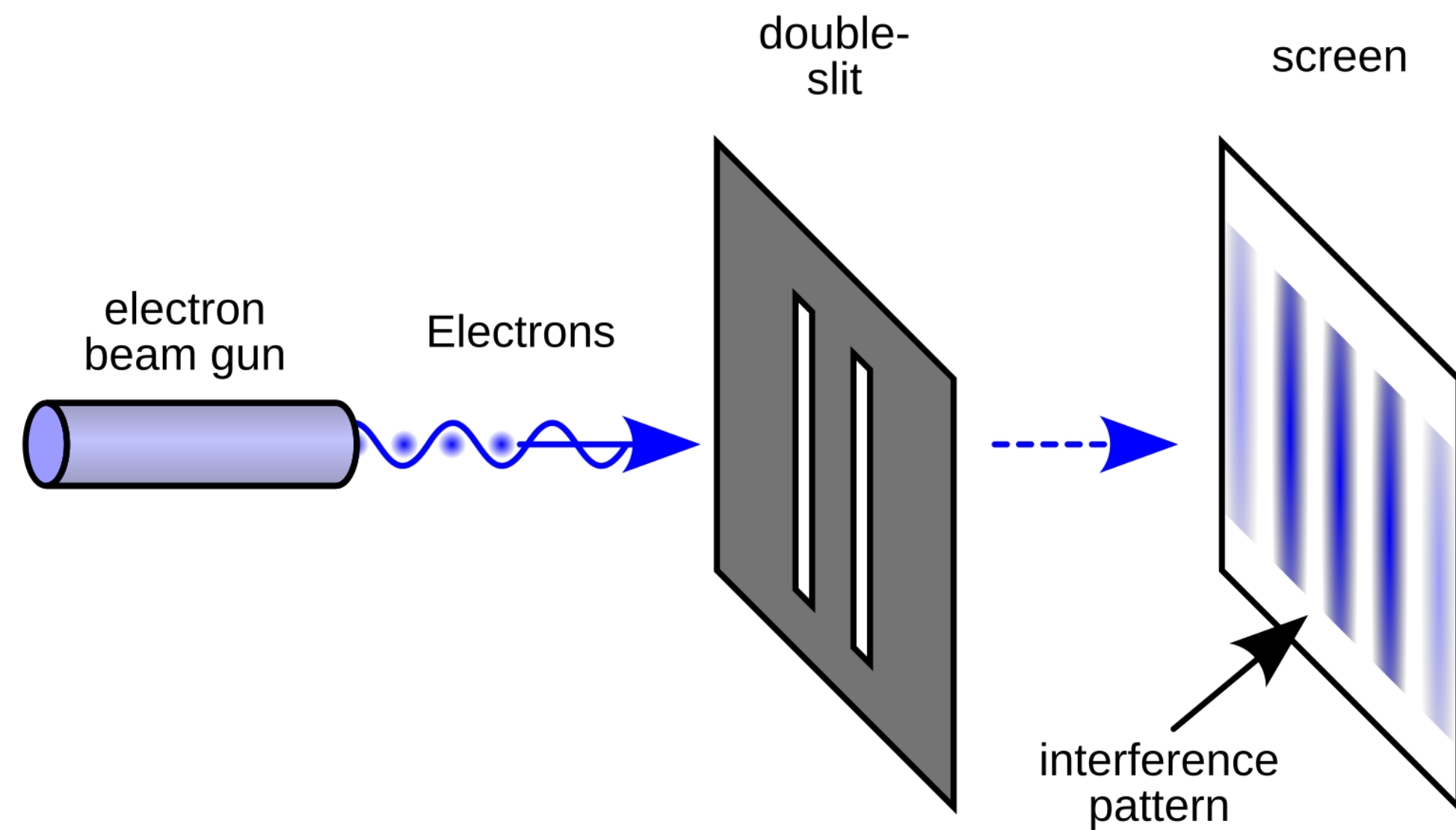
arXiv:2603.02382

In Collaboration with:

Lua F. T. Airoidi
Pedro Machado
Peter Vander Griend



We live in a quantum world...



But we often don't see its quantumness

$$\frac{1}{\sqrt{2}}$$



+

$$\frac{1}{\sqrt{2}}$$



But we often don't see its quantumness

$$\frac{1}{\sqrt{2}}$$



+

$$\frac{1}{\sqrt{2}}$$



~~≠~~



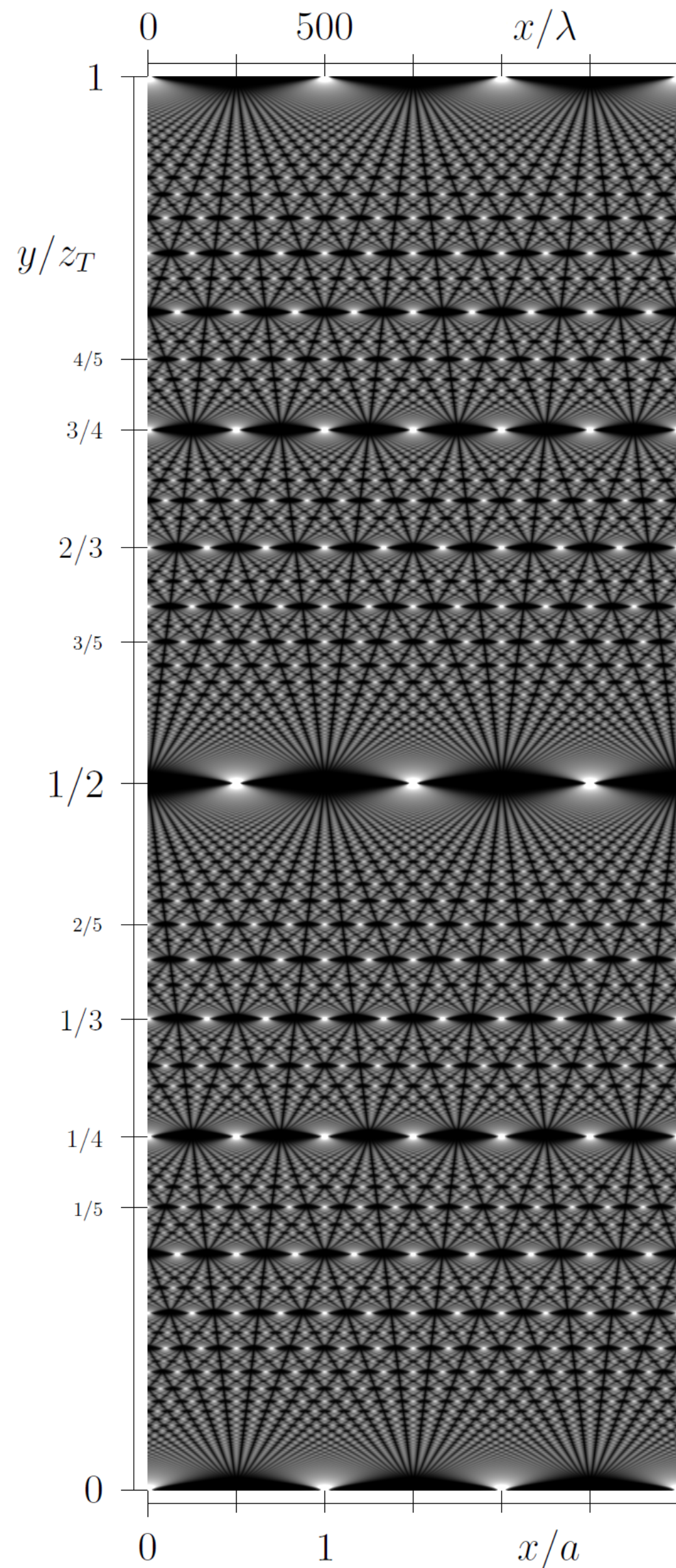
Superpositions tend to decohere

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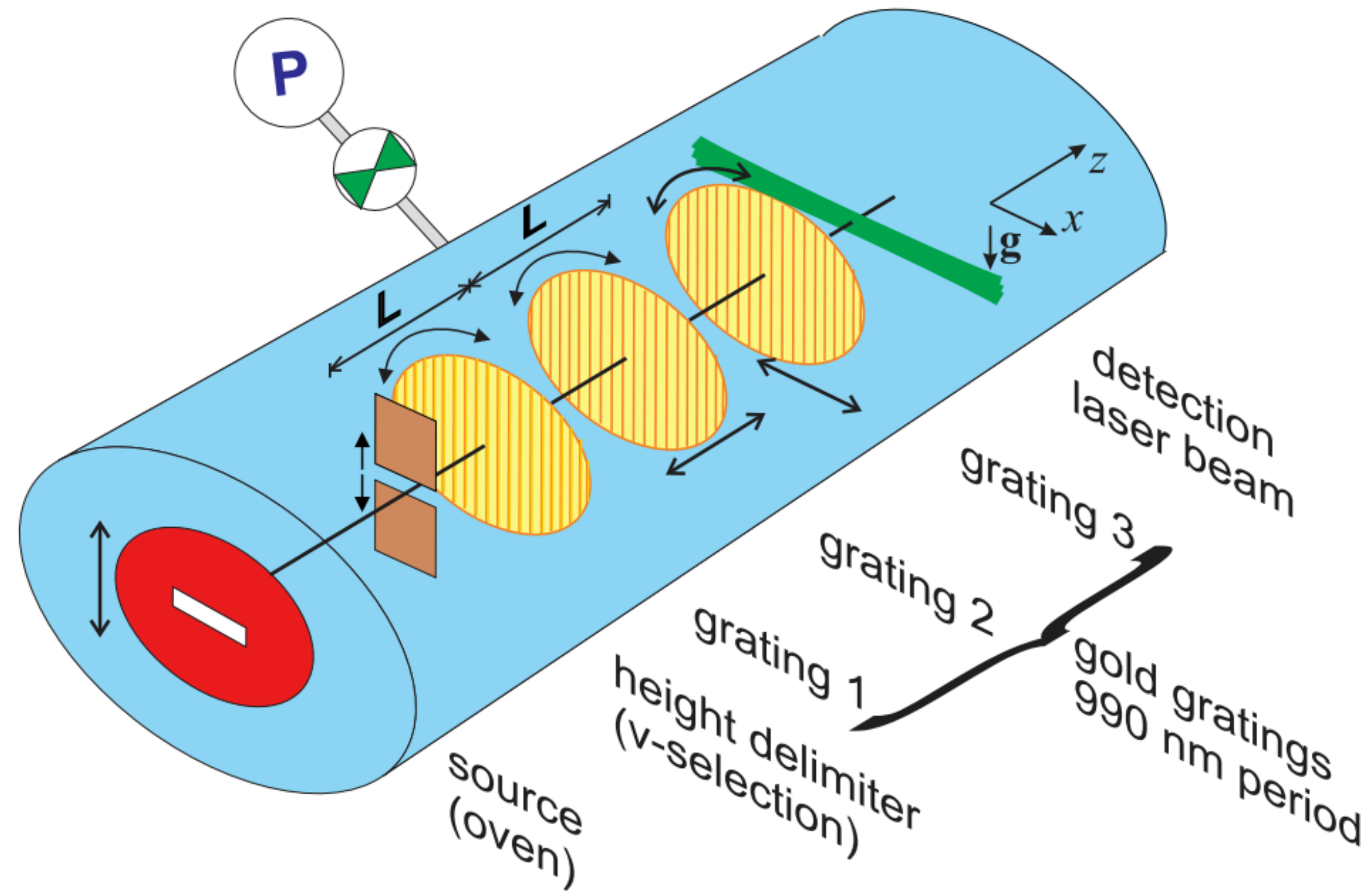
“The loss of visibility in the interference pattern due to the coupling of a quantum system to its environment.”

Appl. Phys. B 77 (2003) 781-787, Zeilinger, et al.

Superpositions tend to decohere

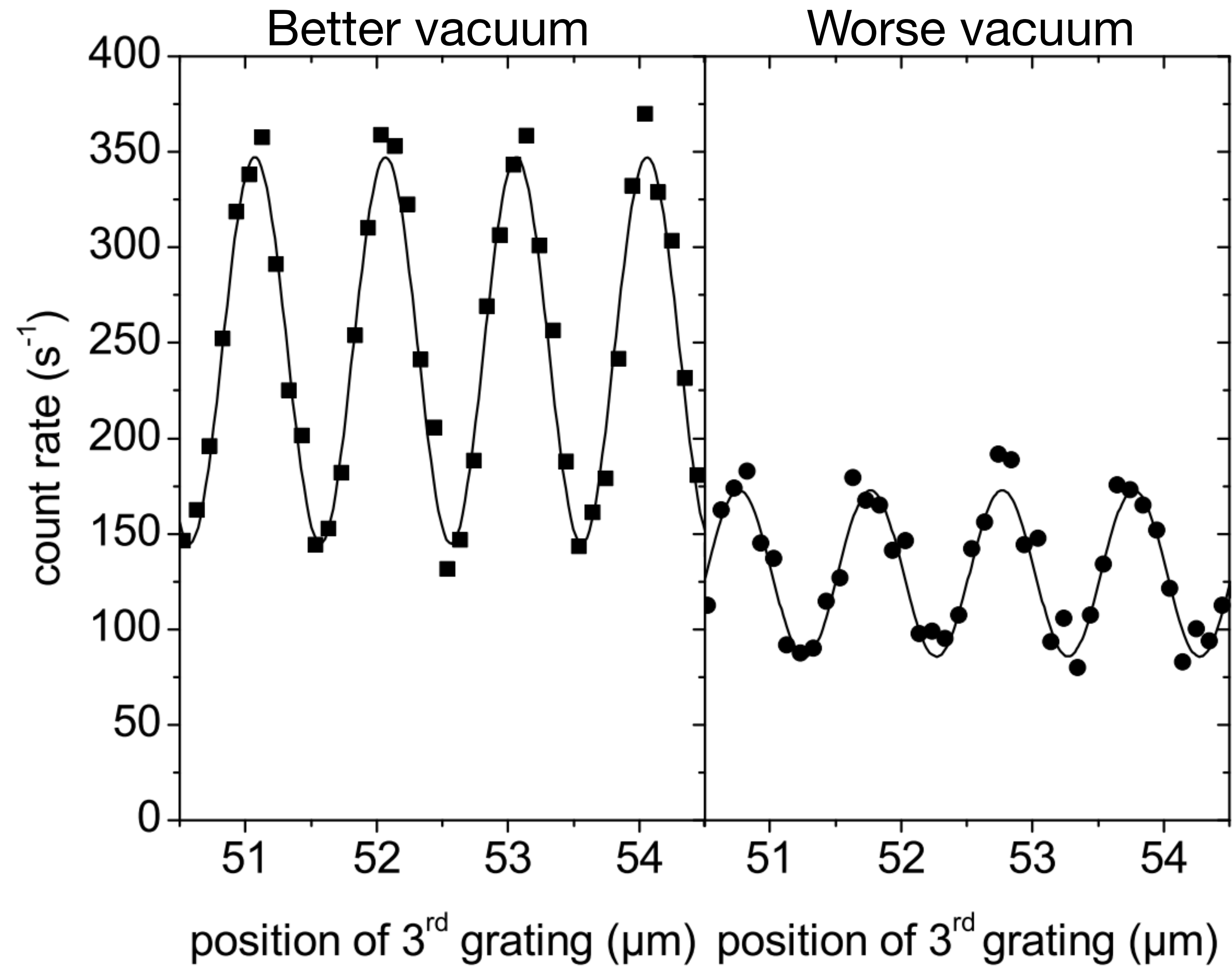


Wikipedia, The Talbot effect

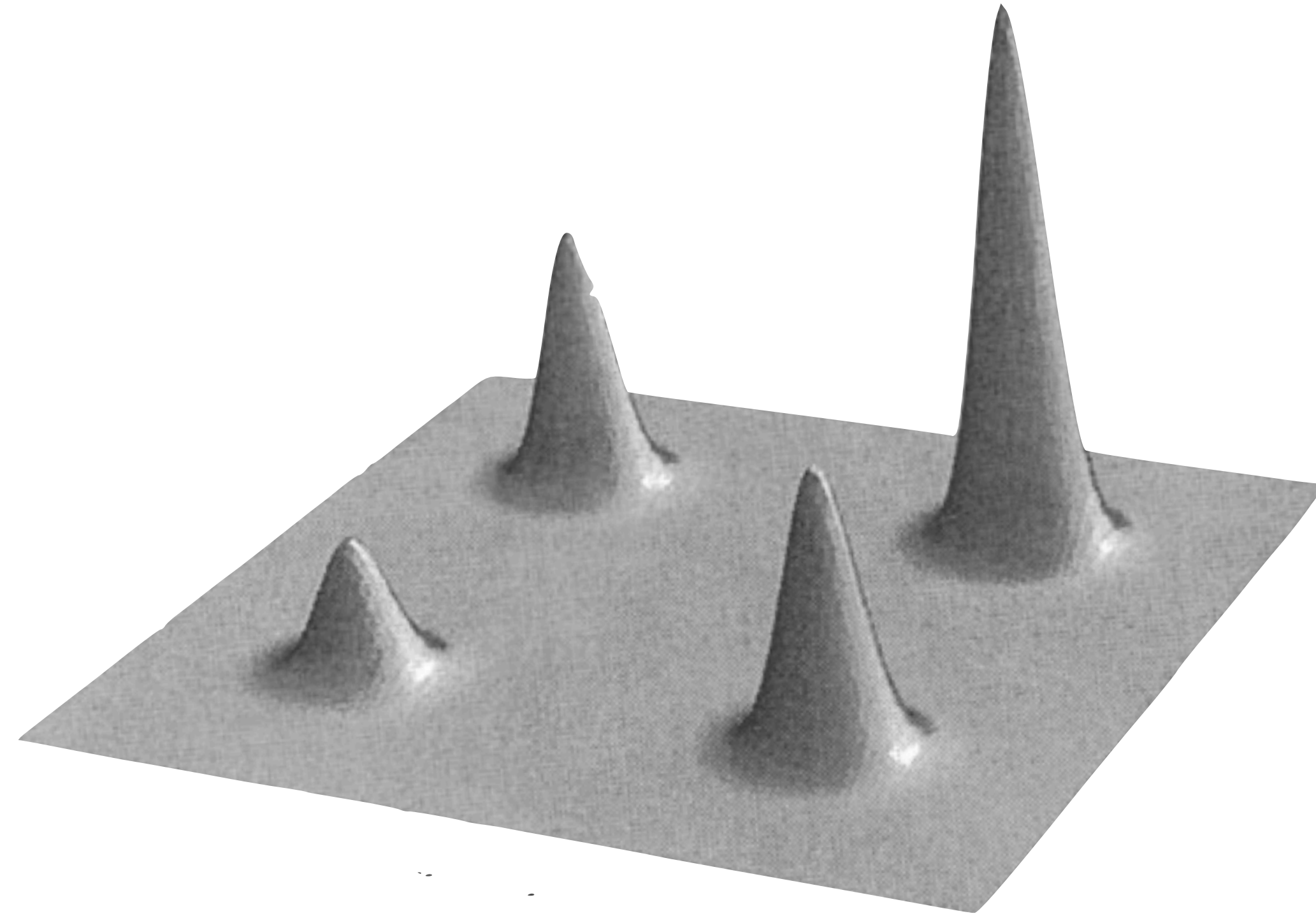


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Superpositions tend to decohere

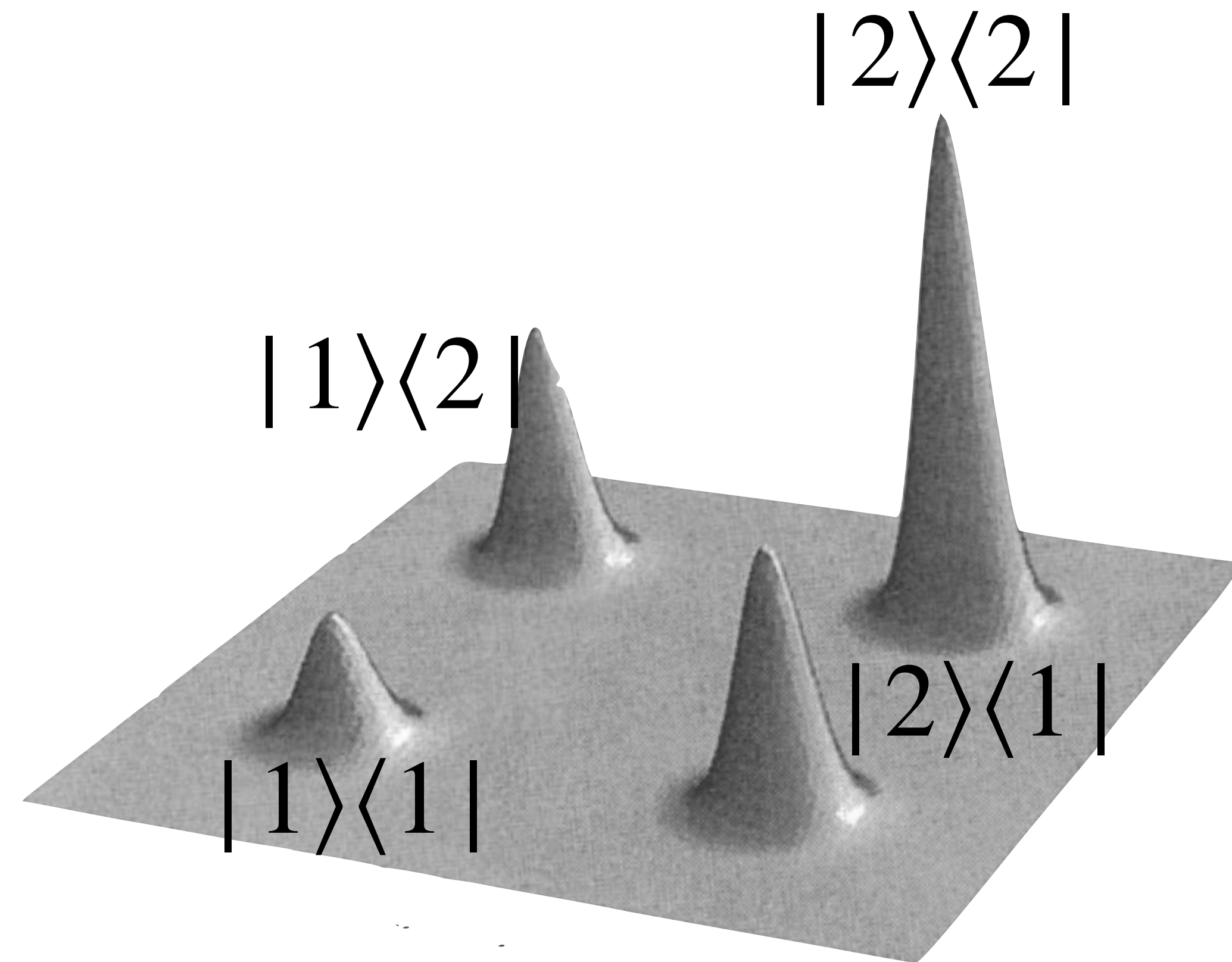


Preparing a superposition



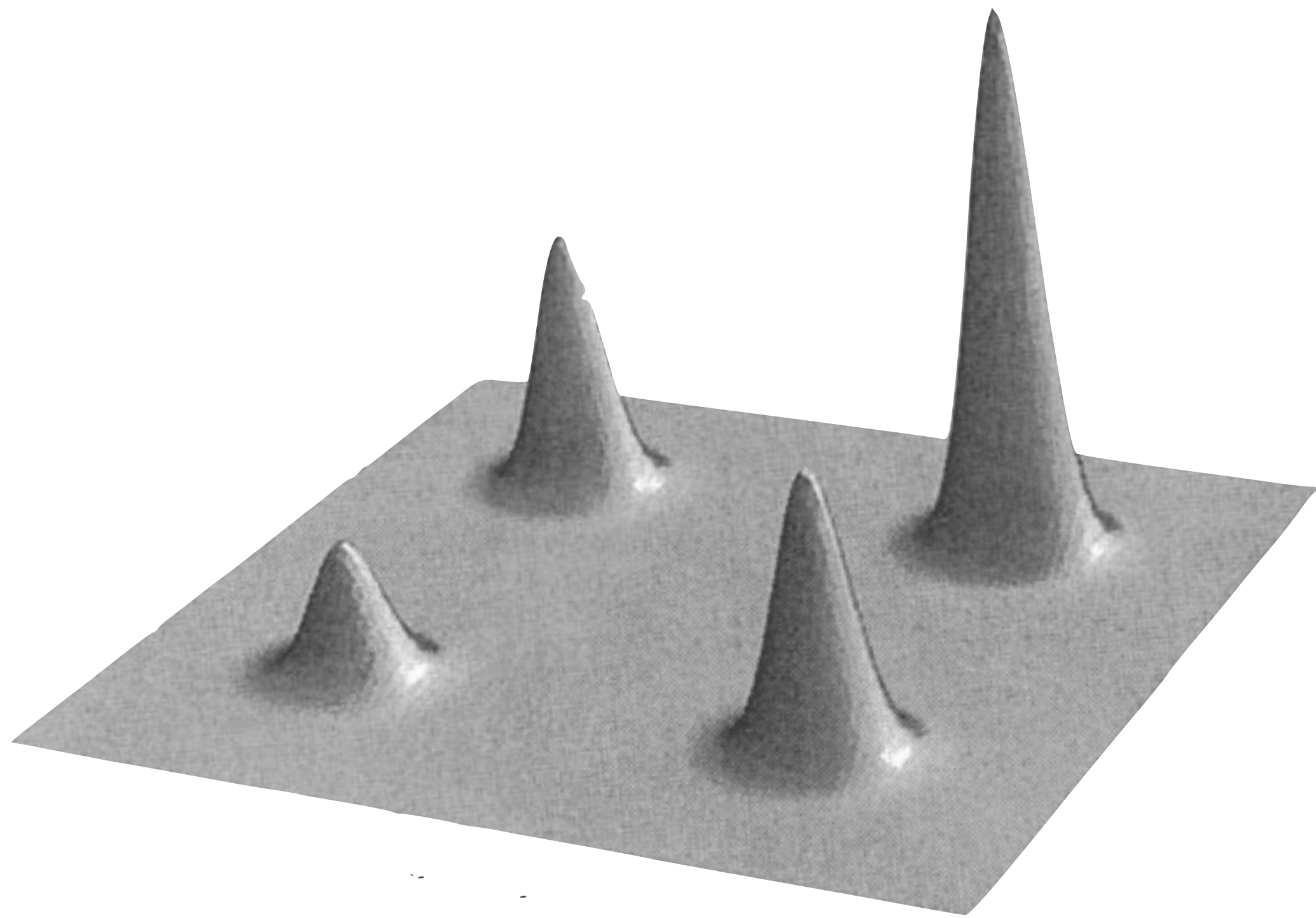
$$|\psi\rangle = a|1\rangle + b|2\rangle$$

Preparing a superposition



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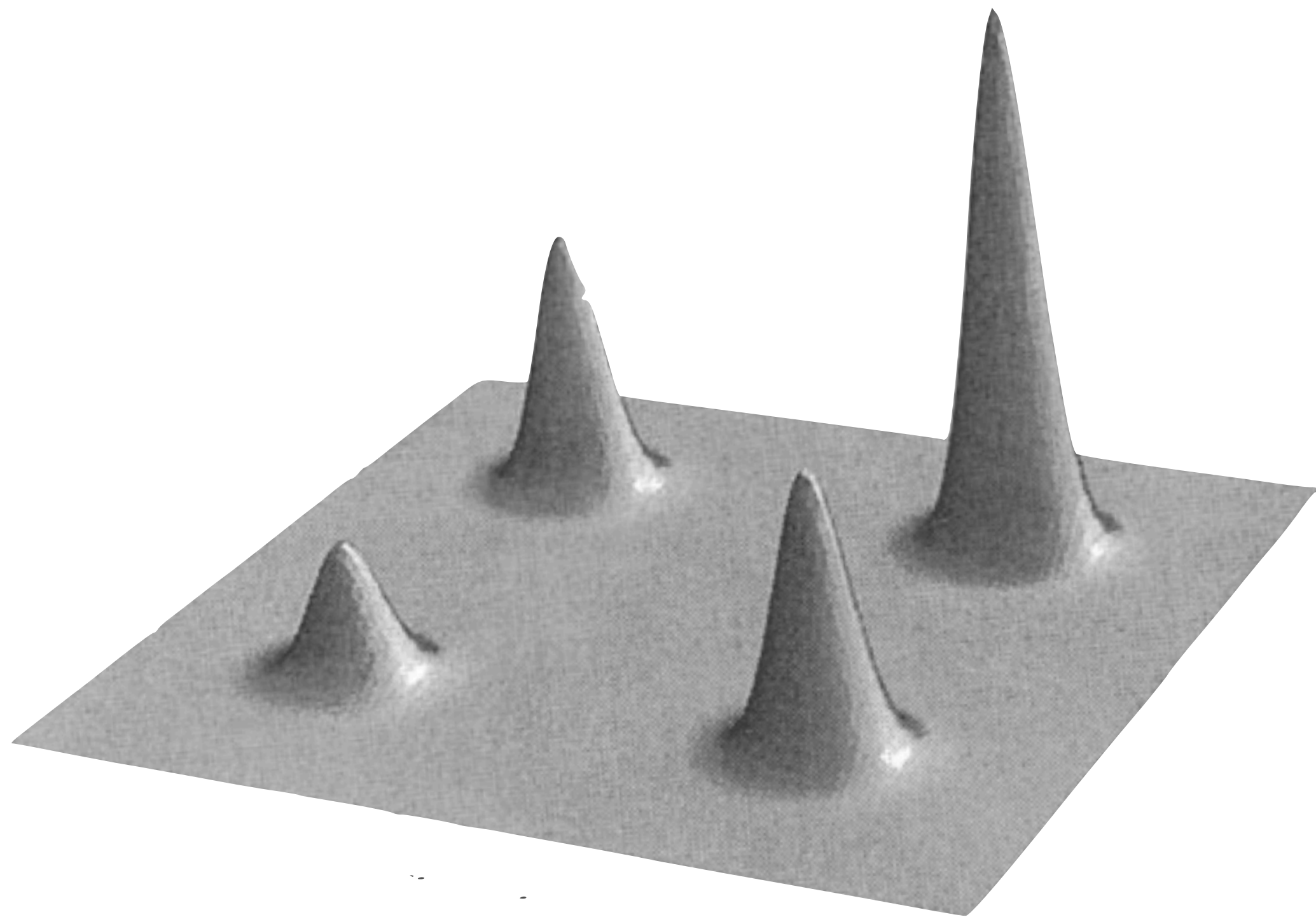
Interference pattern



$$|\langle \phi | \psi \rangle|^2 = |a|^2 |\langle \phi | 1 \rangle|^2$$

$$|\psi\rangle = a|1\rangle + b|2\rangle$$

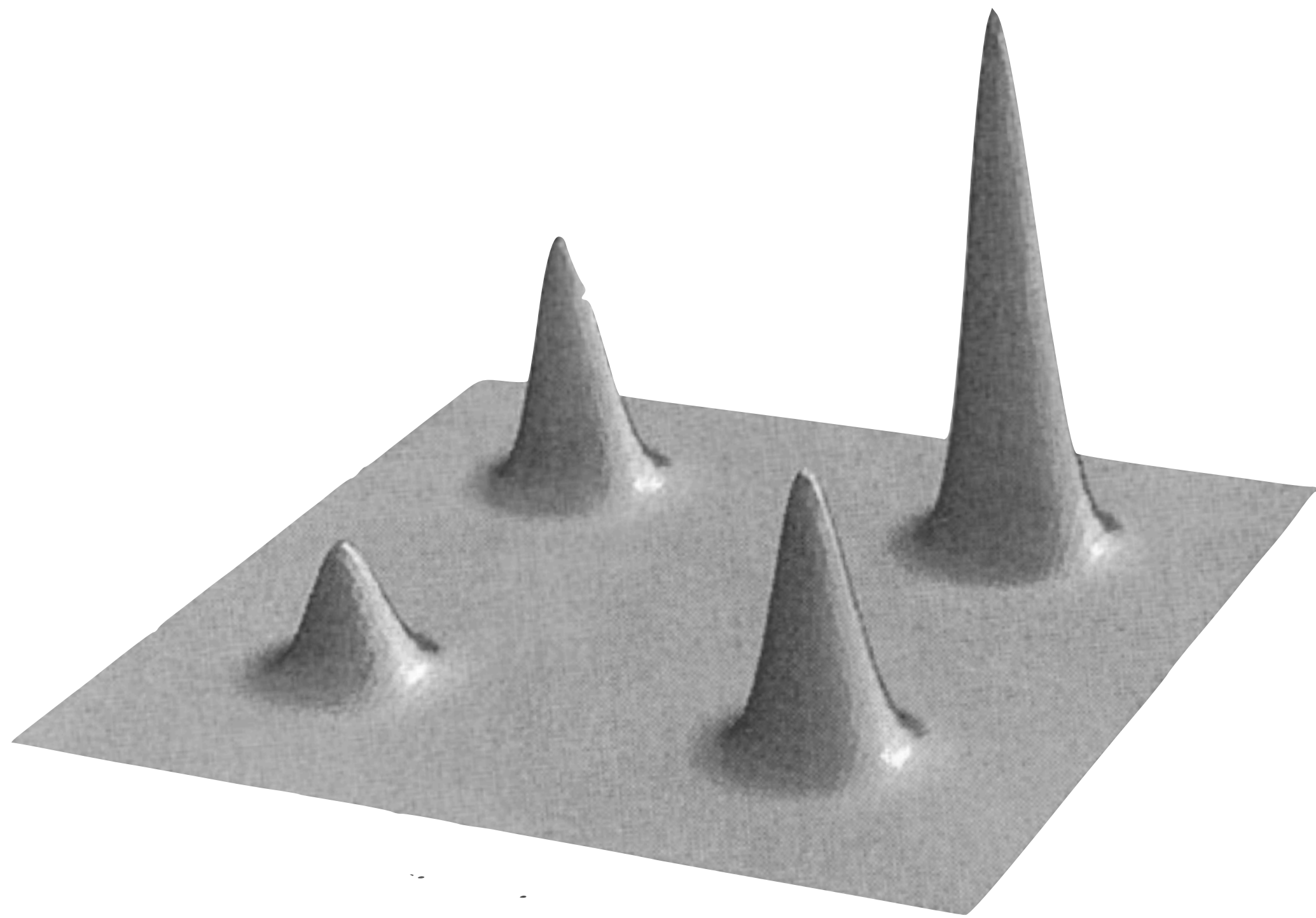
Interference pattern



$$|\langle \phi | \psi \rangle|^2 = |a|^2 |\langle \phi | 1 \rangle|^2 + |b|^2 |\langle \phi | 2 \rangle|^2$$

$$|\psi\rangle = a|1\rangle + b|2\rangle$$

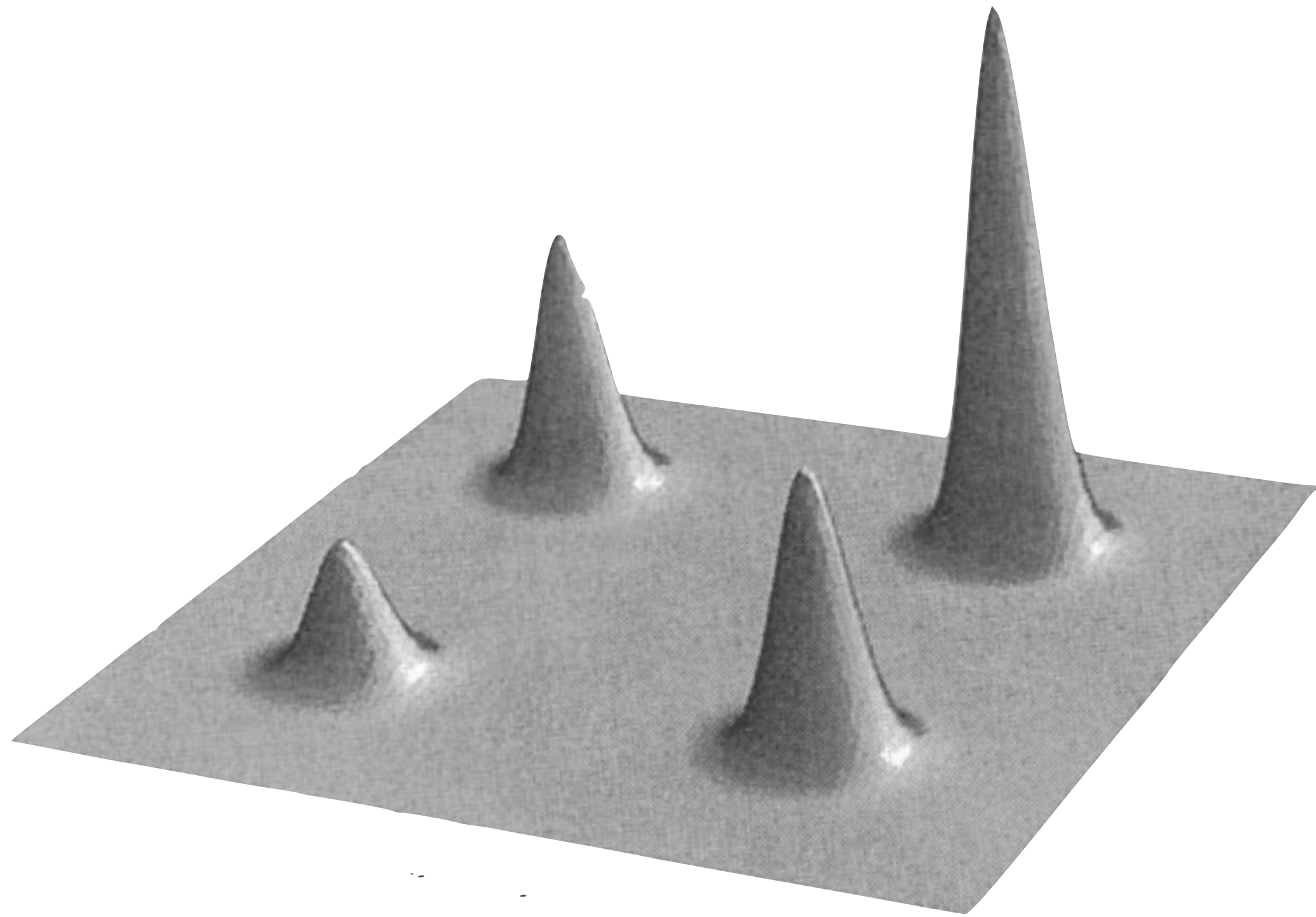
Interference pattern



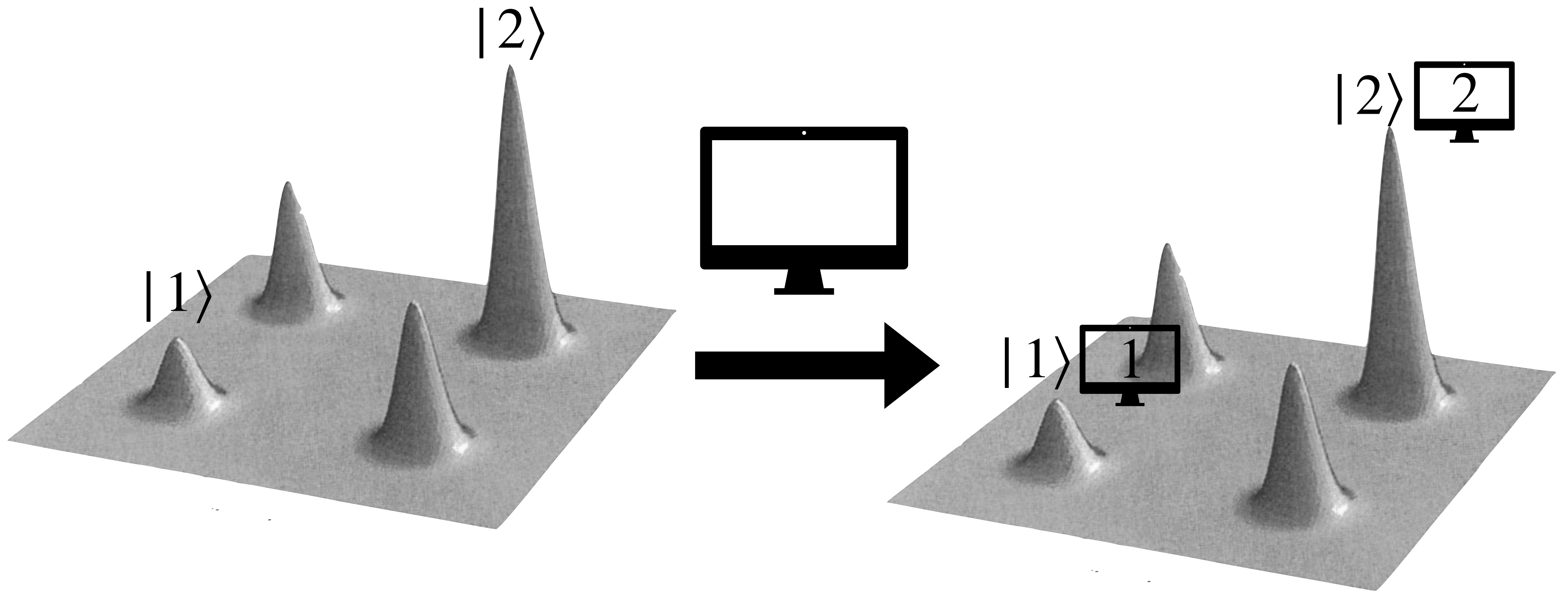
$$\begin{aligned} |\langle \phi | \psi \rangle|^2 &= |a|^2 |\langle \phi | 1 \rangle|^2 \\ &\quad + |b|^2 |\langle \phi | 2 \rangle|^2 \\ &\quad + 2\Re [ab^* \langle \phi | 1 \rangle (\langle \phi | 2 \rangle)^*] \end{aligned}$$

$$|\psi\rangle = a|1\rangle + b|2\rangle$$

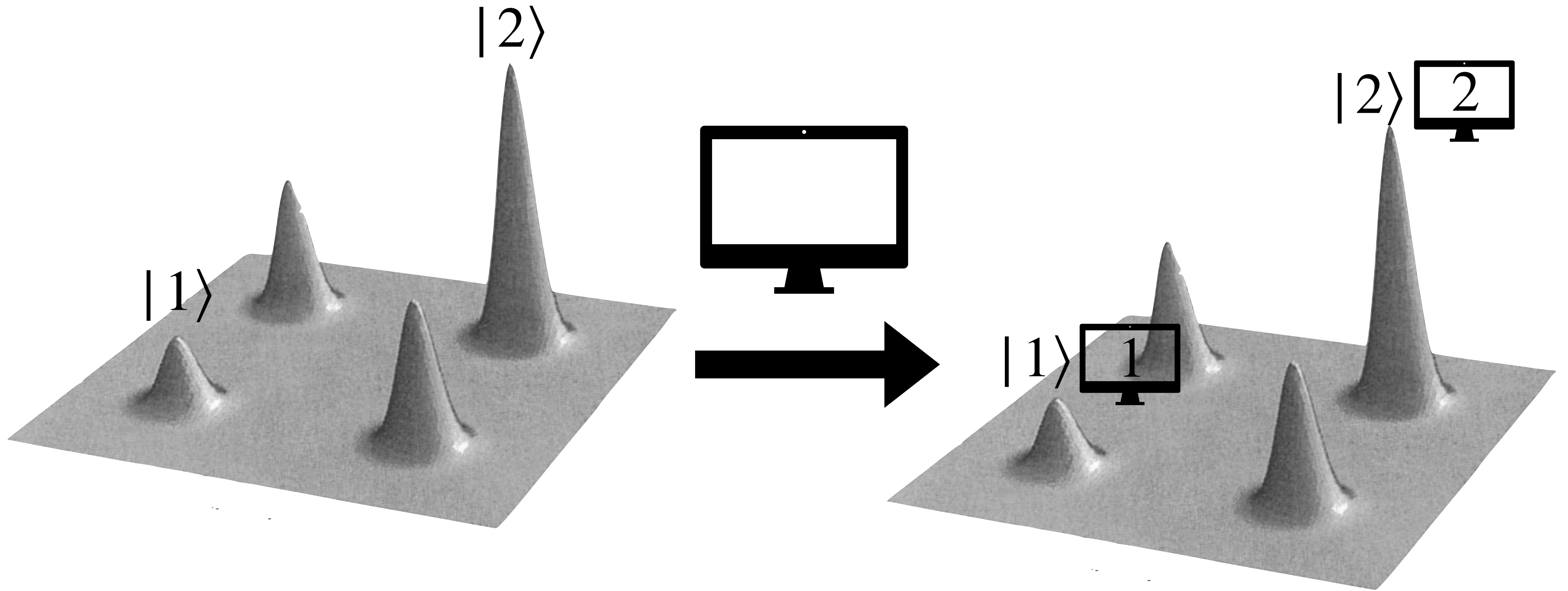
Decoherence mechanism



Decoherence mechanism




Decoherence mechanism



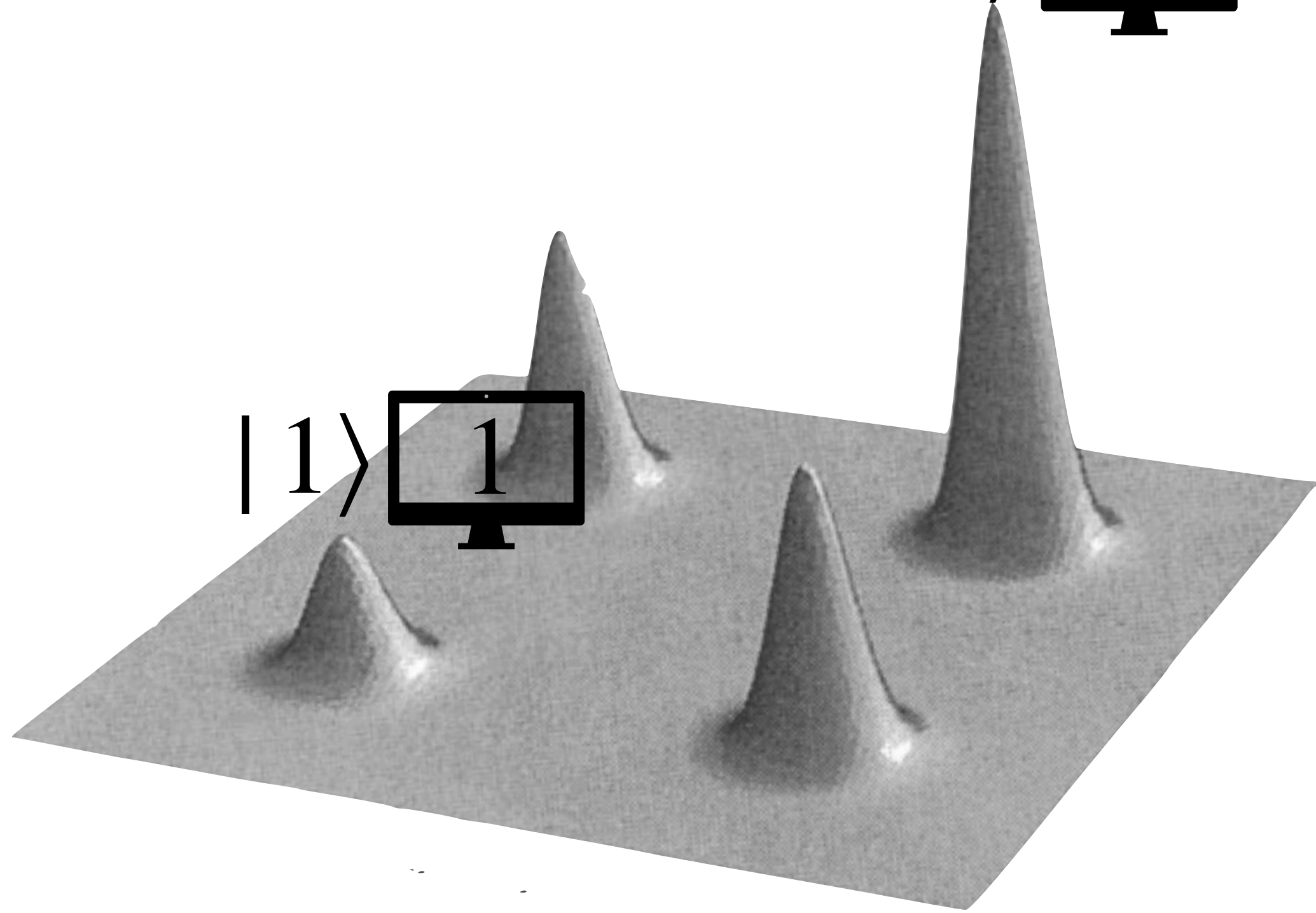
$$|\psi\rangle \otimes |B_0\rangle \rightarrow a |1\rangle |B_1\rangle + b |2\rangle |B_2\rangle$$

Decoherence is everywhere

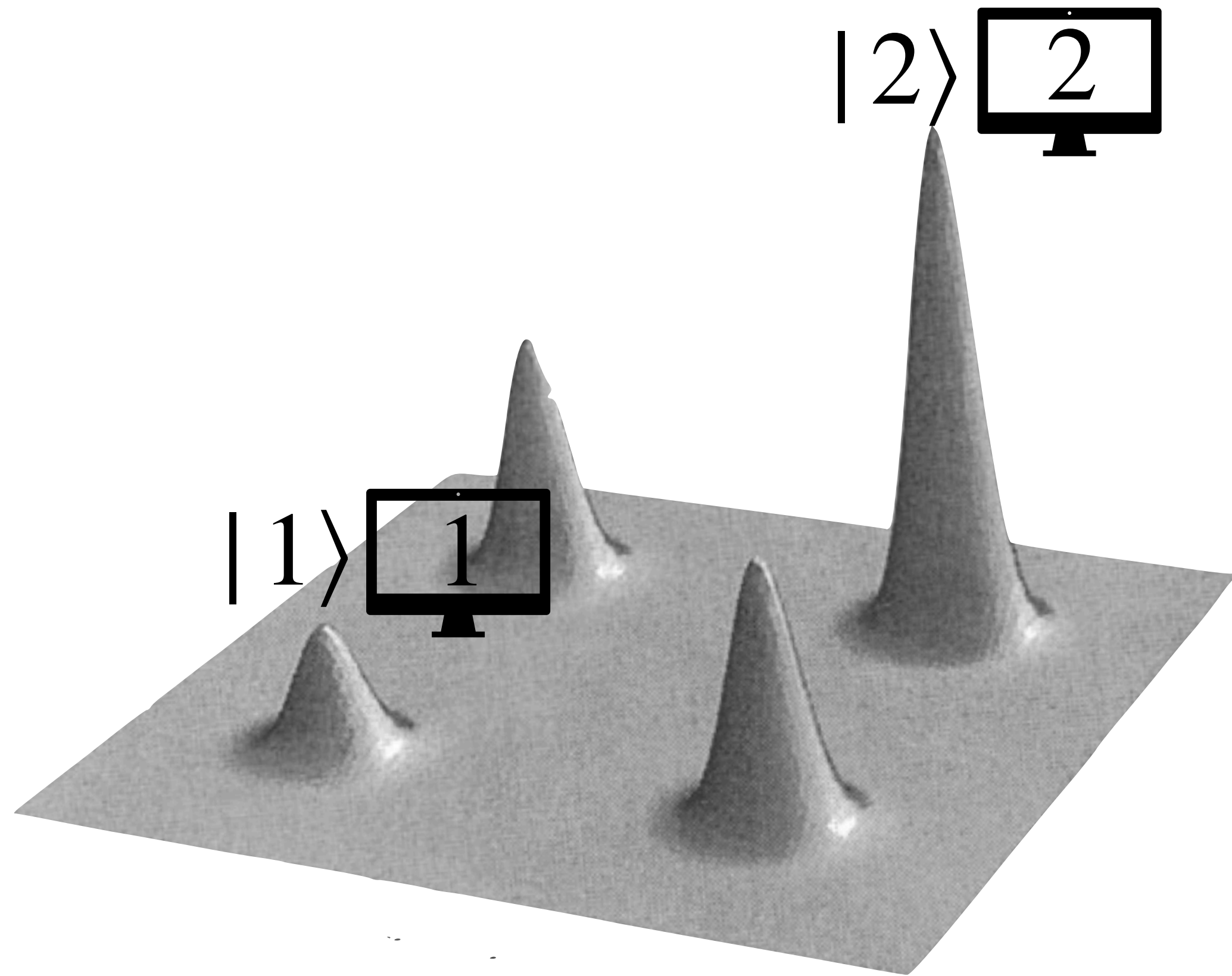
$|2\rangle$ 

$$|\langle \phi | \psi \rangle|^2 = |a|^2 |\langle \phi | 1 \rangle|^2$$

$|1\rangle$ 

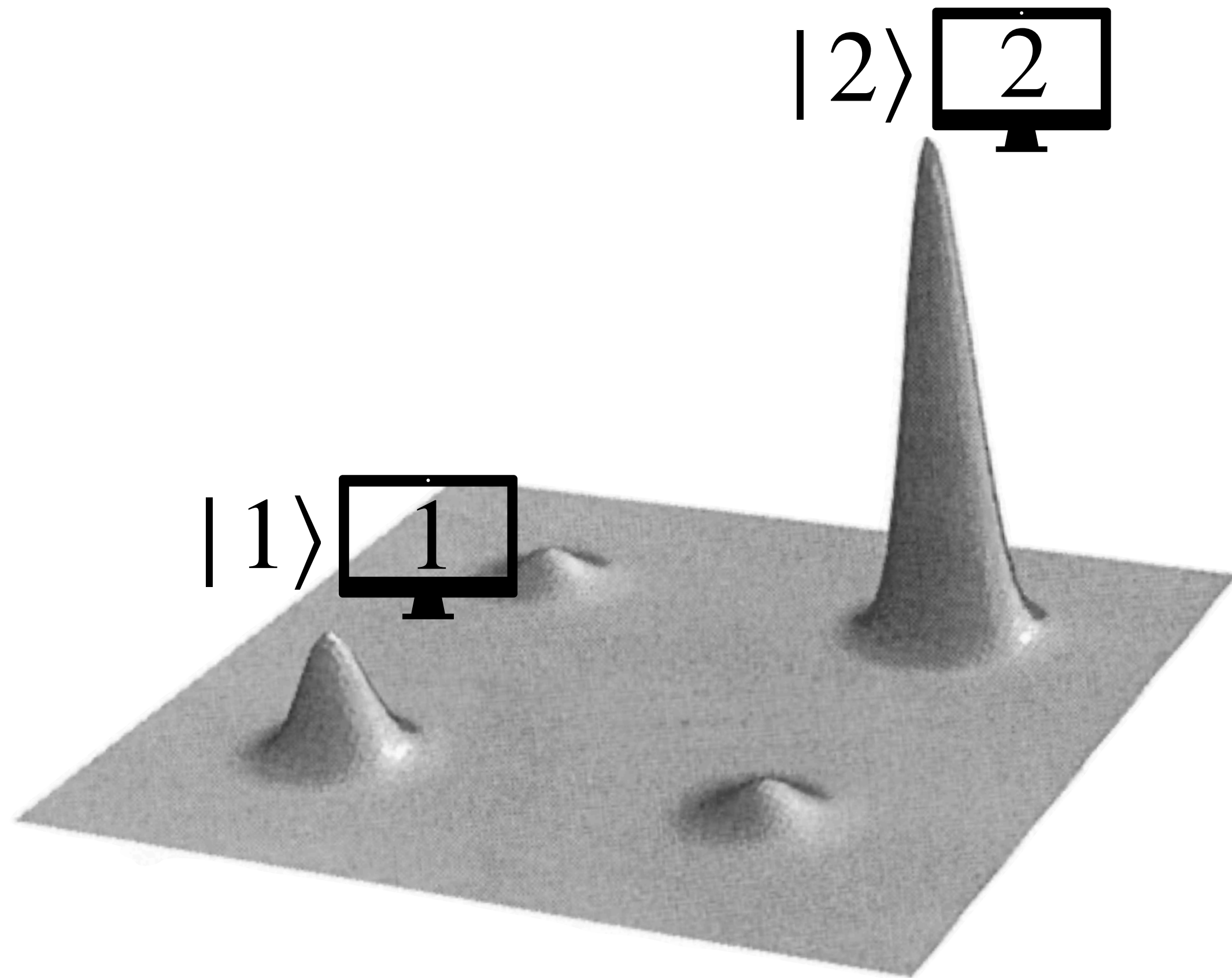


Decoherence is everywhere



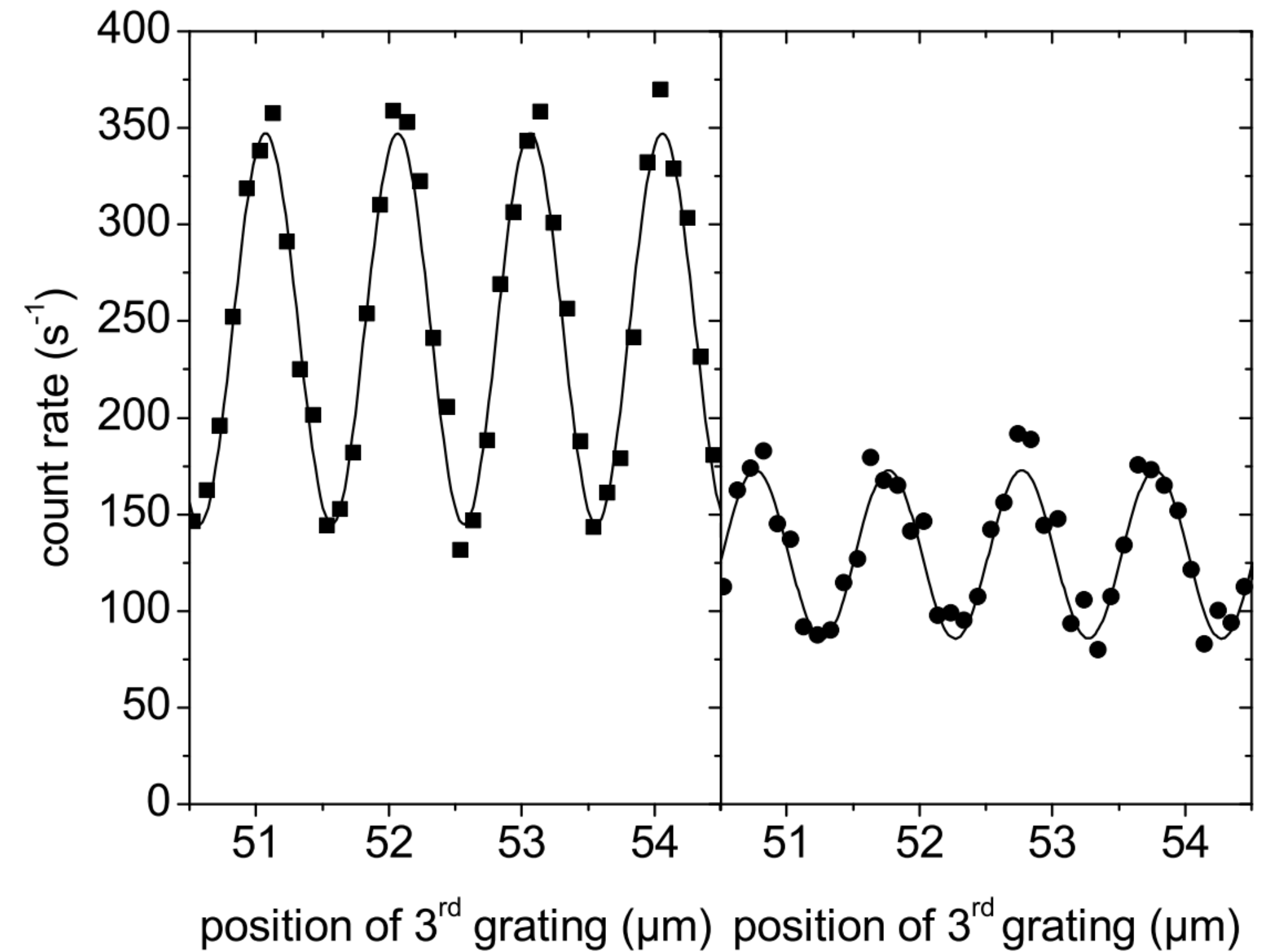
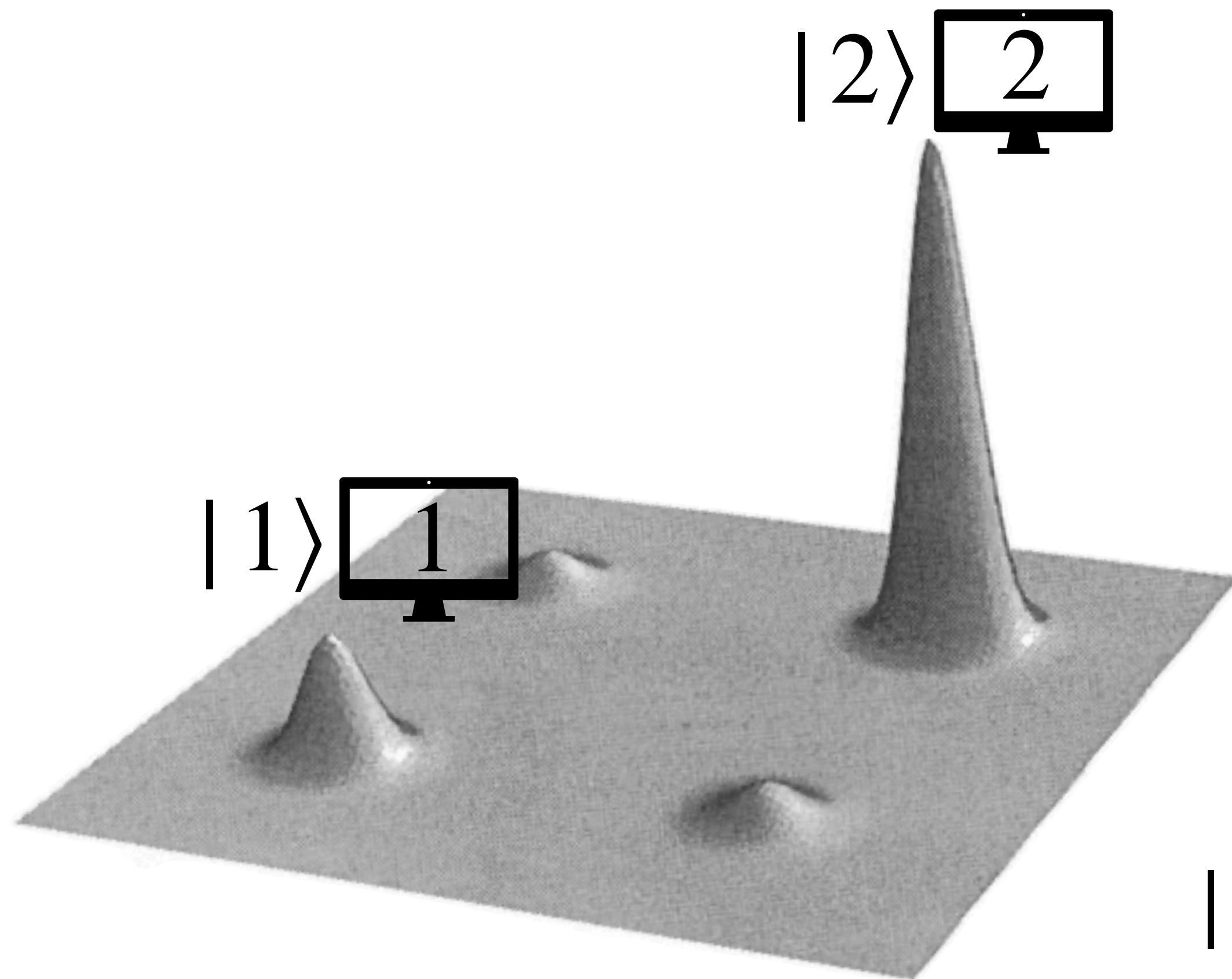
$$|\langle \phi | \psi \rangle|^2 = |a|^2 |\langle \phi | 1 \rangle|^2 + |b|^2 |\langle \phi | 2 \rangle|^2$$

Decoherence is everywhere



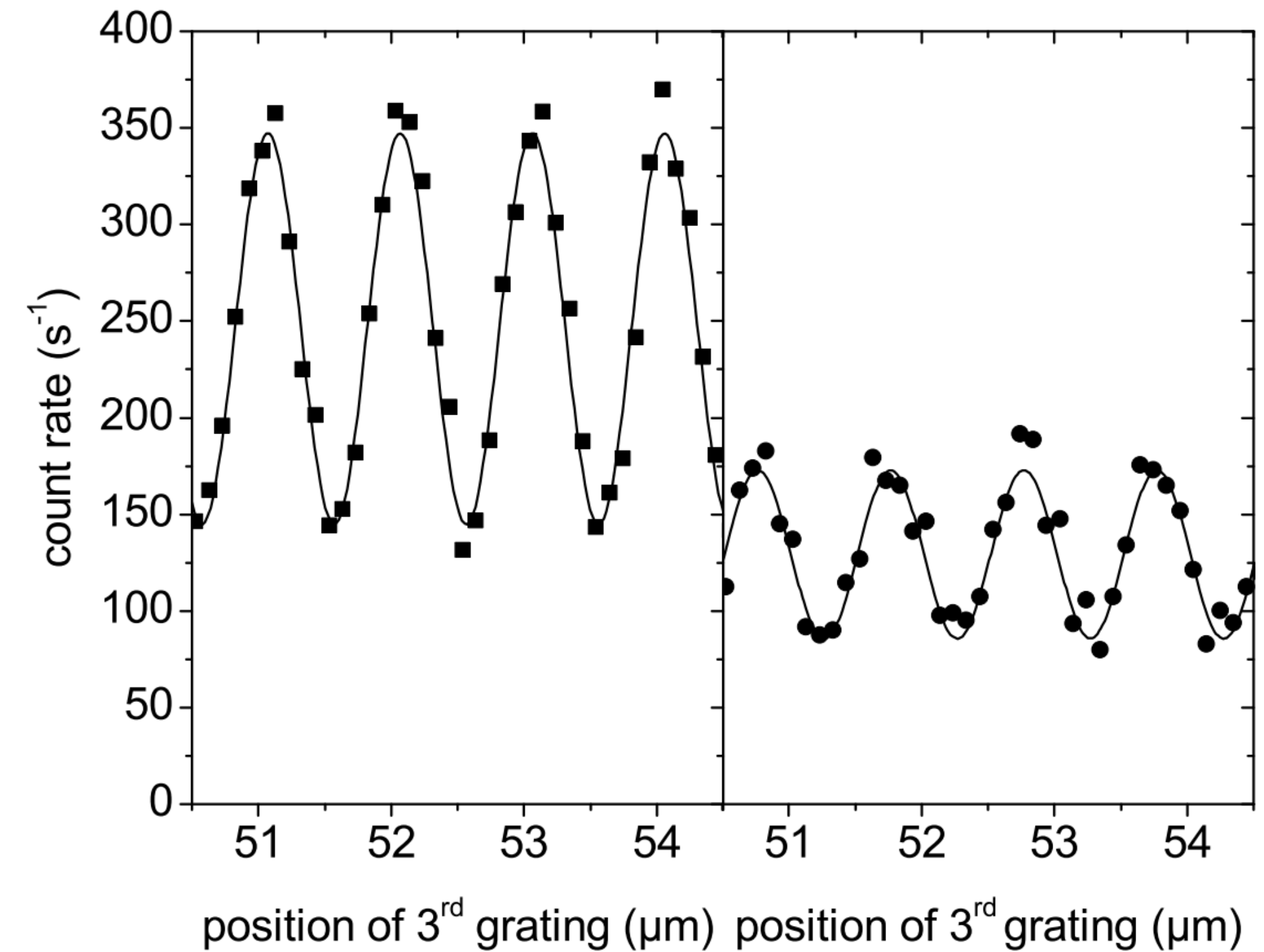
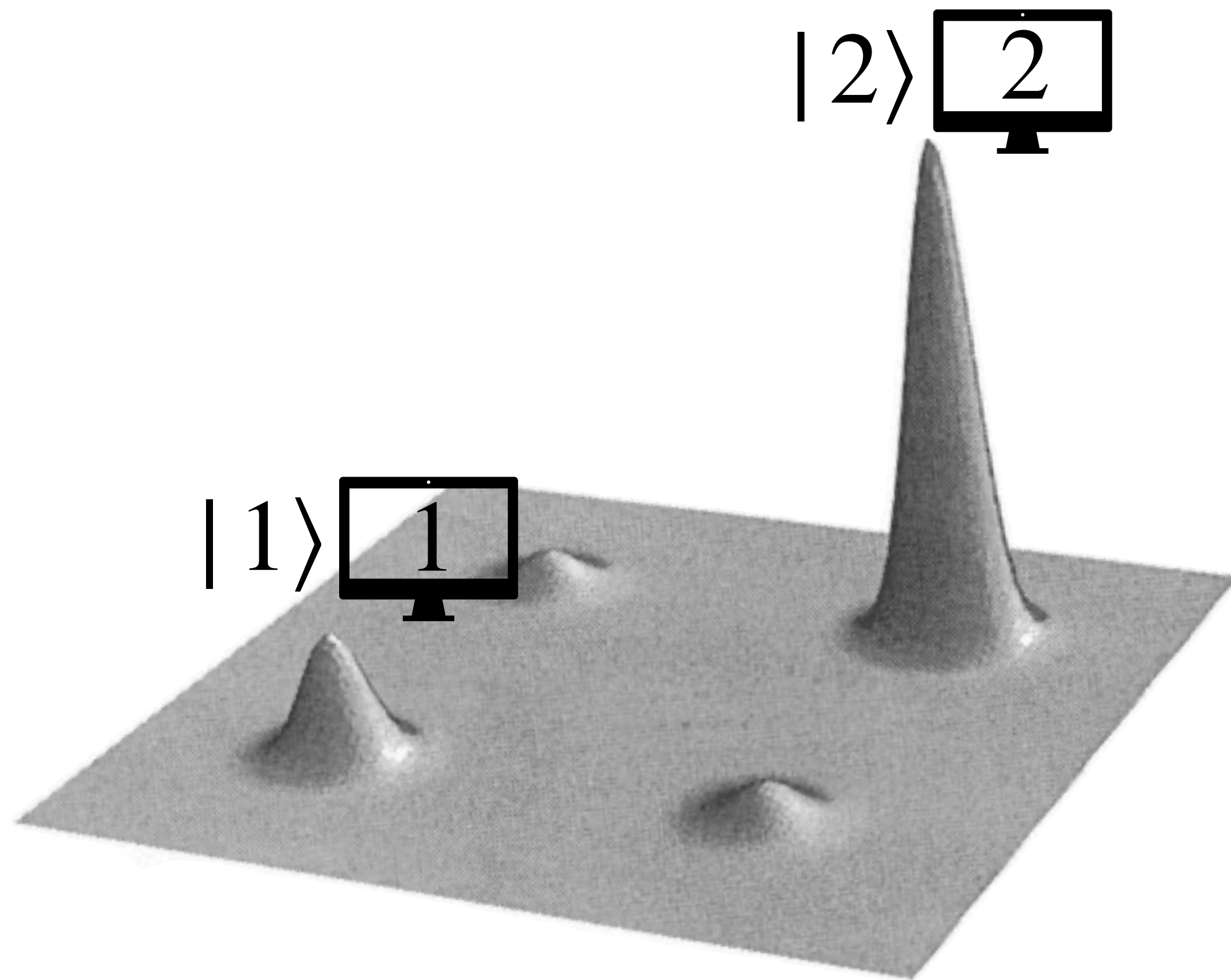
$$|\langle\phi|\psi\rangle|^2 = |a|^2|\langle\phi|1\rangle|^2 + |b|^2|\langle\phi|2\rangle|^2 + 2\Re [ab^*\langle\phi|1\rangle(\langle\phi|2\rangle)^*\langle B_2|B_1\rangle]$$

Decoherence is everywhere



$$\begin{aligned}
 |\langle \phi | \psi \rangle|^2 &= |a|^2 |\langle \phi | 1 \rangle|^2 \\
 &+ |b|^2 |\langle \phi | 2 \rangle|^2 \\
 &+ 2\Re [ab^* \langle \phi | 1 \rangle (\langle \phi | 2 \rangle)^* \langle B_2 | B_1 \rangle]
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Decoherence is everywhere



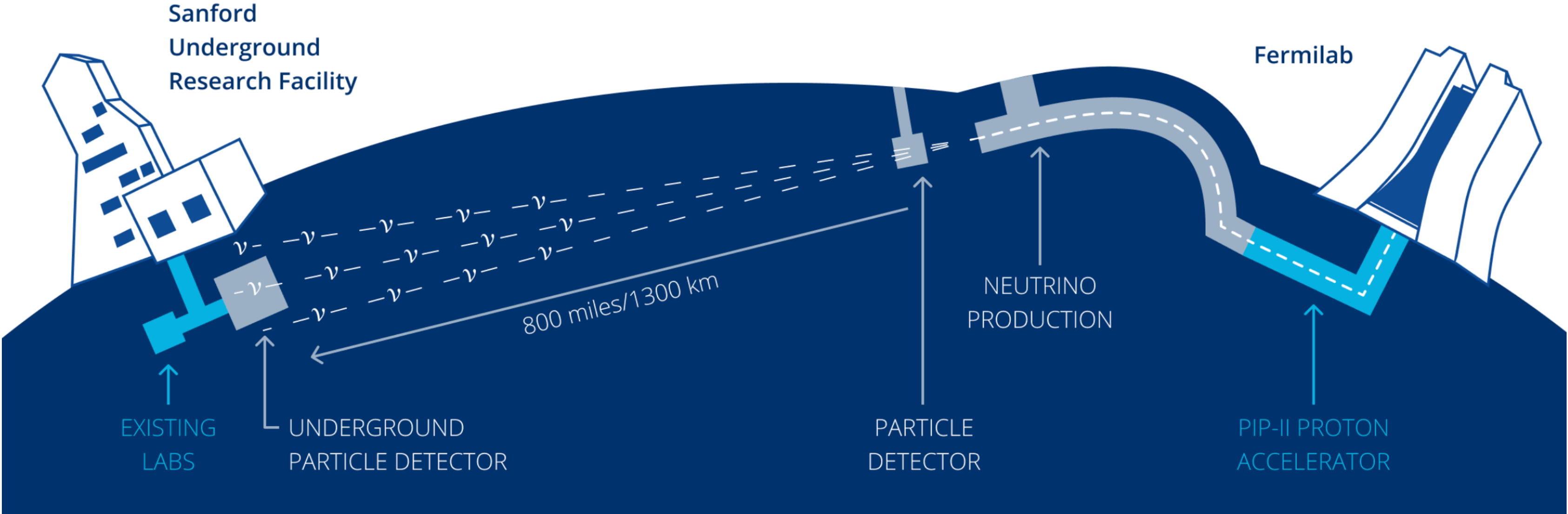
- All these systems require very careful preparation and controlled conditions.
- It is hard to maintain quantum superpositions

Nature knows better

Imagine preparing a quantum state in a superposition in Chicago and detecting it in South Dakota, shooting it through the Earth.

Nature knows better

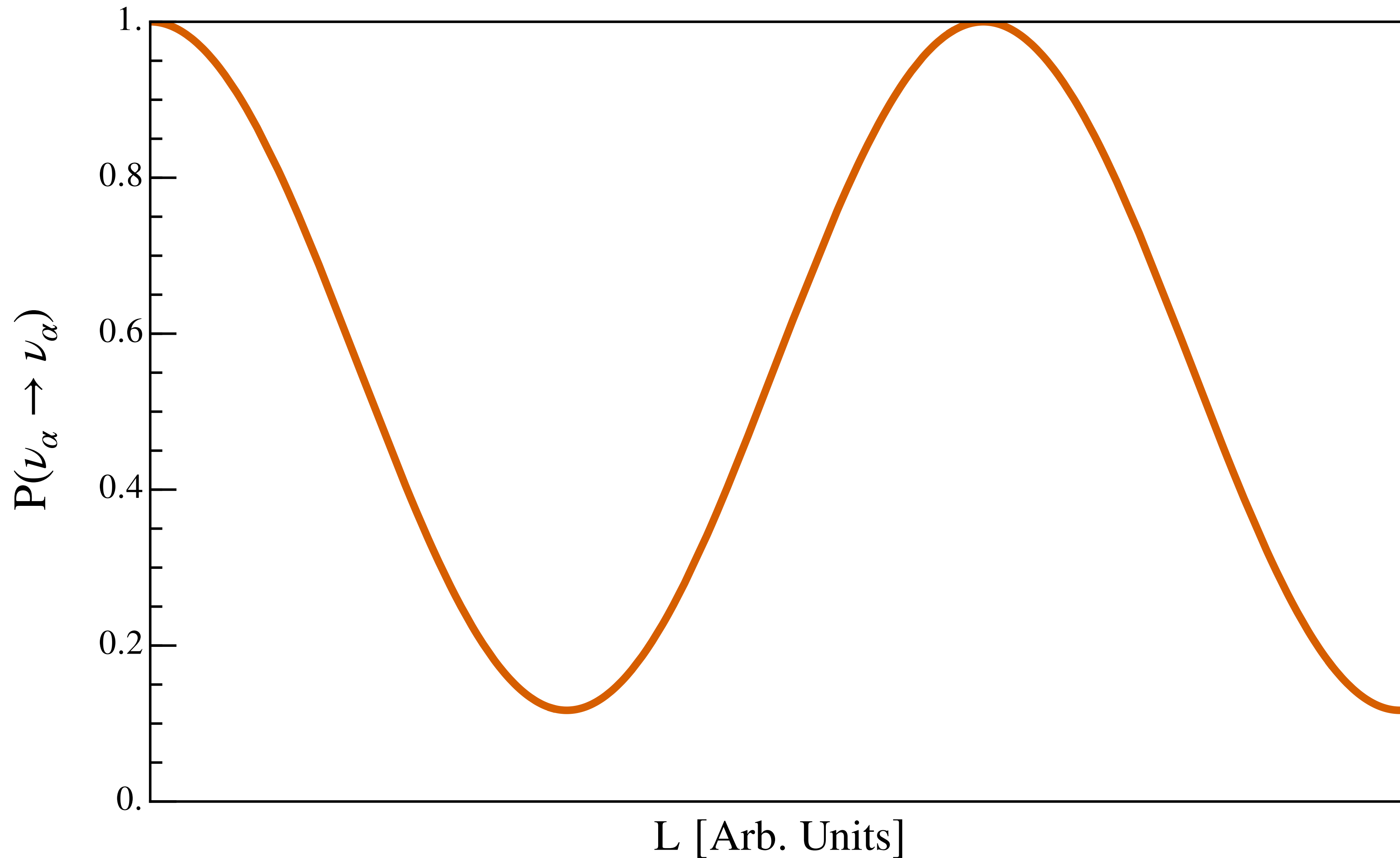
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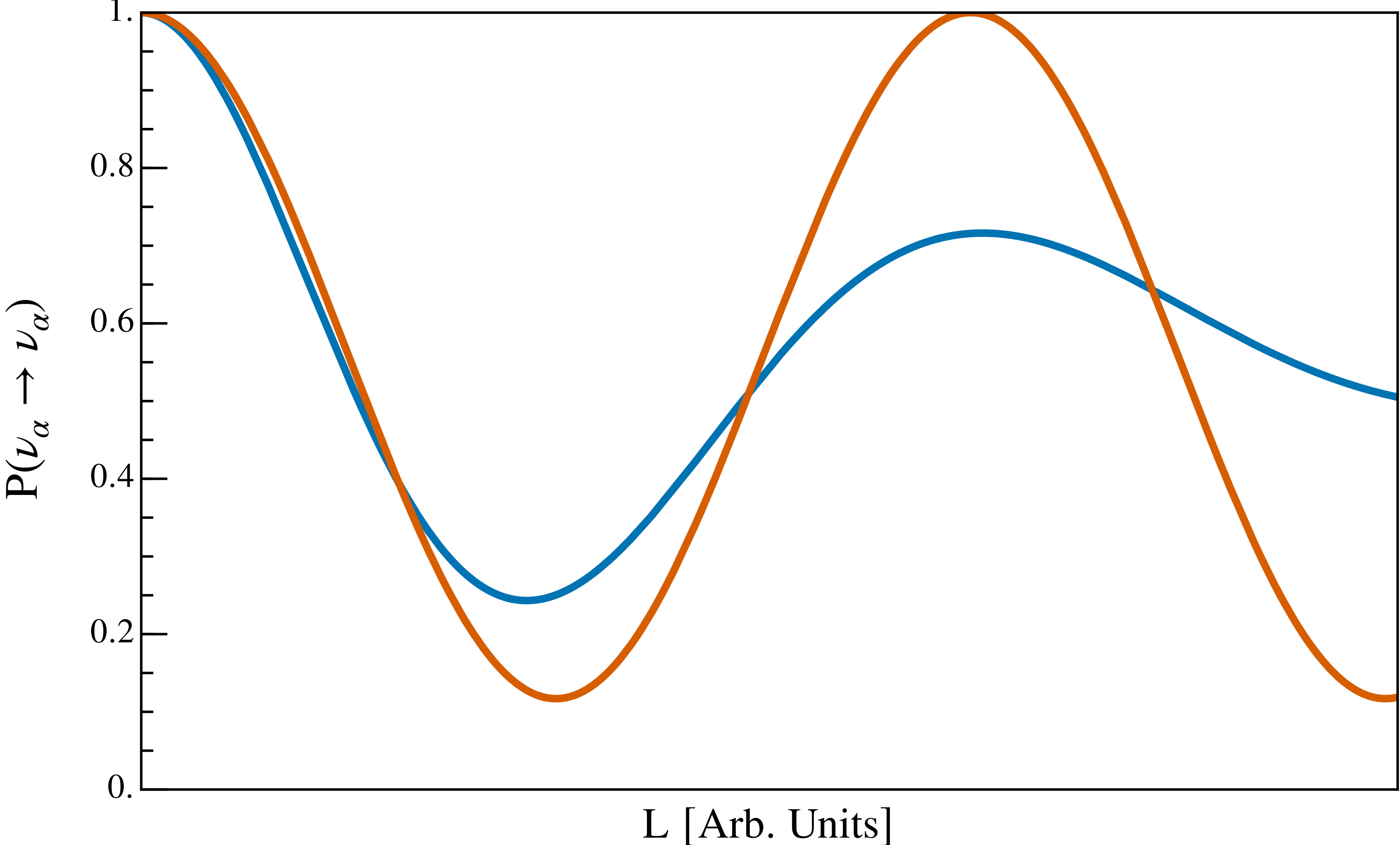
Neutrinos oscillations are the “free lunch” of quantum interference



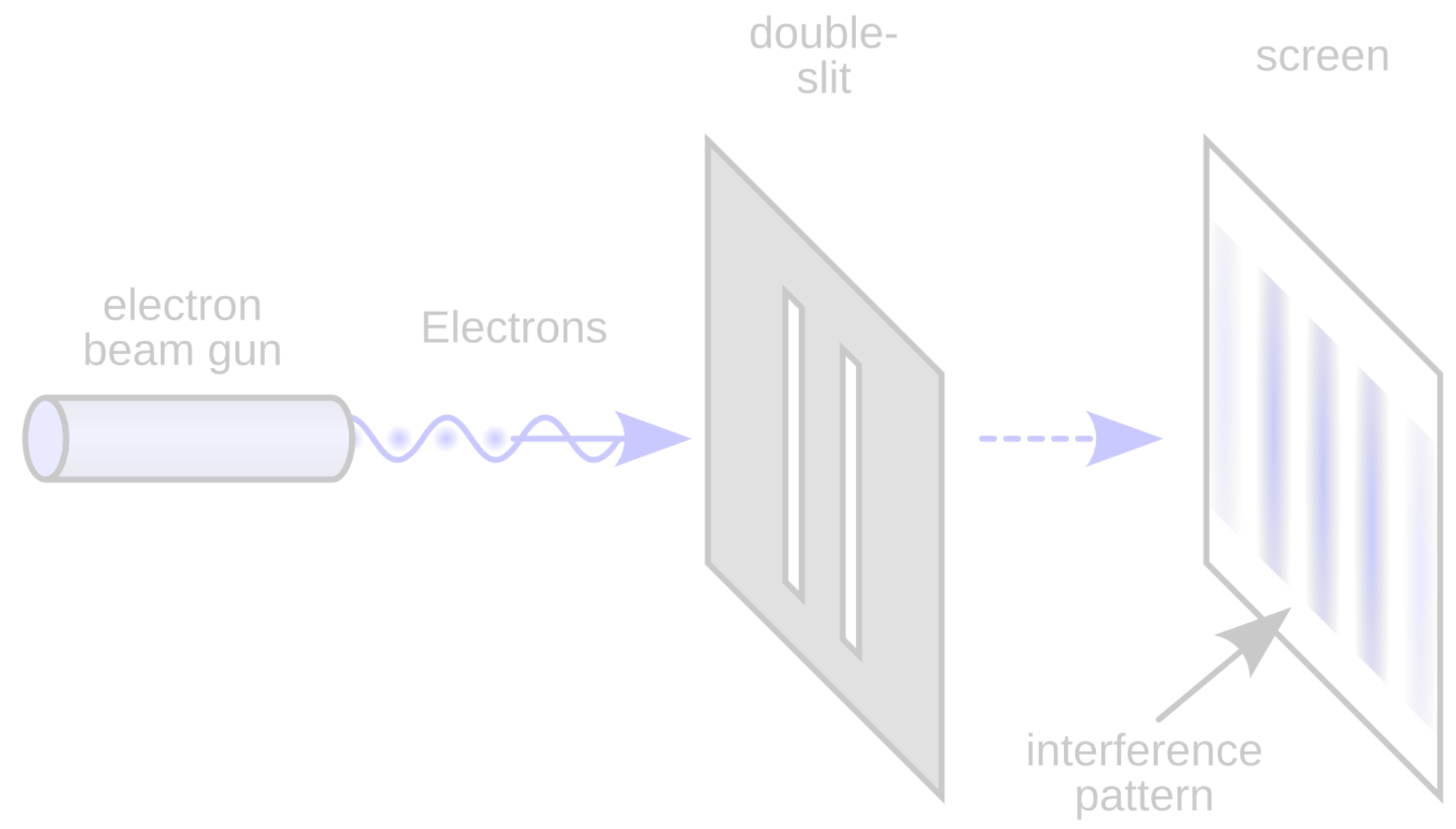
“Visibility” = oscillation pattern



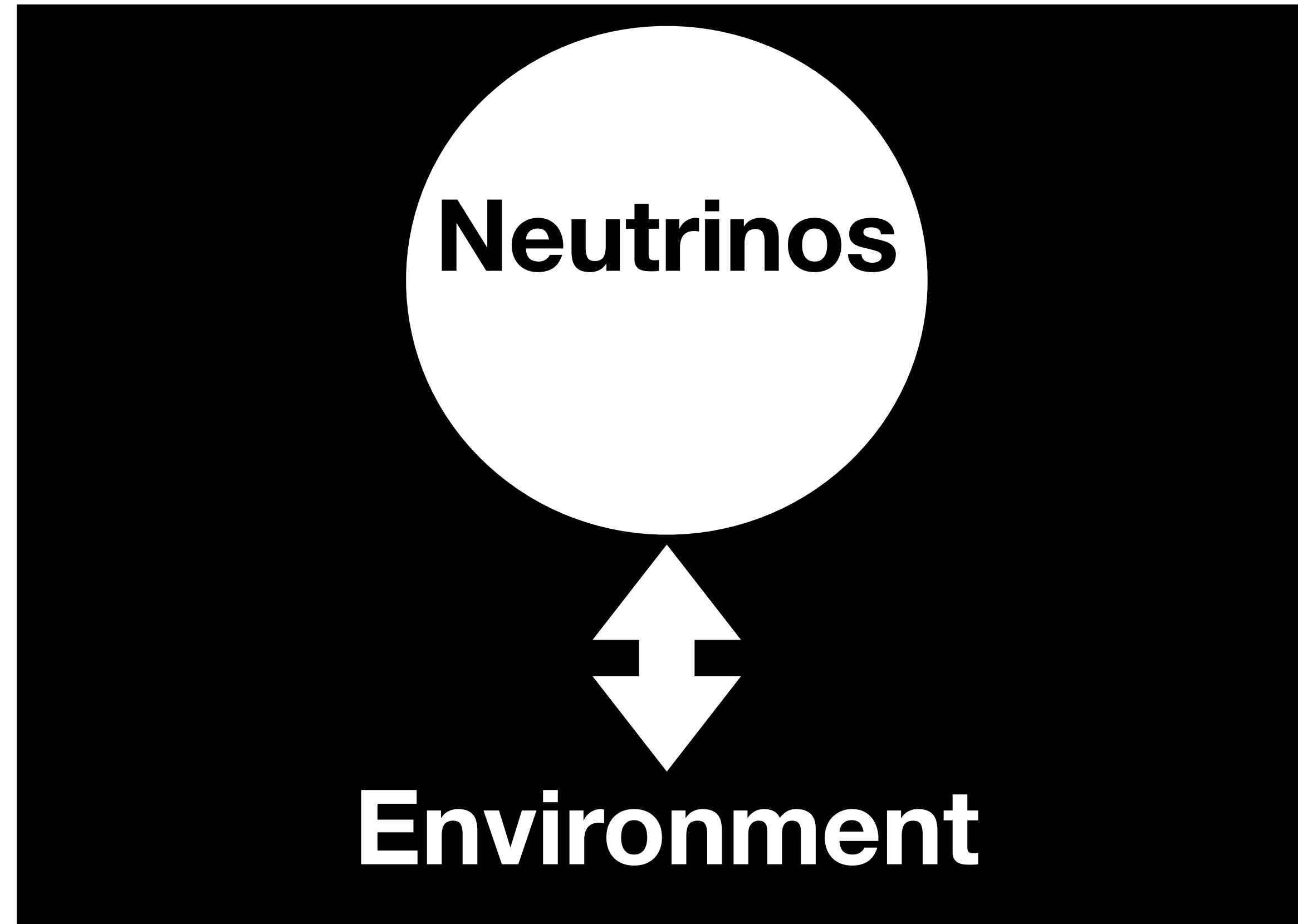
...but what if?



Decoherence in neutrino oscillations?



The framework



“Agnostic way to look for new effects”

Outline

Neutrino oscillations are a quantum interference phenomenon

Outline

Neutrino oscillations are a quantum interference phenomenon

Decoherence dampens interference patterns

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Neutrino oscillations are a quantum interference phenomenon

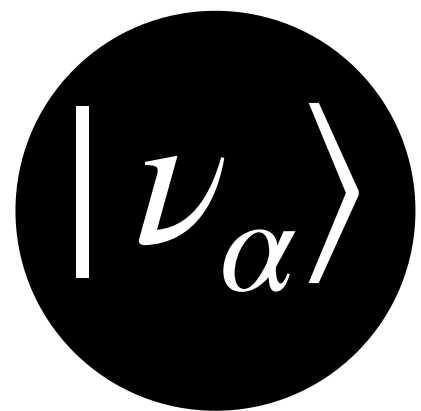
Decoherence dampens interference patterns

What can we learn from decoherence in neutrino oscillations, and how?

Neutrino oscillations in a nutshell

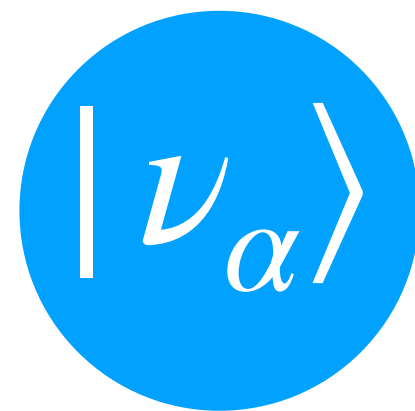
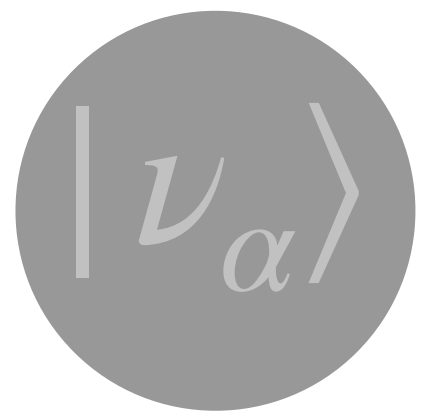
Production

$$|\nu_\alpha(0)\rangle = |\nu_e(0)\rangle = \cos\theta |\nu_1\rangle + \sin\theta |\nu_2\rangle$$



Propagation

$$|\nu_\alpha(t)\rangle = \cos \theta e^{-iE_1 t} |\nu_1\rangle + \sin \theta e^{-iE_2 t} |\nu_2\rangle$$



Detection

$$P(\nu_\alpha \rightarrow \nu_\alpha) = |\langle \nu_\alpha(t) | \nu_\alpha(0) \rangle|^2 = 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$



Neutrinos interact very little

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 - Quantum coherence survives over thousands of kilometers

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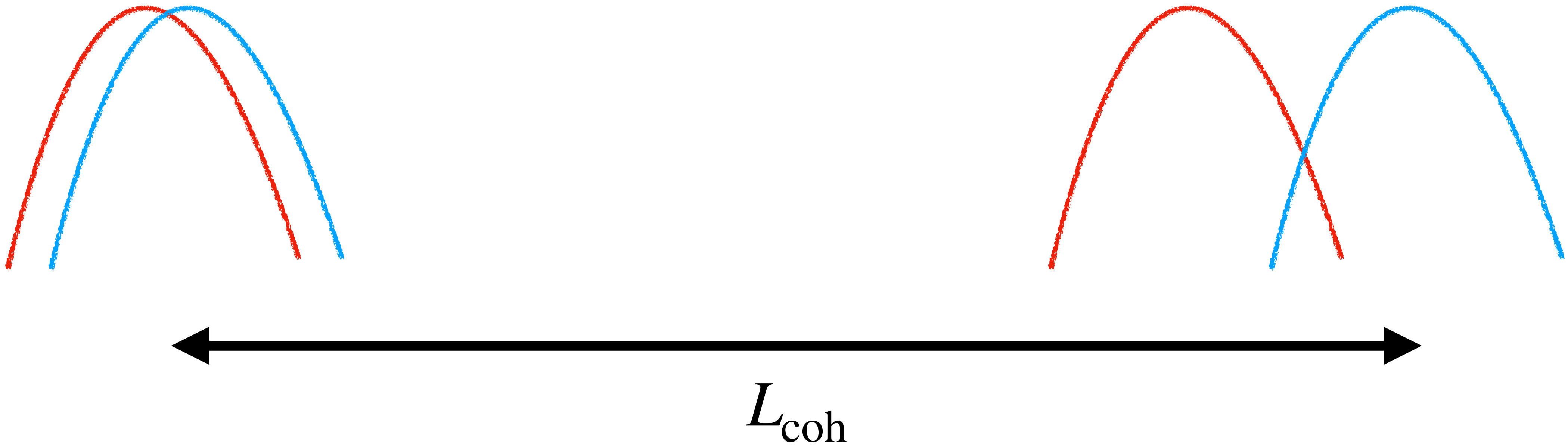
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- The oscillation pattern is an interference effect between mass eigenstates

Neutrinos interact very little

- Neutrinos interact only via the weak force
 - Quantum coherence survives over thousands of kilometers
- The oscillation pattern is an interference effect between mass eigenstates
- Any new physics that modifies neutrino propagation **will leave an imprint on the oscillation pattern**

One type of decoherence

Wave packet decoherence



Open Quantum Systems

Review of density operators

Review of density operators

The density operator satisfies

$$\rho = \rho^\dagger \quad \text{Tr}(\rho) = 1 \quad \rho \geq 0$$

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Given a ket $|\psi\rangle$ the density operator for the state is

$$\rho = |\psi\rangle\langle\psi|$$

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Given a ket $|\psi\rangle$ the density operator for the state is

$$\rho = |\psi\rangle\langle\psi|$$

This is a pure state.

Any pure state satisfies $\text{Tr}(\rho^2) = 1$

Review of density operators

Simple example

$$|\nu_e\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle$$

$$\rho_e = |\nu_e\rangle\langle\nu_e| = \begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{pmatrix}$$

Review of density operators

Mixed states have the form

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

With $\text{Tr}(\rho^2) < 1$

Review of density operators

Simple example

$$\rho = \cos^2 \theta |\nu_1\rangle\langle\nu_1| + \sin^2 \theta |\nu_2\rangle\langle\nu_2|$$

$$\rho = \begin{pmatrix} \cos^2 \theta & 0 \\ 0 & \sin^2 \theta \end{pmatrix}$$

$$\text{Tr}(\rho^2) < 1$$

Review of density operators

Time evolution

$$\partial_t \rho(t) = -i[H, \rho(t)]$$

$$\rho(t) = U(t)\rho(0)U^\dagger(t)$$

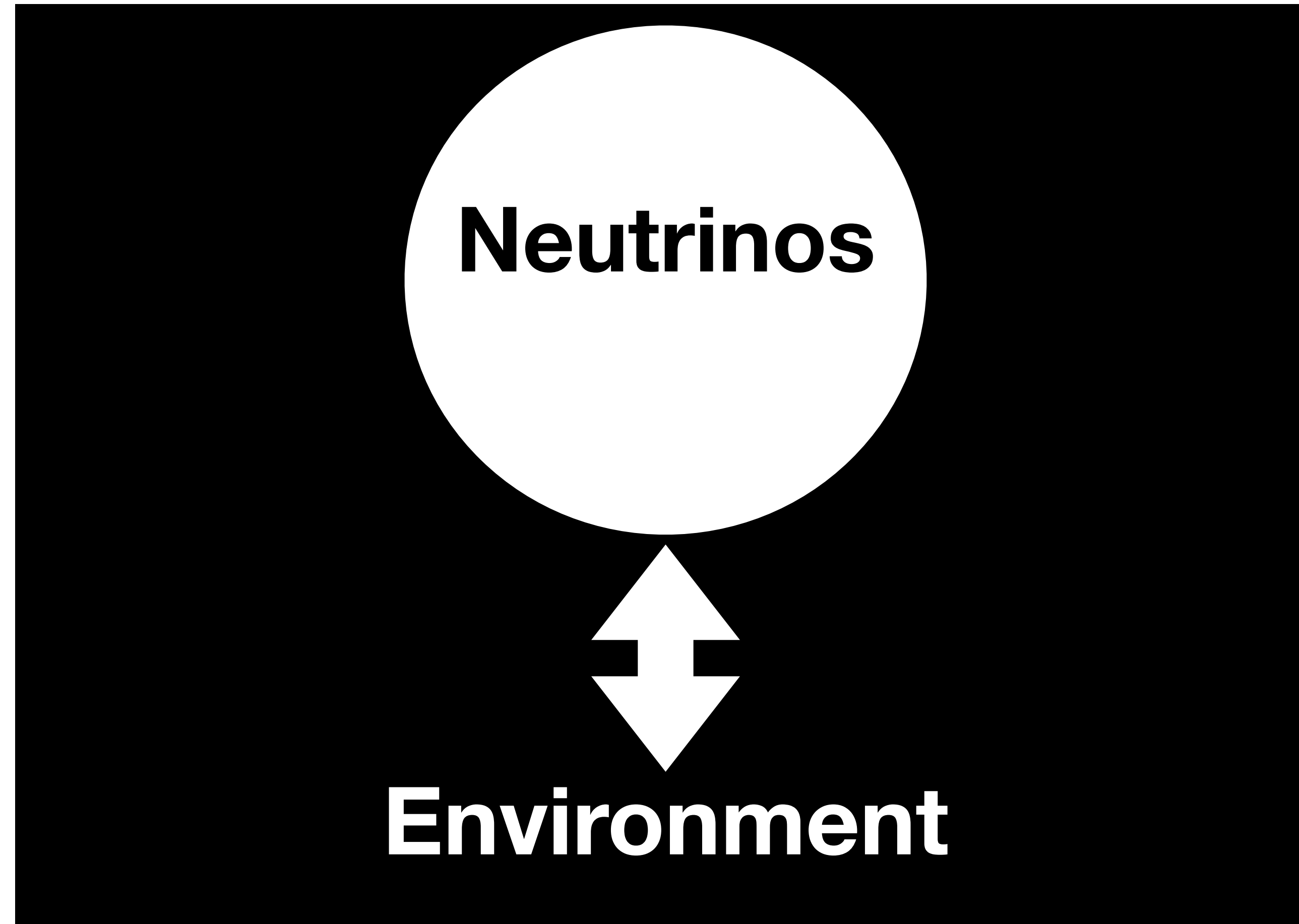
Review of density operators

Time evolved neutrino state

$$|\nu_e(t)\rangle = \cos \theta e^{-iE_1 t} |\nu_1\rangle + \sin \theta e^{-iE_2 t} |\nu_2\rangle$$

$$\rho_e(t) = \begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta e^{-i\Delta E_{12} t} \\ \cos \theta \sin \theta e^{i\Delta E_{12} t} & \sin^2 \theta \end{pmatrix}$$

Open system time evolution



Open system time evolution

One example:

$$\rho_e(t) = \begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta e^{-i\Delta E_{12}t - \Gamma L} \\ \cos \theta \sin \theta e^{i\Delta E_{12}t - \Gamma L} & \sin^2 \theta \end{pmatrix}$$

Open system time evolution

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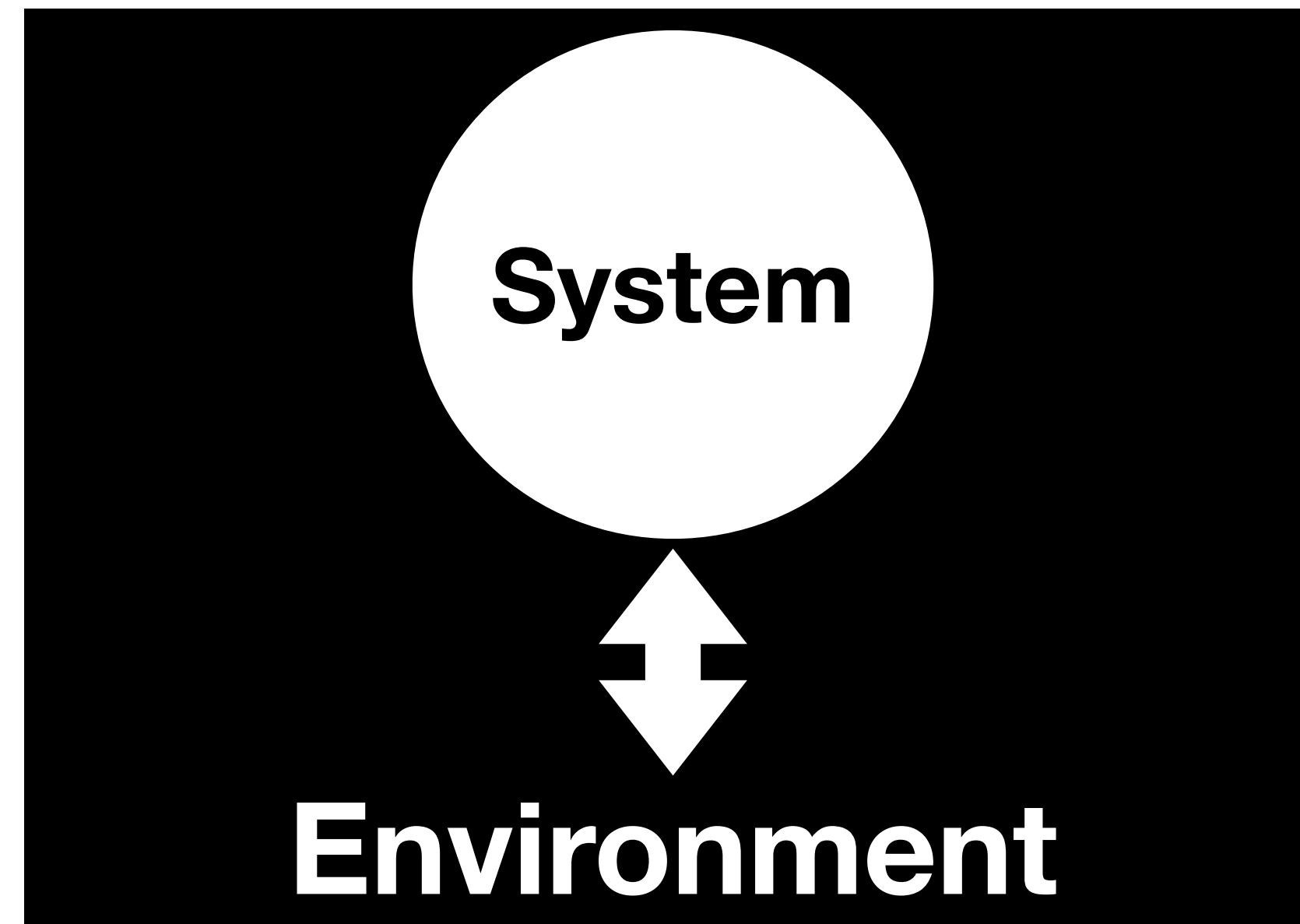
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For large L

$$\rho_e(t) = \begin{pmatrix} \cos^2 \theta & 0 \\ 0 & \sin^2 \theta \end{pmatrix}$$

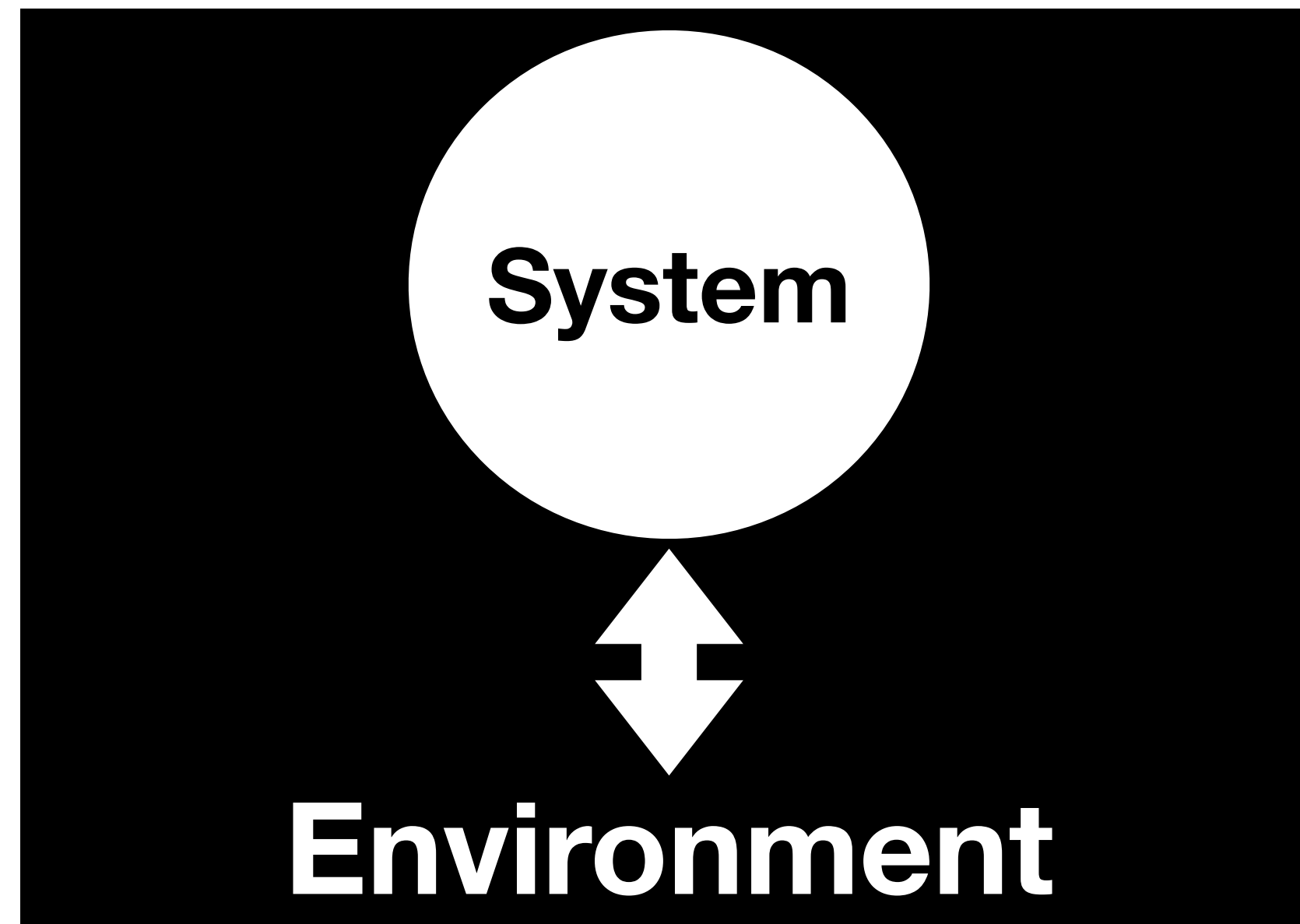
The framework

$$\rho_{\text{tot}}(t_0) = \rho_V(t_0) \otimes \rho_B(t_0)$$



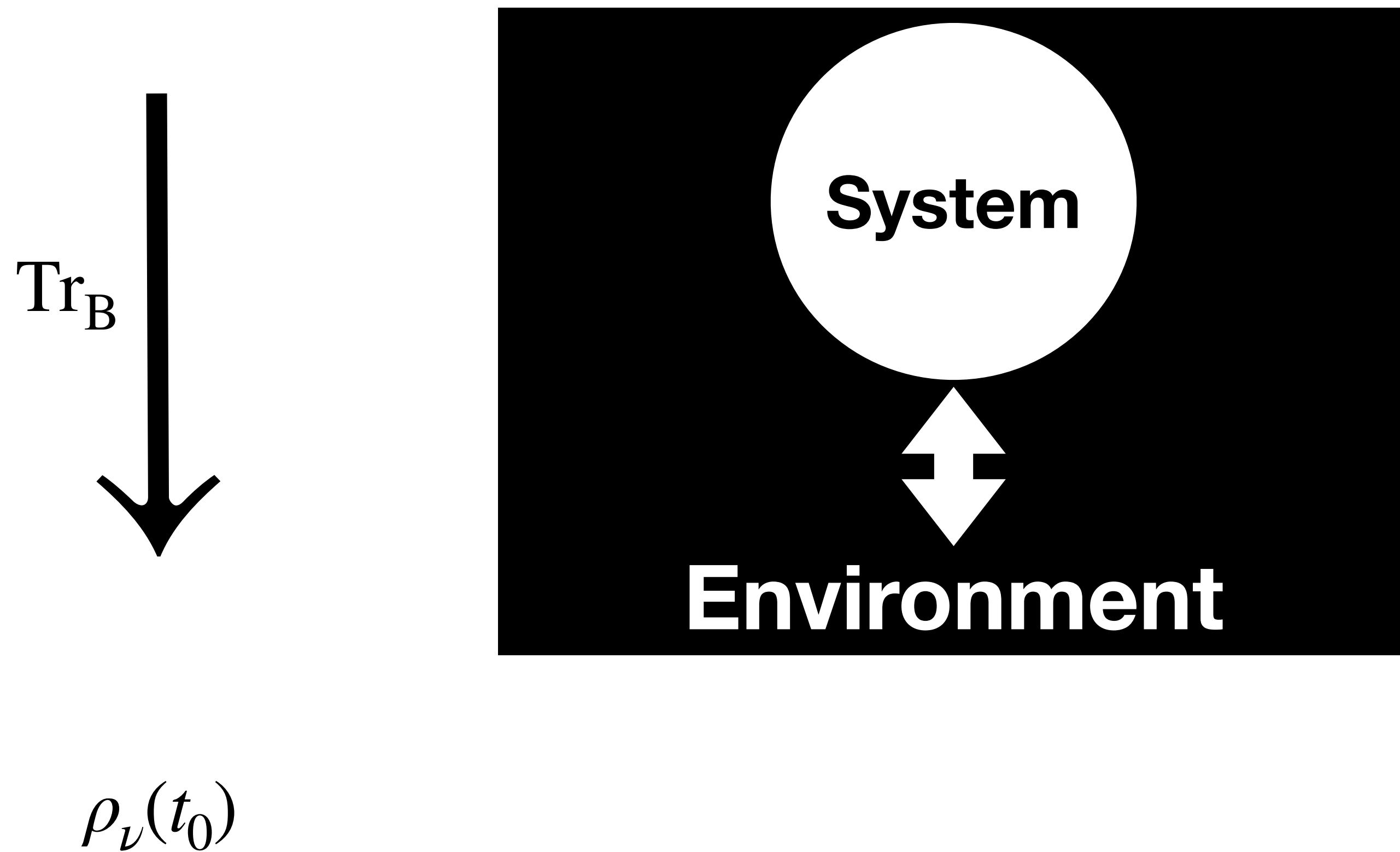
The framework

$$\rho_{\text{tot}}(t_0) = \rho_V(t_0) \otimes \rho_B(t_0) \xrightarrow{\text{Unitary evolution}} \rho_{\text{tot}}(t) = U(t, t_0)[\rho_V(t_0) \otimes \rho_B(t_0)]U^\dagger(t, t_0)$$



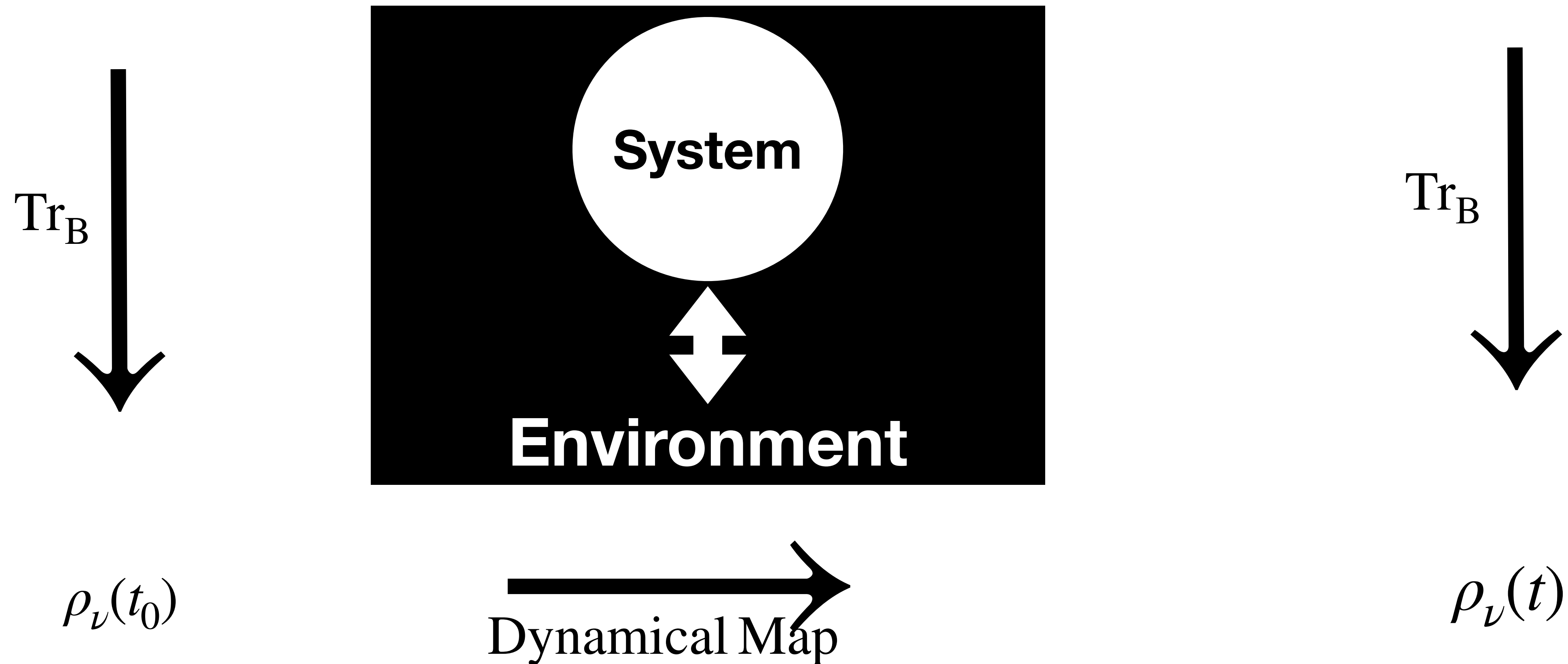
The framework

$$\rho_{\text{tot}}(t_0) = \rho_{\nu}(t_0) \otimes \rho_B(t_0) \xrightarrow{\text{Unitary evolution}} \rho_{\text{tot}}(t) = U(t, t_0)[\rho_{\nu}(t_0) \otimes \rho_B(t_0)]U^\dagger(t, t_0)$$



The framework

$$\rho_{\text{tot}}(t_0) = \rho_{\nu}(t_0) \otimes \rho_B(t_0) \xrightarrow{\text{Unitary evolution}} \rho_{\text{tot}}(t) = U(t, t_0) [\rho_{\nu}(t_0) \otimes \rho_B(t_0)] U^\dagger(t, t_0)$$



Open system time evolution

Final result: Lindblad equation

$$\begin{aligned} \partial_t \rho(t) = & -i[H, \rho(t)] \\ & -L^\dagger L \rho(t) - \rho(t) L^\dagger L + 2L \rho(t) L^\dagger \end{aligned}$$

Open system time evolution

Final result: Lindblad equation

$$\begin{aligned} \partial_t \rho(t) = & -i[H, \rho(t)] \\ & -L^\dagger L \rho(t) - \rho(t) L^\dagger L + 2L \rho(t) L^\dagger \end{aligned}$$

Conserves the trace and keeps eigenvalues positive

How has the framework been applied to neutrino physics?

Open system open possibilities

Article | Published: 26 March 2024

Search for decoherence from quantum gravity with atmospheric neutrinos

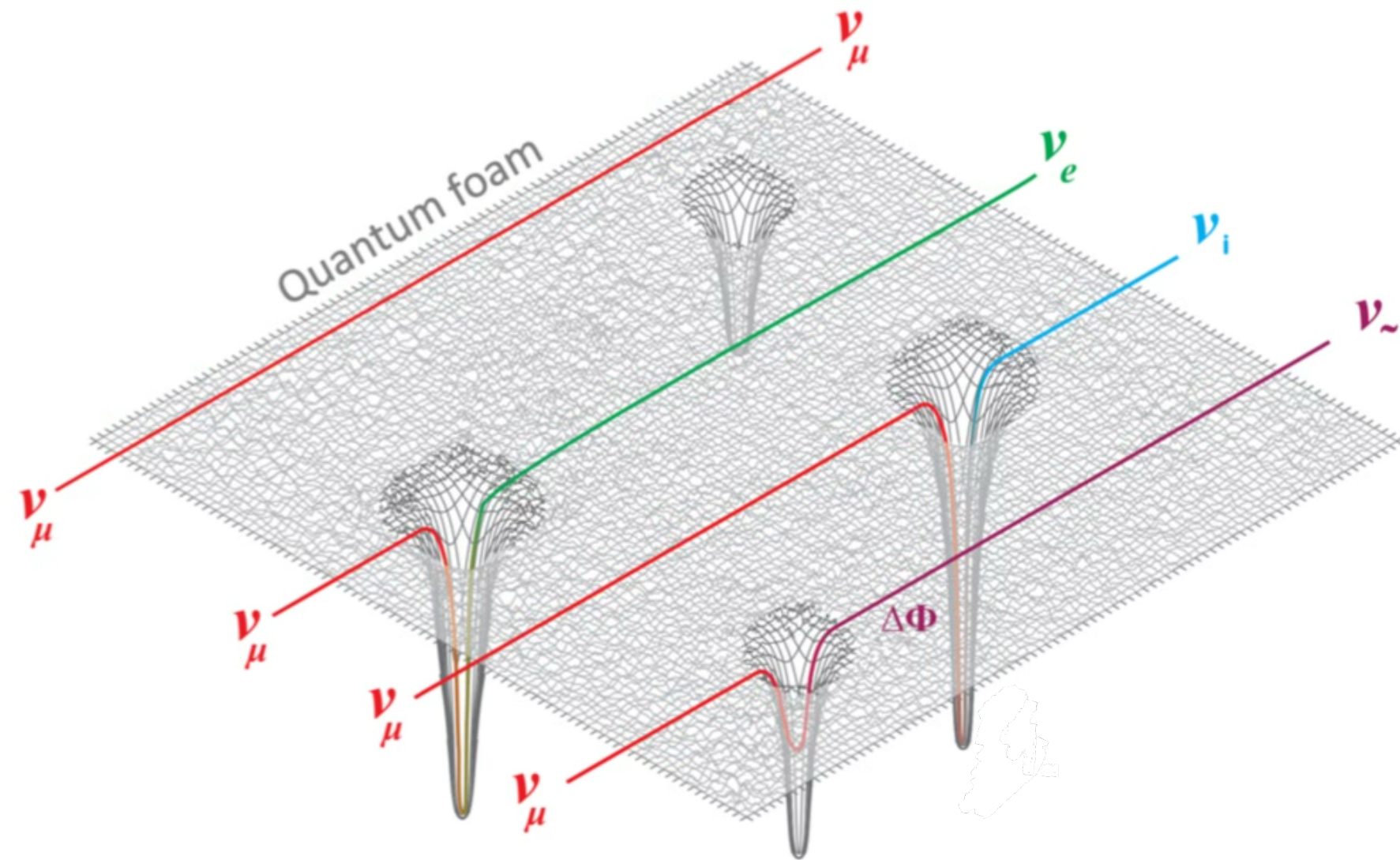
[The IceCube Collaboration](#)

[Nature Physics](#) **20**, 913–920 (2024) | [Cite this article](#)

3483 Accesses | **4** Citations | **315** Altmetric | [Metrics](#)

How the framework is applied to neutrino physics

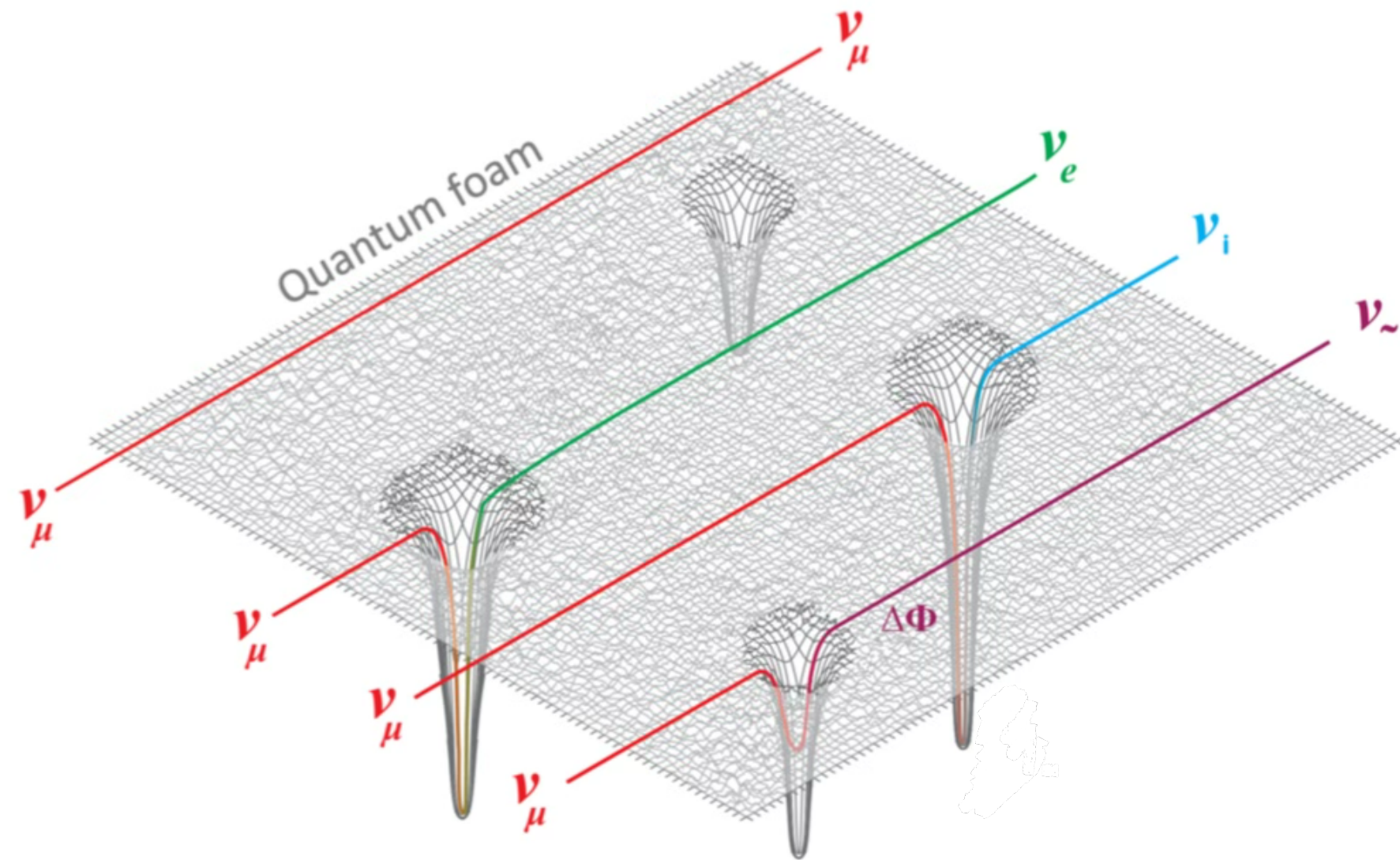
Motivation:



From: The IceCube collaboration, Nature Physics 20, 913-920

How the framework is applied to neutrino physics

Motivation:



From: The IceCube collaboration, Nature Physics 20, 913-920

Method:

$$\dot{\rho} = -i[H, \rho] - \mathcal{D}[\rho]$$

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

Quick detour

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$



Quick detour

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

The “trick” (vectorization) in two generations

$$\rho = \rho_0 \mathbf{1} + \rho_i \sigma_i$$



Quick detour

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

The “trick” (vectorization) in two generations

$$\rho = \rho_0 \mathbf{1} + \rho_i \sigma_i$$

Such that

$$\partial_t \begin{pmatrix} \rho_0 \\ \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} \\ M_{10} & M_{11} & M_{12} & M_{13} \\ M_{20} & M_{21} & M_{22} & M_{23} \\ M_{30} & M_{31} & M_{32} & M_{33} \end{pmatrix} \begin{pmatrix} \rho_0 \\ \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix}$$

Quick detour

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

In three generations: $\rho = \rho_0 \mathbf{1} + \rho_i \lambda_i$



Quick detour

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

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$$\begin{pmatrix} \rho_0 \\ \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \\ \rho_5 \\ \rho_6 \\ \rho_7 \\ \rho_8 \end{pmatrix}$$



Quick detour

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

In three generations: $\rho = \rho_0 \mathbf{1} + \rho_i \lambda_i$

$$\begin{pmatrix} \rho_0 \\ \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \\ \rho_5 \\ \rho_6 \\ \rho_7 \\ \rho_8 \end{pmatrix} \quad \mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Quick detour

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

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Diagonal of the density matrix

$$\text{diag}(\rho) = \left(\rho_0 + \rho_3 + \frac{1}{\sqrt{3}} \rho_8, \rho_0 - \rho_3 + \frac{1}{\sqrt{3}} \rho_8, \rho_0 - \frac{2}{\sqrt{3}} \rho_8 \right)$$

Quick detour

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

In three generations: $\rho = \rho_0 \mathbf{1} + \rho_i \lambda_i$

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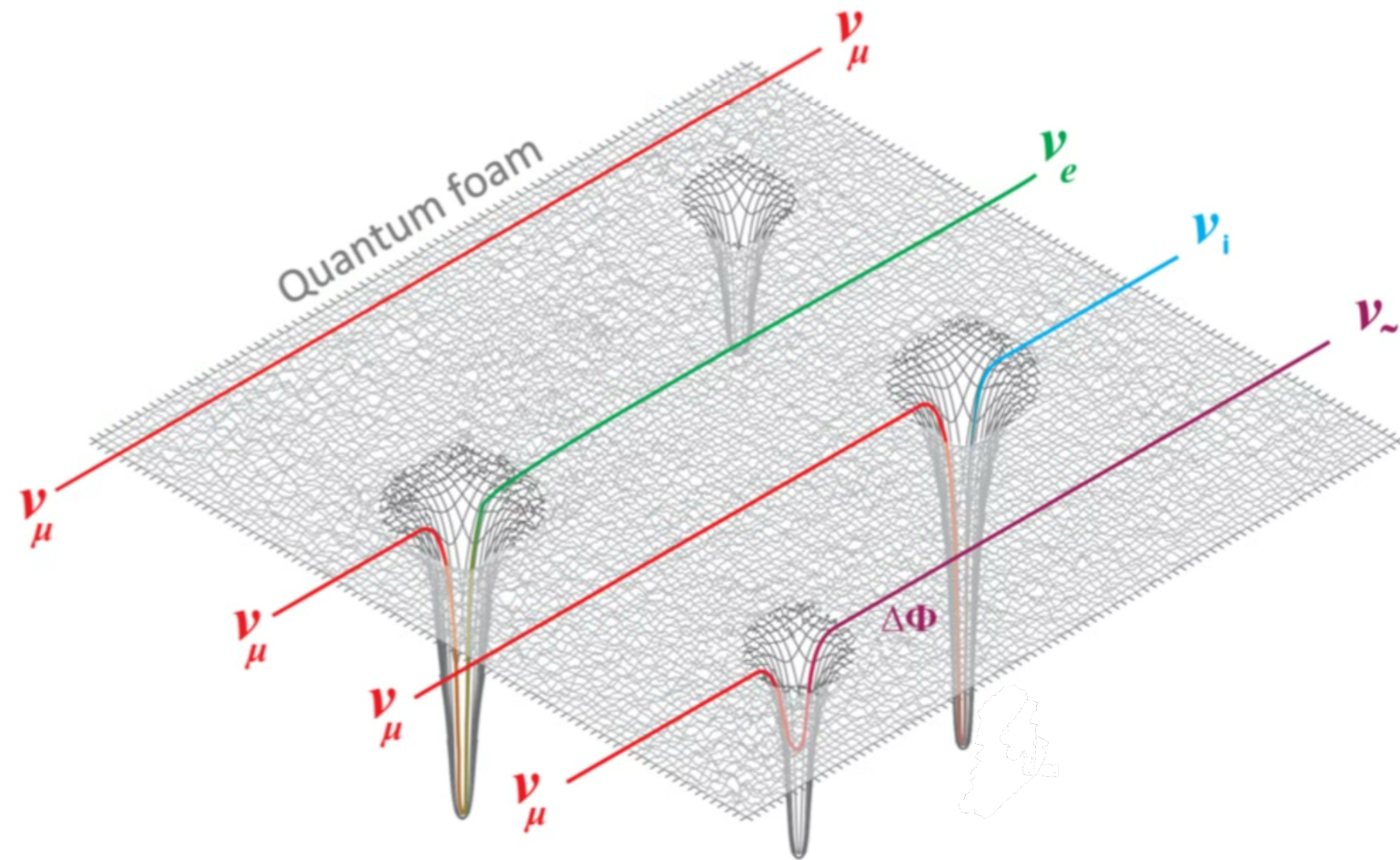
Diagonal of the density matrix

$$\text{diag}(\rho) = \left(\rho_0 + \rho_3 + \frac{1}{\sqrt{3}} \rho_8, \rho_0 - \rho_3 + \frac{1}{\sqrt{3}} \rho_8, \rho_0 - \frac{2}{\sqrt{3}} \rho_8 \right)$$

$$\text{Tr}(\rho) = 3\rho_0$$

How the framework is applied to neutrino physics

Motivation:



From: The IceCube collaboration, Nature Physics 20, 913-920

Method:

$$\dot{\rho} = -i[H, \rho] - \mathcal{D}[\rho]$$

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

Pheno:

$$\Gamma(E_\nu) = \Gamma_0 \left(\frac{E_\nu}{E_0} \right)^n$$

How the framework is applied to neutrino physics

Pheno:

$$\Gamma(E_\nu) = \Gamma_0 \left(\frac{E_\nu}{E_0} \right)^n$$

Scaling:

$$\rho_e(t) = \begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta e^{-i\Delta E_{12}t - \Gamma L} \\ \cos \theta \sin \theta e^{i\Delta E_{12}t - \Gamma L} & \sin^2 \theta \end{pmatrix}$$

2018

		$n = -2$	$n = -1$	$n = 0$	$n = 1$	$n = 2$
Normal Ordering	IceCube (this work)					
	Atmospheric ($\gamma_{31} = \gamma_{32}$)	$2.8 \cdot 10^{-18}$	$4.2 \cdot 10^{-21}$	$4.0 \cdot 10^{-24}$	$1.0 \cdot 10^{-27}$	$1.0 \cdot 10^{-31}$
	Solar I ($\gamma_{31} = \gamma_{21}$)	$6.8 \cdot 10^{-19}$	$1.2 \cdot 10^{-21}$	$1.3 \cdot 10^{-24}$	$3.5 \cdot 10^{-28}$	$1.9 \cdot 10^{-32}$
	Solar II ($\gamma_{32} = \gamma_{21}$)	$5.2 \cdot 10^{-19}$	$9.2 \cdot 10^{-22}$	$9.7 \cdot 10^{-25}$	$2.4 \cdot 10^{-28}$	$9.0 \cdot 10^{-33}$
	DeepCore (this work)					
	Atmospheric ($\gamma_{31} = \gamma_{32}$)	$4.3 \cdot 10^{-20}$	$2.0 \cdot 10^{-21}$	$8.2 \cdot 10^{-23}$	$3.0 \cdot 10^{-24}$	$1.1 \cdot 10^{-25}$
	Solar I ($\gamma_{31} = \gamma_{21}$)	$1.2 \cdot 10^{-20}$	$5.4 \cdot 10^{-22}$	$2.1 \cdot 10^{-23}$	$6.6 \cdot 10^{-25}$	$2.0 \cdot 10^{-26}$
Solar II ($\gamma_{32} = \gamma_{21}$)	$7.5 \cdot 10^{-21}$	$3.5 \cdot 10^{-22}$	$1.4 \cdot 10^{-23}$	$4.2 \cdot 10^{-25}$	$1.1 \cdot 10^{-26}$	

GeV*

Coloma, et al., *Eur.Phys.J.C* 78 (2018) 8, 614

IceCube collaboration, *Nature Physics* 20, 913-920

n	Phase Perturbation Γ_{90}	State Selection Γ_{90}
0	$1.18 \cdot 10^{-15}$ eV	$1.17 \cdot 10^{-15}$ eV
1	$6.89 \cdot 10^{-16}$ eV	$6.67 \cdot 10^{-16}$ eV
2	$9.80 \cdot 10^{-18}$ eV	$9.48 \cdot 10^{-18}$ eV
3	$1.58 \cdot 10^{-19}$ eV	$1.77 \cdot 10^{-19}$ eV

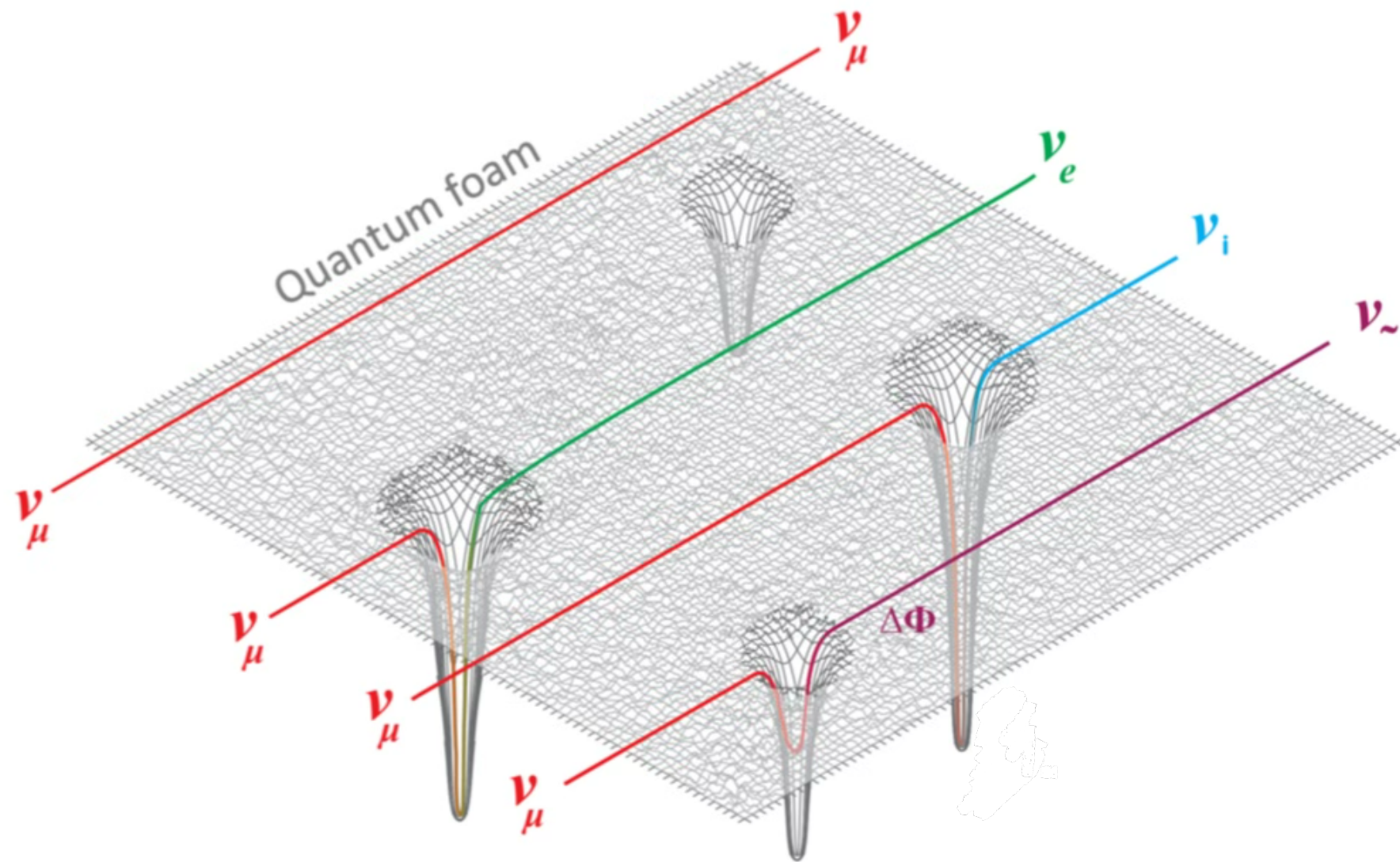
2024

TABLE I. The 90% CL upper limits on Γ_{90} for each n in $E_0=1$ TeV in the state selection (SS) and phase perturbation (PS) models.

**How close is IceCube to probe
decoherence for a concrete model?**

How the framework is applied to neutrino physics

Motivation:



From: The IceCube collaboration, Nature Physics 20, 913-920

Method:

$$\dot{\rho} = -i[H, \rho] - \mathcal{D}[\rho]$$

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

Pheno:

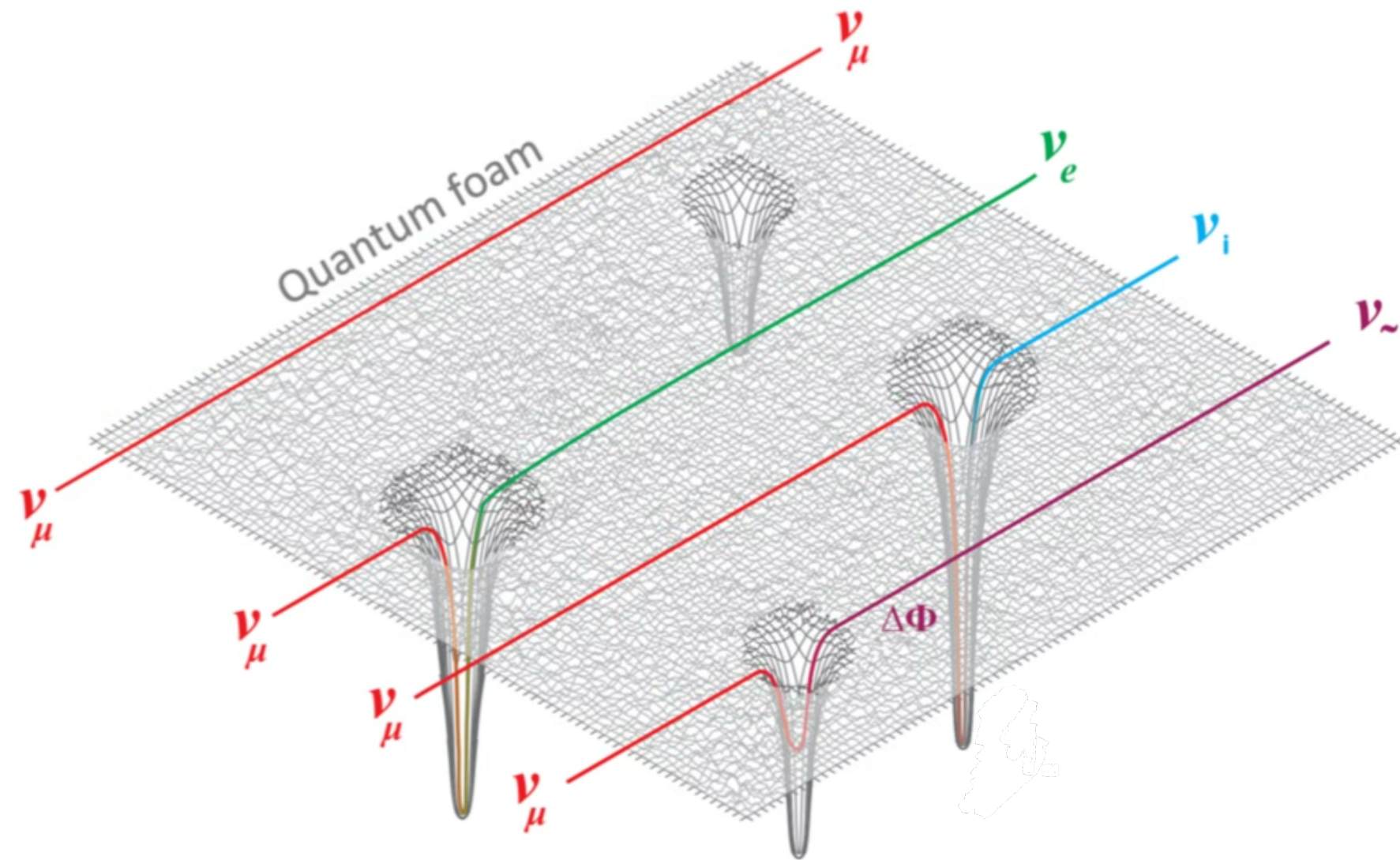
$$\Gamma(E_\nu) = \Gamma_0 \left(\frac{E_\nu}{E_0} \right)^n$$

What is Γ ?

**Where does it come from? How?
Complete framework?**

How the framework is applied to neutrino physics

Motivation:



From: The IceCube collaboration, Nature Physics 20, 913-920

Method:

$$\dot{\rho} = -i[H, \rho] - \mathcal{D}[\rho]$$

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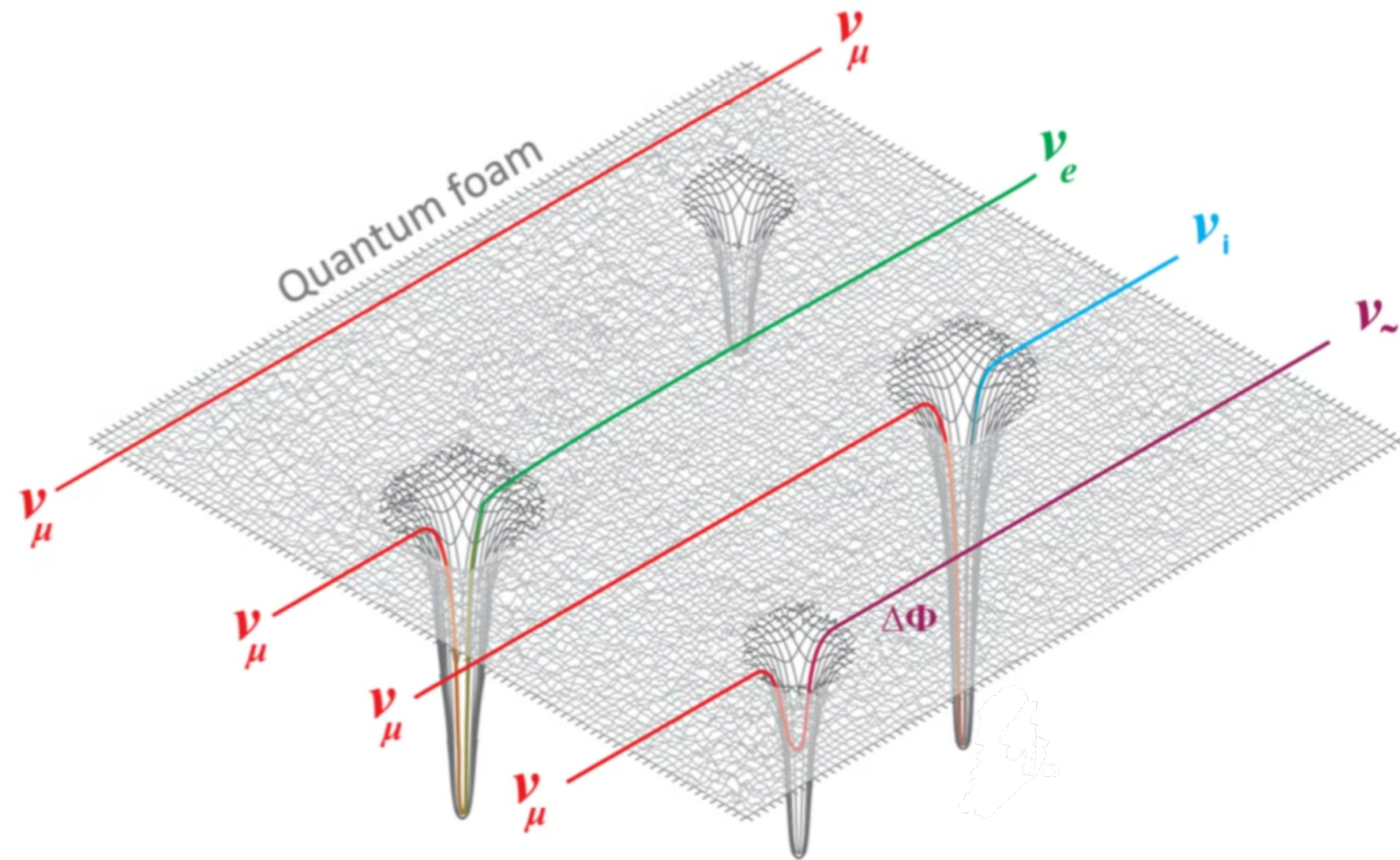
Pheno:

$$\Gamma(E_\nu) = \Gamma_0 \left(\frac{E_\nu}{E_0} \right)^n$$

“Agnostic way to look for new effects”

How the framework is applied to neutrino physics

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Method:

$$\dot{\rho} = -i[H, \rho] - \mathcal{D}[\rho]$$

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

Pheno:

$$\Gamma(E_\nu) = \Gamma_0 \left(\frac{E_\nu}{E_0} \right)^n$$

“Agnostic way to look for new effects”

Now, however, the agnostic wants some proof

I will give you one concrete example

Open system approach to neutrinos propagating in an ultralight scalar background

Lua F. T. Airoldi,^{1,2,*} Gustavo F. S. Alves,^{2,3,†} Pedro A. N. Machado,^{2,‡} and Peter Vander Griend^{2,4,§}

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Open system approach to neutrinos propagating in an ultralight scalar background

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- 1. Mapping between model parameters and open system framework**
- 2. This simple model eludes previous analyses**

Ultralight scalar background

Ultralight scalars are compelling dark matter candidates

Hui, et al., Phys. Rev. D 95, 043541

Hu, et al., Phys. Rev. Lett. 85, 1158

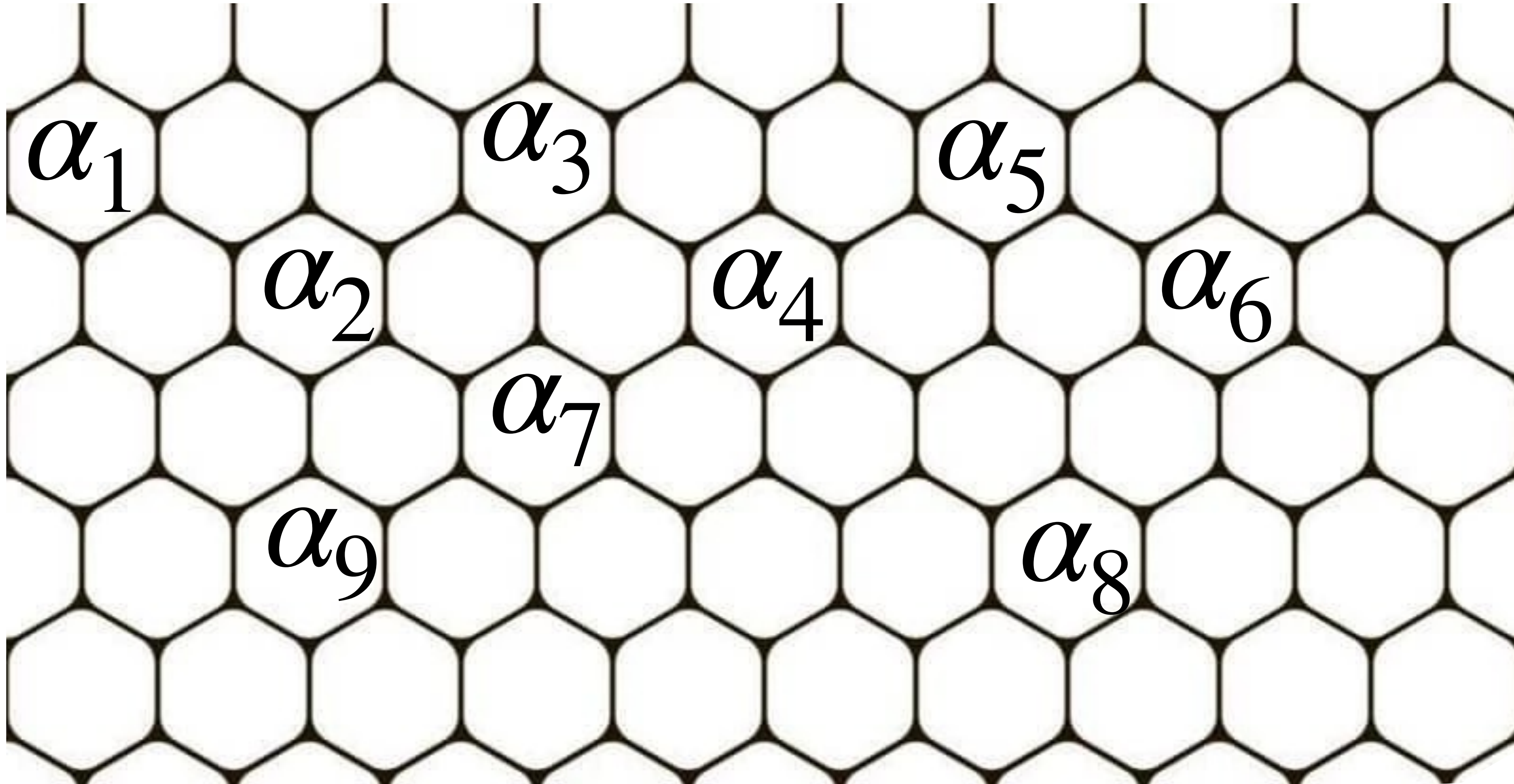
Ferreira, Astron.Astrophys.Rev. 29 (2021) 1, 7.

Chadha-Day et al., Sci. Adv. 8, sciadv.abj3618

Arvanitaki, Phys. Rev. D 81, 123530

...

The scalar field can be seen as



$$\alpha_i = \sqrt{n} e^{i\theta_i}$$

Time hierarchy

▶ \mathcal{V}



Time

Time hierarchy

▶ \mathcal{V}

.....▶ **Scalar modulation**



Time

Time hierarchy

► \mathcal{V}

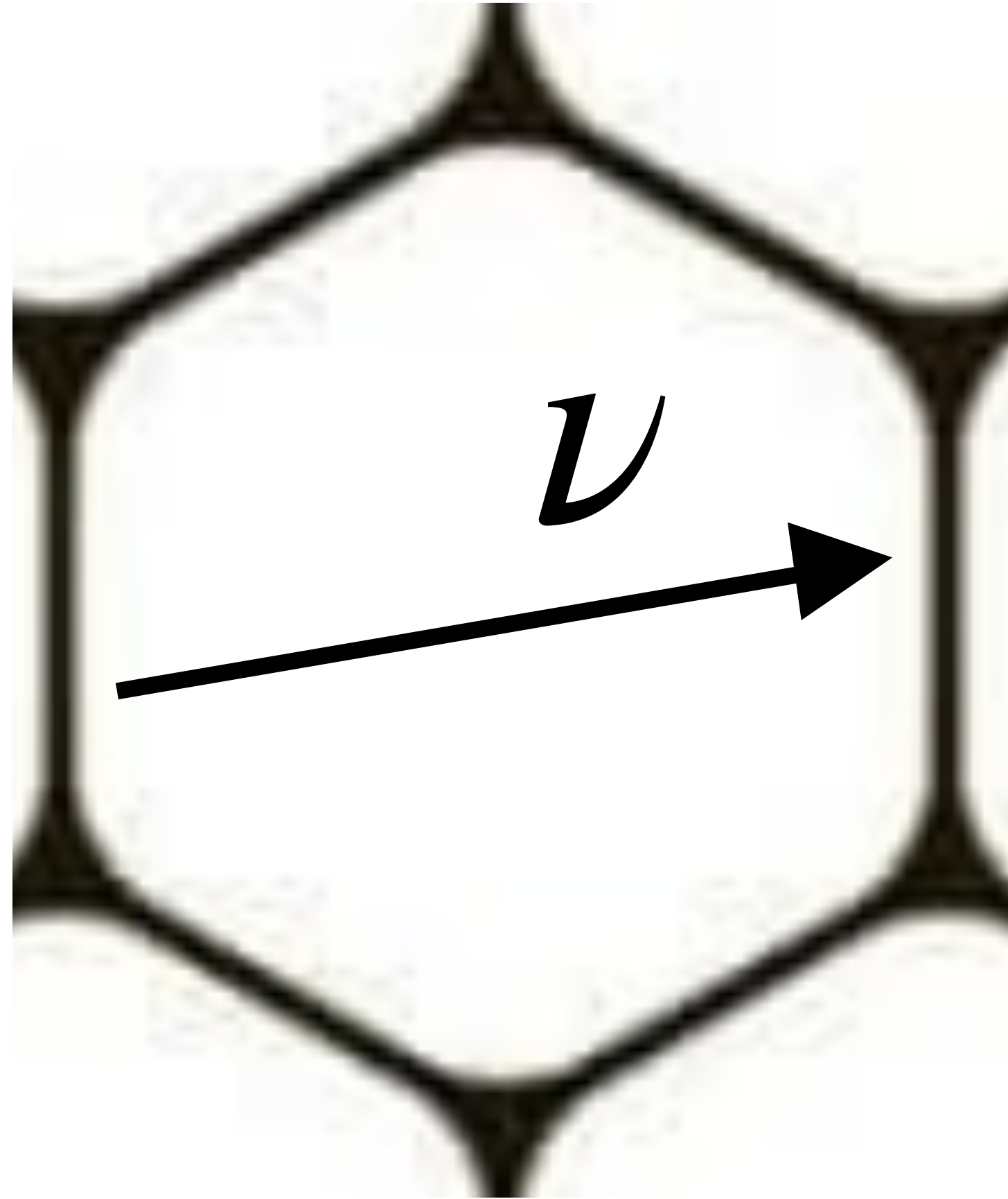
..... ► **Scalar modulation**



Time

$$10^{-23} \text{ eV} < m_\phi < 10^{-11} \text{ eV}$$

The scales determine the scalar state



Field state

$|\alpha\rangle$

The scales determine the scalar state

$$\phi(x, t) = \sum_k \frac{1}{\sqrt{2VE_k}} (a_k e^{-ikx} + a_k^\dagger e^{ikx})$$

The scales determine the scalar state

$$\phi(x, t) = \sum_k \frac{1}{\sqrt{2VE_k}} (a_k e^{-ikx} + a_k^\dagger e^{ikx})$$

Ultralight

$$\phi(x, t) = \sum_k \frac{1}{\sqrt{2VE_k}} (\alpha_k e^{-ikx} + \alpha_k^\dagger e^{ikx})$$

The scales determine the scalar state

$$\phi(x, t) = \sum_k \frac{1}{\sqrt{2VE_k}} (a_k e^{-ikx} + a_k^\dagger e^{ikx})$$

Ultralight

$$\phi(x, t) = \sum_k \frac{1}{\sqrt{2VE_k}} (\alpha_k e^{-ikx} + \alpha_k^\dagger e^{ikx})$$

Non relativistic

$$\phi(x, t) \approx \frac{\sqrt{2\rho_\phi}}{m_\phi} \cos(m_\phi t + \theta)$$

Neutrinos interacting with a new scalar

$$-\mathcal{L} \supset g_{\phi,1}\phi\bar{\nu}_1\nu_1 + g_{\phi,2}\phi\bar{\nu}_2\nu_2 + g_{\phi,3}\phi\bar{\nu}_3\nu_3$$

Neutrinos interacting with a new scalar

$$-\mathcal{L} \supset g_{\phi,1}\phi\bar{\nu}_1\nu_1 + g_{\phi,2}\phi\bar{\nu}_2\nu_2 + g_{\phi,3}\phi\bar{\nu}_3\nu_3$$

Neutrino masses get modified

$$m_{\nu_i} = m_{\nu_i}(g_{\phi} = 0) + g_{\phi,i}\phi$$

Neutrino masses become time dependent

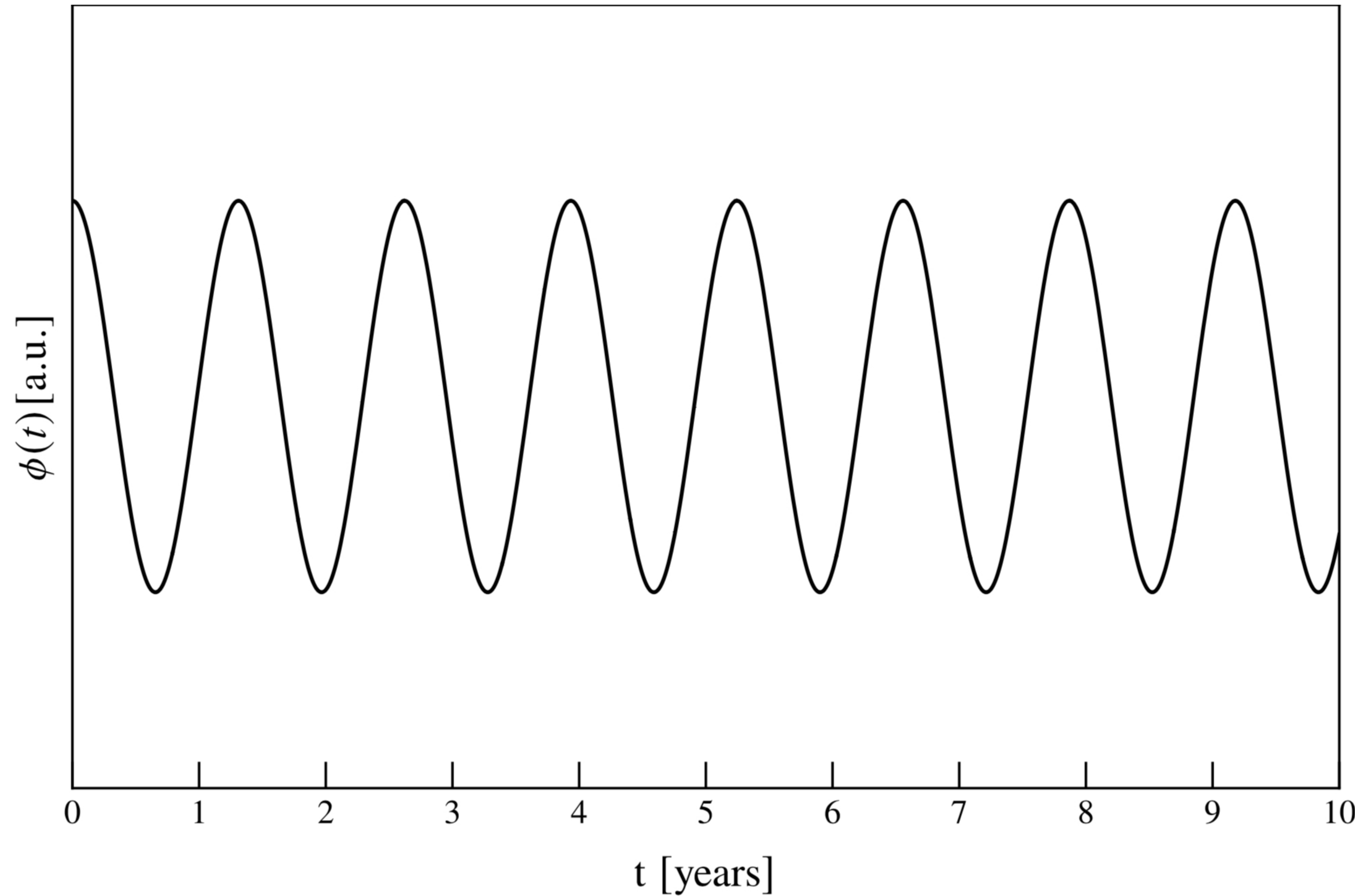
$$m_{\nu_i}(t) = m_{\nu_i}^0 + g_{\phi,i} \phi_0 \cos(m_\phi t + \theta)$$

Neutrino oscillation frequency is modified

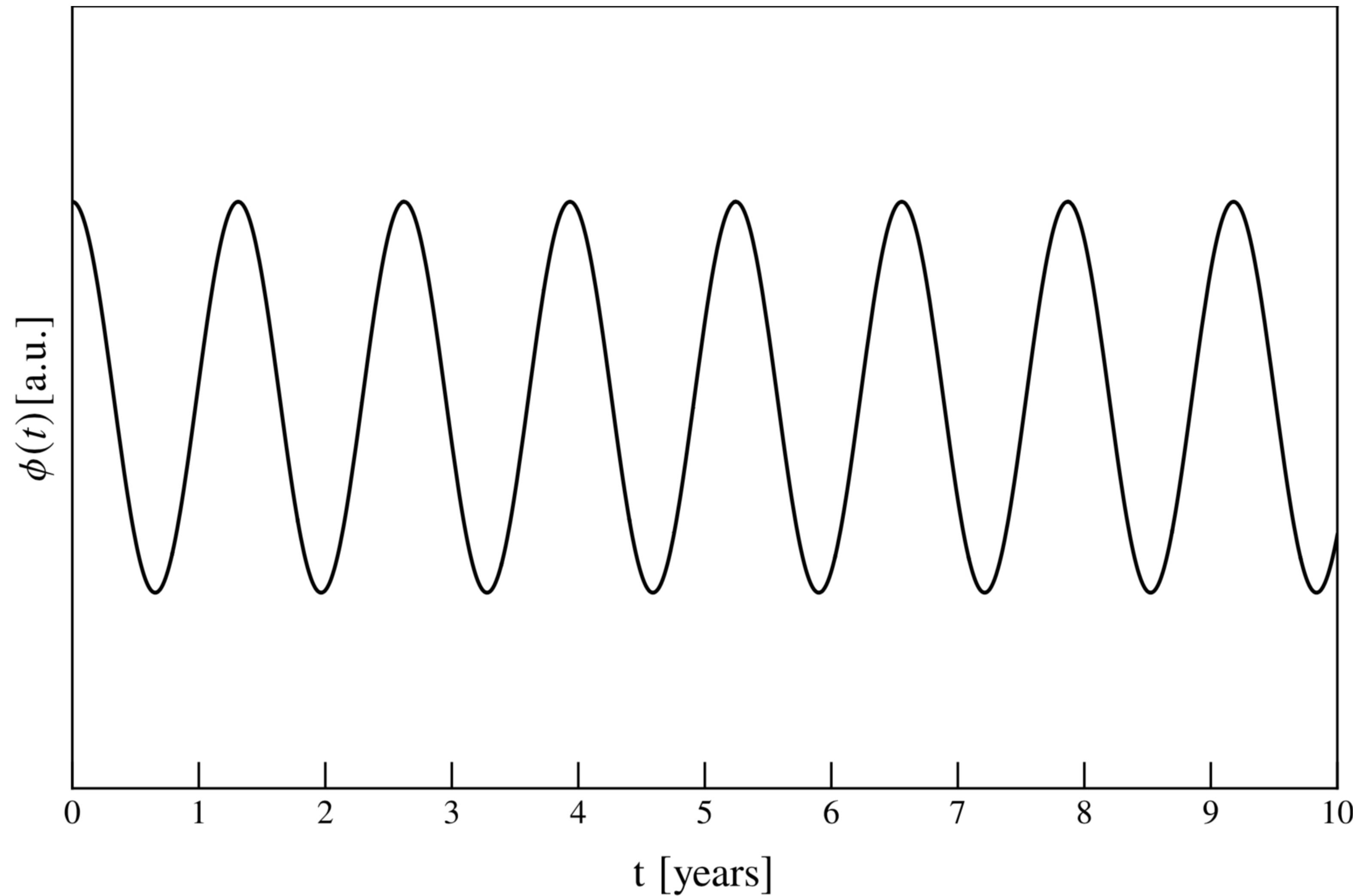
$$\frac{\Delta m_{ij}^2}{2E} \rightarrow \frac{\Delta m_{ij}^2}{2E} + \frac{\Delta m_{ij}^2(t)}{2E}$$

$$\Delta m_{ij}^2(t) = 2(g_i m_i - g_j m_j) \phi_0 \cos(m_\phi t + \theta)$$

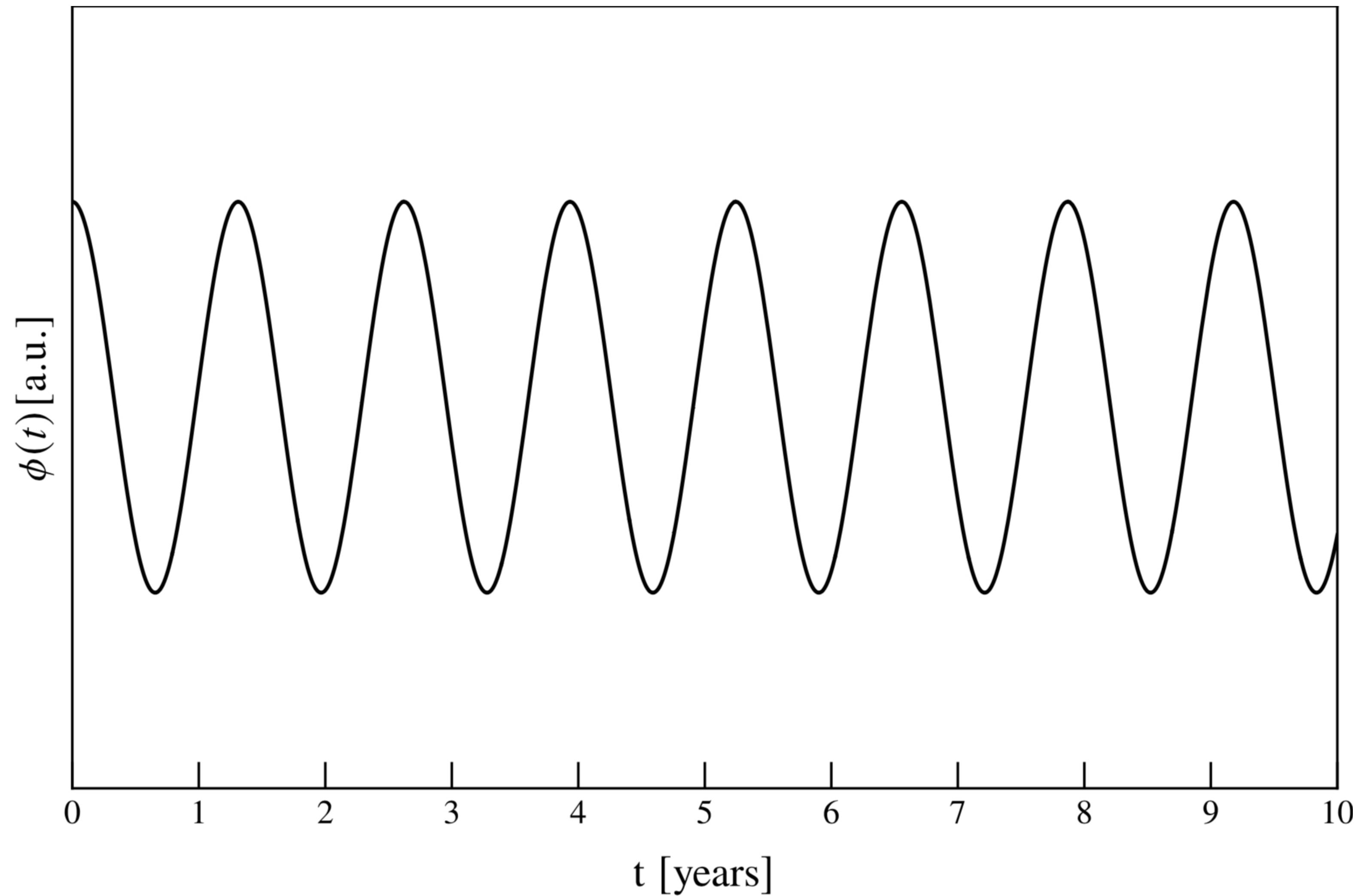
What can happen with neutrinos?



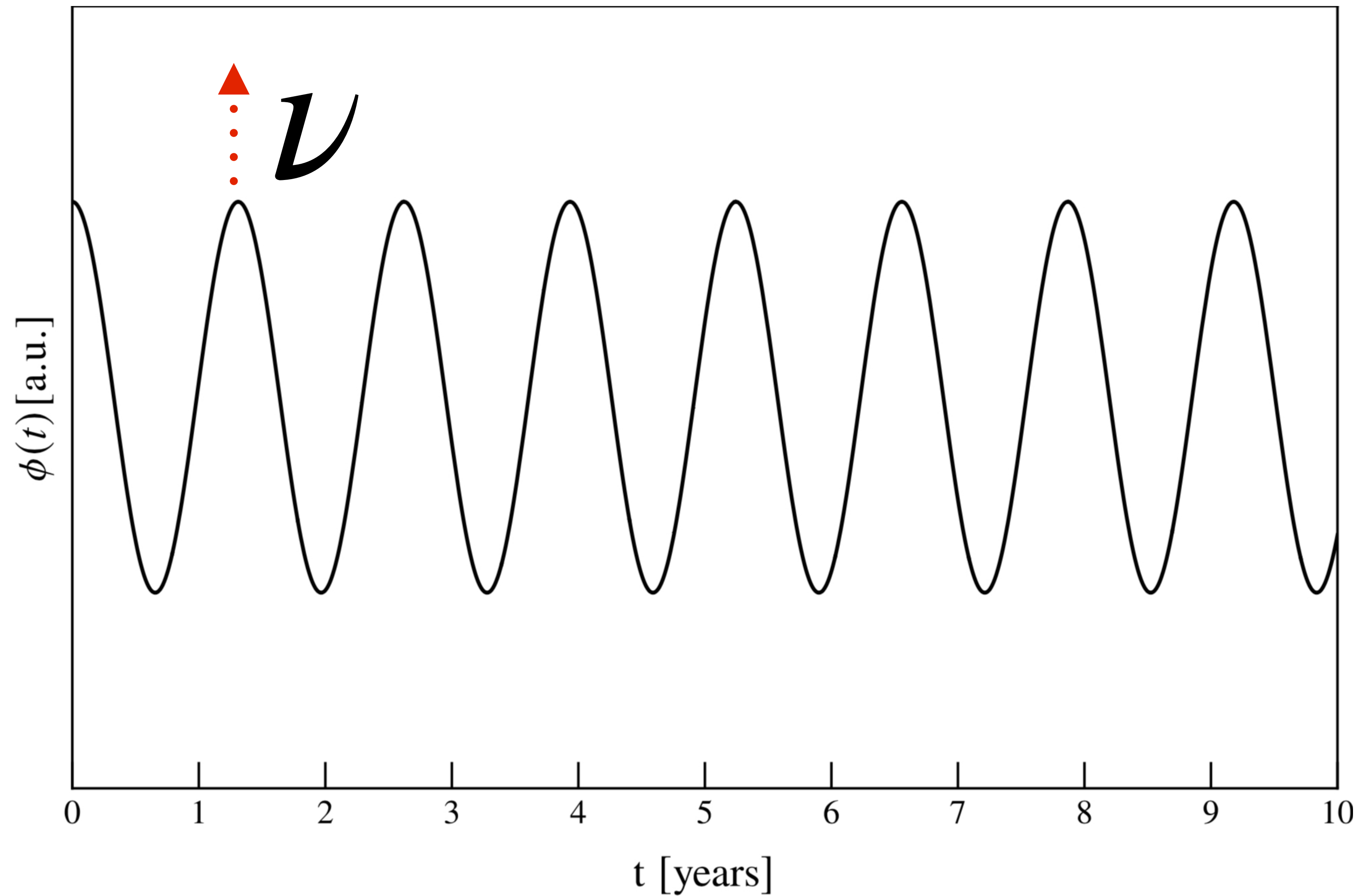
Several cycles over the experimental livetime



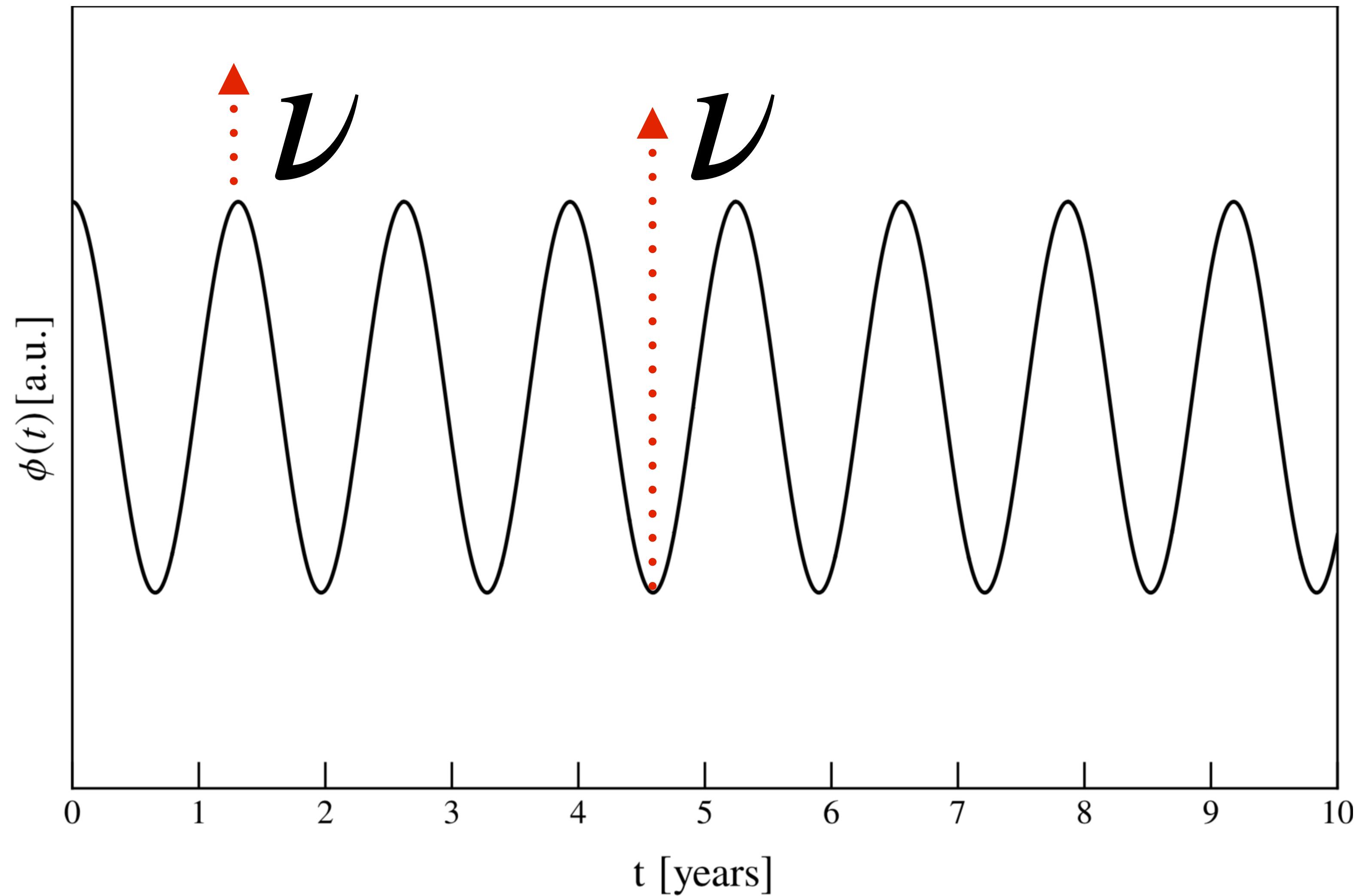
Neutrinos start with different initial conditions



Neutrinos start with different initial conditions



Neutrinos start with different initial conditions





Most explored scenario

Neutrino Oscillations as a Probe of Light Scalar Dark Matter

[Asher Berlin](#)

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Phys. Rev. Lett. **117**, 231801 – Published 30 November, 2016

DOI: <https://doi.org/10.1103/PhysRevLett.117.231801>

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Distorted neutrino oscillations from time varying cosmic fields

[Gordan Krnjaic](#)^{1,*}, [Pedro A. N. Machado](#)^{1,†}, and [Lina Necib](#)^{2,‡}

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Phys. Rev. D **97**, 075017 – Published 16 April, 2018

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Expo

[Submitted on 20 Dec 2025]

Ultralight dark matter search in a large liquid scintillator detector

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Signatures of ultralight dark matter in neutrino oscillation experiments

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[Luis A. Delgado](#), [O. G. Miranda](#), [Hiroshi Nunokawa](#)

I will show that we can also approach the problem from an open systems perspective

[Home](#) > [Journal of High Energy Physics](#) > Article

Signatures of ultralight dark matter in neutrino oscillation experiments

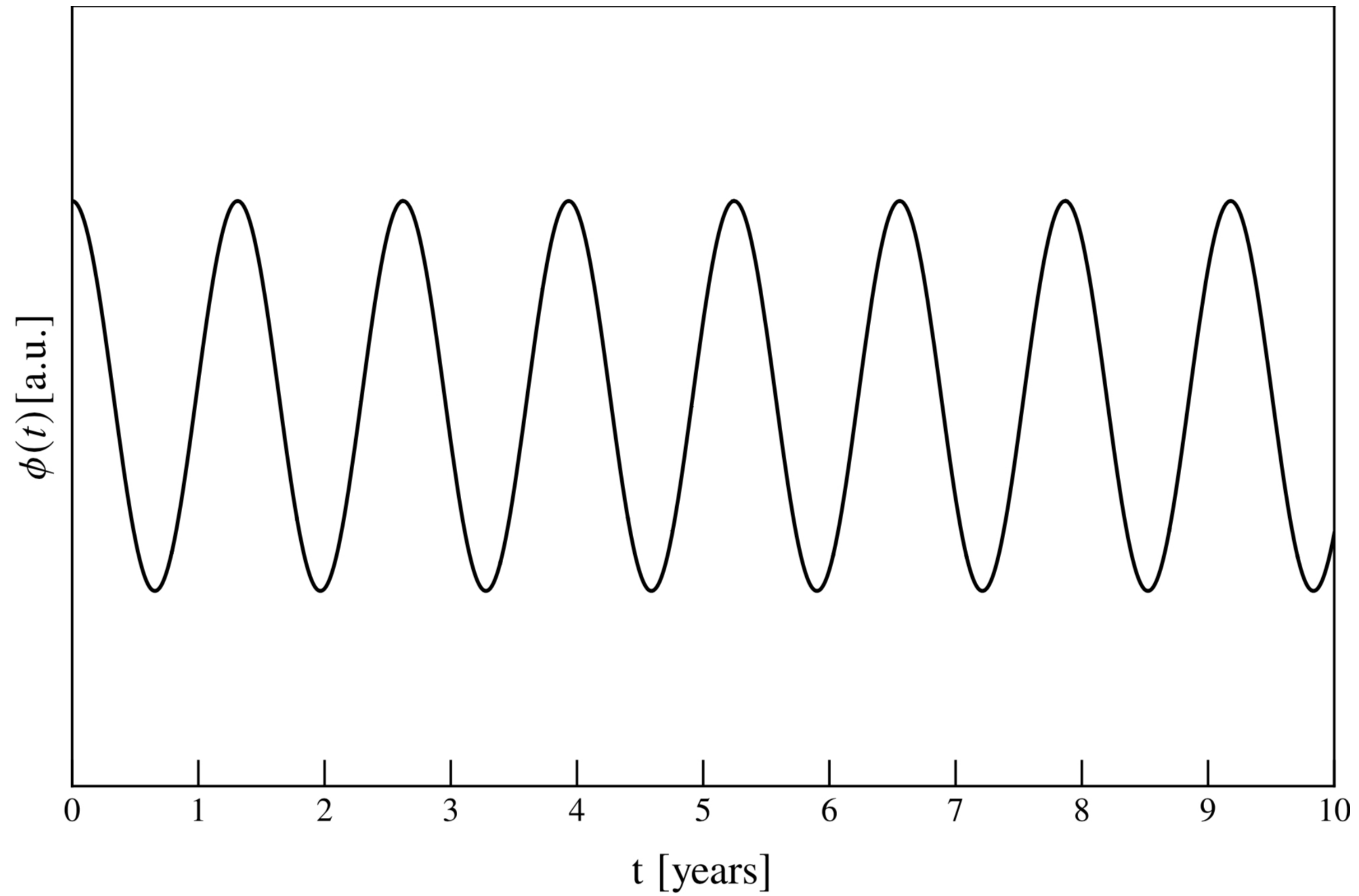
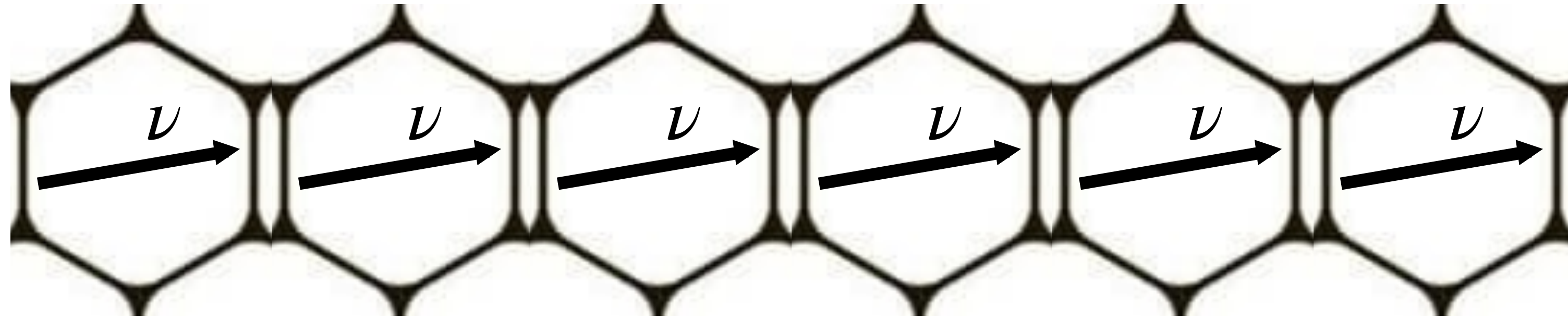
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[Abhish Dev](#) ✉, [Pedro A. N. Machado](#) & [Pablo Martínez-Miravé](#)



The field can be effectively described as

$$\phi(x, t) = \phi_0 \cos(\xi)$$

$$\xi \in [0, 2\pi]$$

Impact on neutrino propagation

Neutrino Hamiltonian

$$H_\nu = H_0 + H_\phi$$

Impact on neutrino propagation

Neutrino Hamiltonian

$$H_\nu = H_0 + H_\phi$$

$$H_0 = \frac{1}{2E} \text{diag}(m_1^2, m_2^2, m_3^2)$$

Impact on neutrino propagation

Neutrino Hamiltonian

$$H_\nu = H_0 + H_\phi$$

$$H_0 = \frac{1}{2E} \text{diag}(m_1^2, m_2^2, m_3^2)$$

$$H_\phi = \frac{\phi_0}{E} \text{diag}((g_\phi m)_1, (g_\phi m)_2, (g_\phi m)_3) \cos \xi$$

How to approach the problem?

Typically:

1. Compute the time evolution for one neutrino.

$$U(t, \xi) = \exp \left(-iH_\nu(\xi)t \right)$$

How to approach the problem?

Typically:

- 1. Compute the time evolution for one neutrino.**
- 2. Compute the oscillation probability. It will be a function of an unknown phase.**

How to approach the problem?

Typically:

1. **Compute the time evolution for one neutrino.**
2. **Compute the oscillation probability. It will be a function of an unknown phase.**
3. **Average over the phase.**

$$\bar{P} = \frac{1}{2\pi} \int_0^{2\pi} d\xi P(\xi)$$

From an open system perspective

Recall

$$\rho(t, \xi) = U(t, \xi) \rho(0) U^\dagger(t, \xi)$$

From an open system perspective

Recall

$$\rho(t, \xi) = U(t, \xi) \rho(0) U^\dagger(t, \xi)$$

Average

$$\bar{\rho}(t) = \frac{1}{2\pi} \int_0^{2\pi} d\xi U(t, \xi) \rho(0) U^\dagger(t, \xi)$$

How to get the master equation?

Define

$$\Delta U = U(t, \xi) - \bar{U}(t) \quad \bar{U}(t) = \frac{1}{2\pi} \int_0^{2\pi} d\xi U(t, \xi)$$

How to get the master equation?

Define

$$\Delta U = U(t, \xi) - \bar{U}(t) \quad \bar{U}(t) = \frac{1}{2\pi} \int_0^{2\pi} d\xi U(t, \xi)$$

We can rewrite

$$\bar{\rho}(t) = \bar{U}(t)\rho(0)\bar{U}^\dagger(t) + \frac{1}{2\pi} \int_0^{2\pi} d\xi \Delta U(t, \xi)\rho(0)\Delta U^\dagger(t, \xi)$$

How to get the master equation?

Define an effective hamiltonian

$$\partial_t \bar{U}(t) = -iV(t)\bar{U}(t)$$

Burgess, et al., Annals of Physics
Volume 256, Issue 1, 1

How to get the master equation?

Define an effective hamiltonian

$$\partial_t \bar{U}(t) = -iV(t)\bar{U}(t)$$

Burgess, et al., Annals of Physics
Volume 256, Issue 1, 1

We find the time evolution equation

$$\partial_t \bar{\rho}(t) = -i[V(t), \bar{\rho}(t)] + \partial_t \left(\frac{1}{2\pi} \int_0^{2\pi} d\xi \Delta U(t, \xi) \rho(0) \Delta U^\dagger(t, \xi) \right)$$

How to get the master equation?

To second order in perturbation theory

$$U(t, \xi) \approx \mathbf{1} - i \int_0^t d\tau H(\tau, \xi) - \int_0^t d\tau \int_0^\tau d\tau' H(\tau, \xi) H(\tau', \xi)$$

We can determine $V(t)$ and $\Delta U(t, \xi)$

The master equation

$$\partial_t \bar{\rho}(t) = -i[H_0, \bar{\rho}(t)] - \frac{\phi_0^2 t}{2E^2} ((g_\phi \hat{m}_\nu)^2 \bar{\rho}(t) + \bar{\rho}(t) (g_\phi \hat{m}_\nu)^2 - 2g_\phi \hat{m}_\nu \bar{\rho}(t) g_\phi \hat{m}_\nu)$$

$$\hat{m}_\nu = \text{diag}(m_1, m_2, m_3)$$

The master equation

$$\partial_t \bar{\rho}(t) = -i[H_0, \bar{\rho}(t)] - \frac{\phi_0^2 t}{2E^2} ((g_\phi \hat{m}_\nu)^2 \bar{\rho}(t) + \bar{\rho}(t) (g_\phi \hat{m}_\nu)^2 - 2g_\phi \hat{m}_\nu \bar{\rho}(t) g_\phi \hat{m}_\nu)$$

Recall

$$\partial_t \rho(t) = -i[H, \rho(t)] - L^\dagger L \rho(t) - \rho(t) L^\dagger L + 2L \rho(t) L^\dagger$$

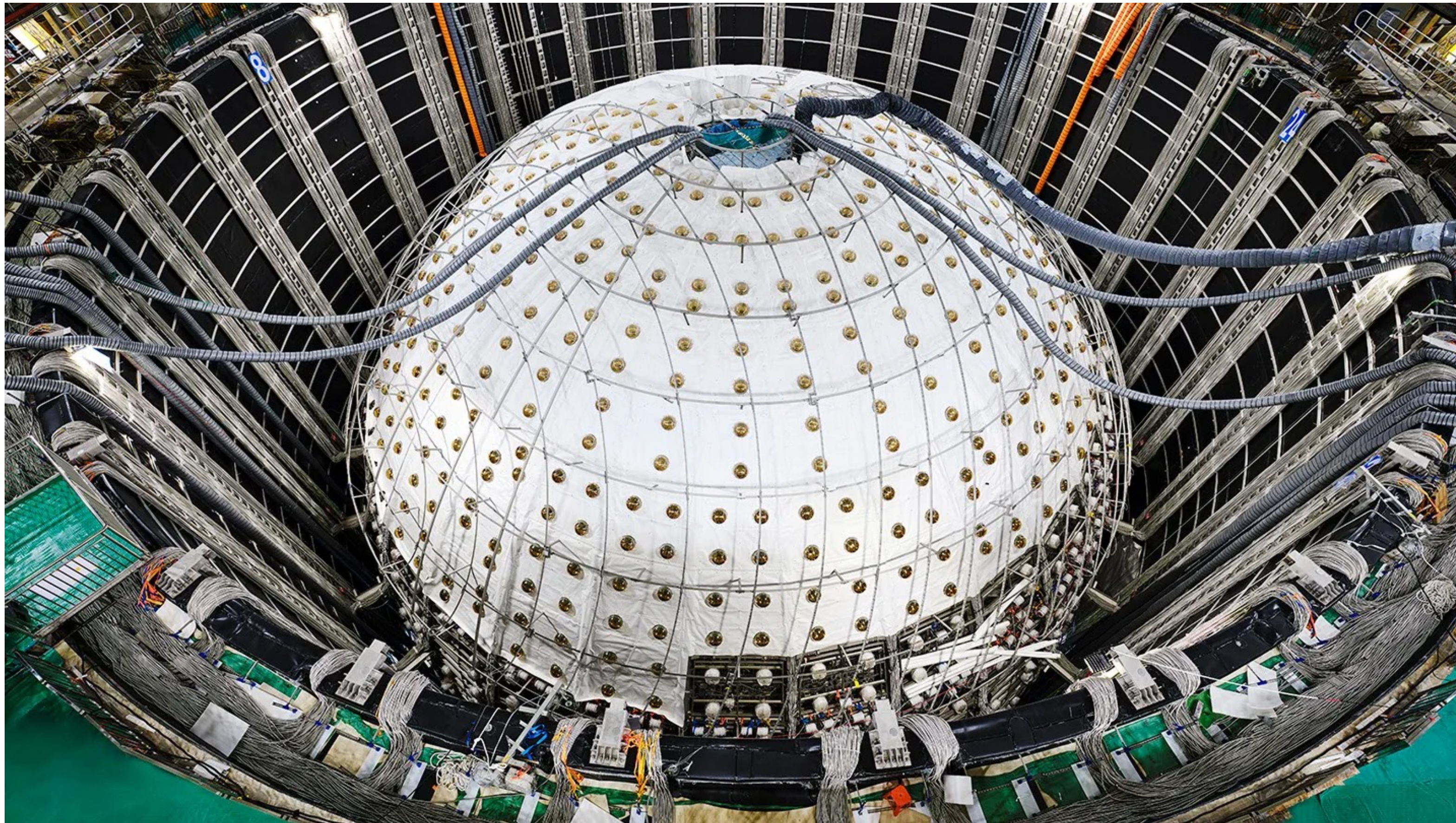
The master equation

$$\partial_t \bar{\rho}(t) = -i[H_0, \bar{\rho}(t)]$$

$$-\frac{\phi_0^2 t}{2E^2} ((g_\phi \hat{m}_\nu)^2 \bar{\rho}(t) + \bar{\rho}(t) (g_\phi \hat{m}_\nu)^2 - 2g_\phi \hat{m}_\nu \bar{\rho}(t) g_\phi \hat{m}_\nu)$$

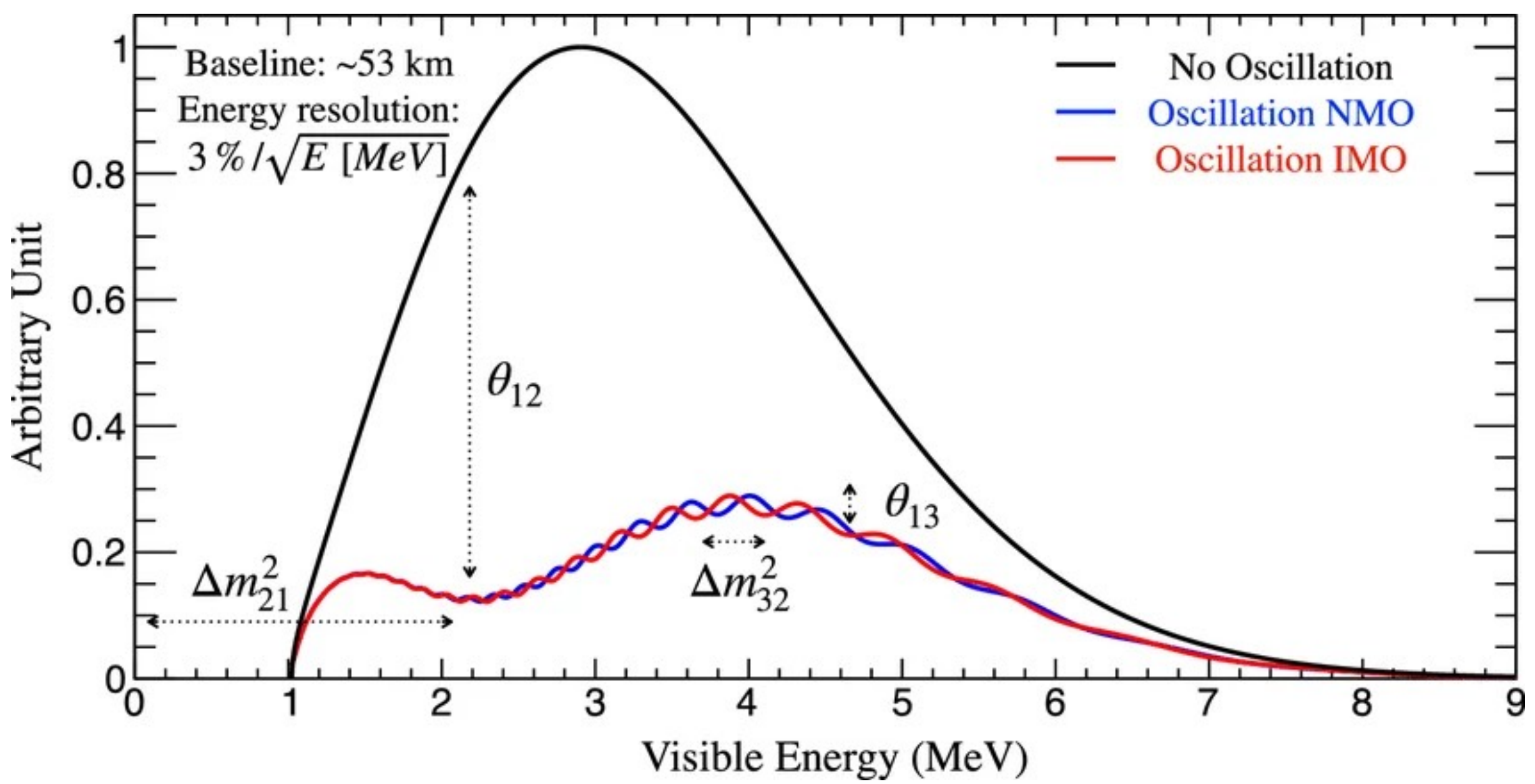
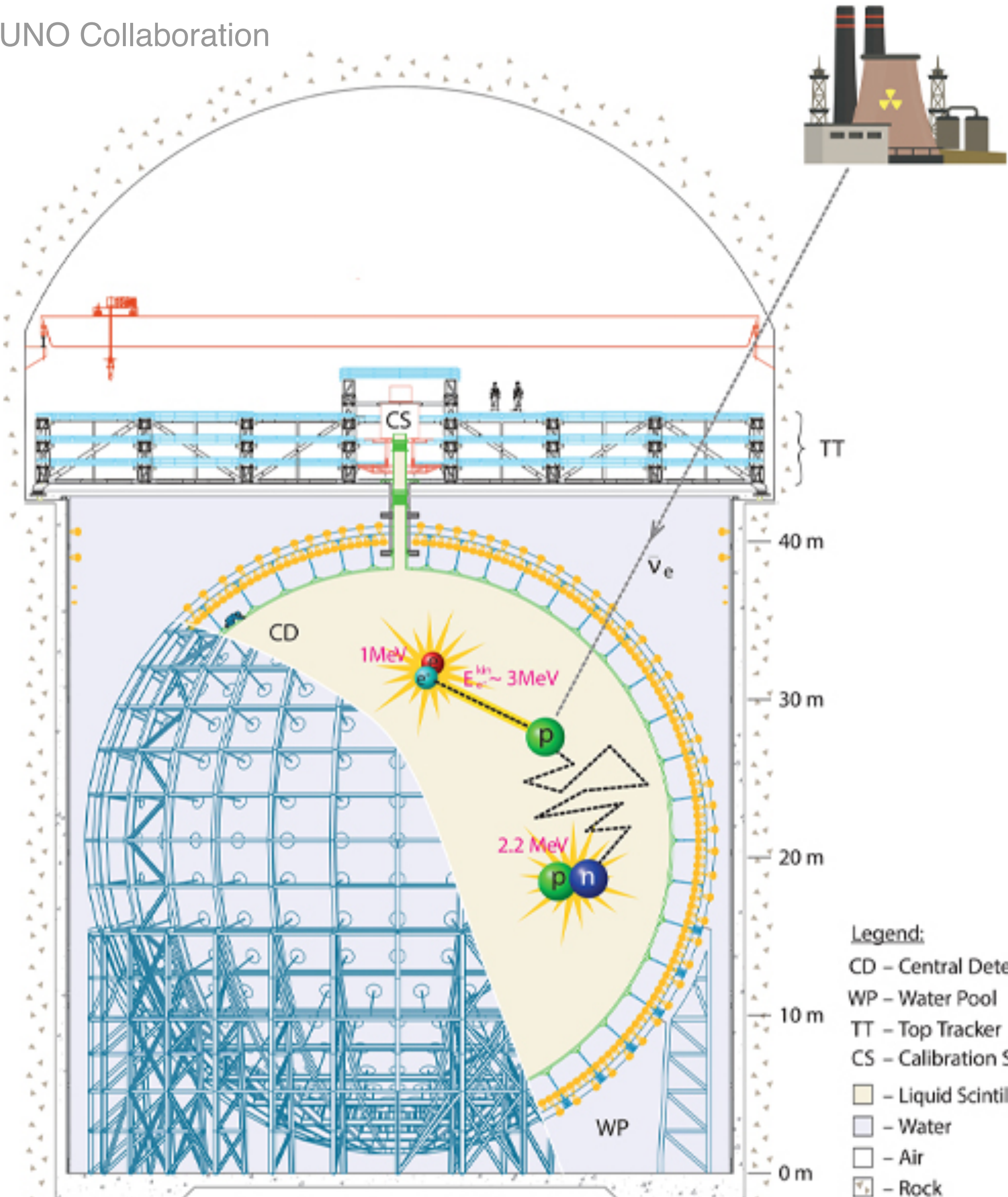
Better sensitivity at lower energies

JUNO experiment



JUNO experiment

Image: JUNO Collaboration



Standard Oscillation probability

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \frac{1}{2} (\sin^2(2\theta_{13}) + \sin^2(2\theta_{12})) \\ - P_{\odot} + \frac{\sin^2(2\theta_{13})}{2} \sqrt{1 - \sin^2(2\theta_{12}) \sin^2 \Delta_{21}} \cos(2|\Delta_{ee}| \pm \Phi_{\odot})$$

Standard Oscillation probability

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$$P_{\odot} \approx \sin^2 2\theta_{12} \sin^2 \Delta_{21}$$

$$\Delta_{ee} = \cos^2 \theta_{12} \Delta_{31} + \sin^2 \theta_{12} \Delta_{32}$$

$$\Phi_{\odot} = \arctan(\cos 2\theta_{12} \tan \Delta_{21}) - \Delta_{21} \cos 2\theta_{12}$$

$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{4E} L$$

$$\cos^4 \theta_{13} \approx 1$$

How does the probability change?

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \frac{1}{2} (\sin^2(2\theta_{13}) + \sin^2(2\theta_{12}))$$
$$- \exp\left(-\frac{(\Delta\phi_{21})^2}{4}\right) P_{\odot}$$
$$+ \exp\left(-\frac{(\Delta\phi_{atm})^2}{4}\right) \frac{\sin^2(2\theta_{13})}{2} \sqrt{1 - \sin^2(2\theta_{12}) \sin^2 \Delta_{21}} \cos(2|\Delta_{ee}| \pm \Phi_{\odot})$$

How does the probability change?

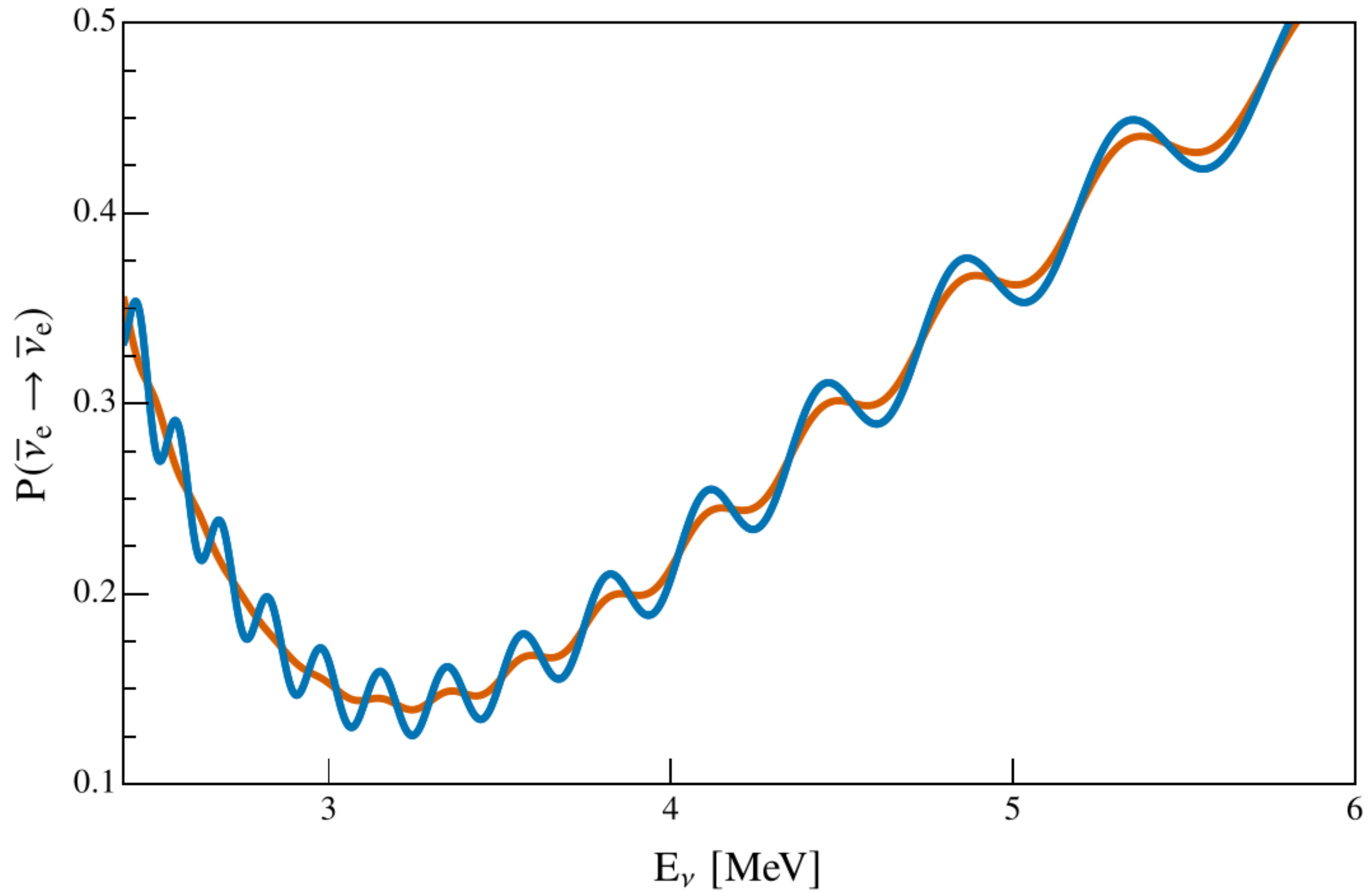
$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \frac{1}{2} (\sin^2(2\theta_{13}) + \sin^2(2\theta_{12}))$$
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$$+ \exp\left(-\frac{(\Delta\phi_{atm})^2}{4}\right) \frac{\sin^2(2\theta_{13})}{2} \sqrt{1 - \sin^2(2\theta_{12}) \sin^2 \Delta_{21}} \cos(2|\Delta_{ee}| \pm \Phi_{\odot})$$

How does the probability change?

$$\begin{aligned}
 P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx & 1 - \frac{1}{2} (\sin^2(2\theta_{13}) + \sin^2(2\theta_{12})) \\
 & - \exp\left(-\frac{(\Delta\phi_{21})^2}{4}\right) P_{\odot} \\
 & + \exp\left(-\frac{(\Delta\phi_{atm})^2}{4}\right) \frac{\sin^2(2\theta_{13})}{2} \sqrt{1 - \sin^2(2\theta_{12}) \sin^2 \Delta_{21}} \cos(2|\Delta_{ee}| \pm \Phi_{\odot})
 \end{aligned}$$

$$(\Delta\phi_{ij})^2 = g_{\phi}^2 \phi_0^2 (m_i - m_j)^2 \frac{L^2}{E^2}$$

Consequences



Consequences

For large L

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \rightarrow 1 - \frac{1}{2} (\sin^2(2\theta_{13}) + \sin^2(2\theta_{12}))$$

**This particular example
is easy enough to solve
without approximations**

The exact solution

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \frac{1}{2} (\sin^2(2\theta_{13}) + \sin^2(2\theta_{12})) \\ - J_0(\Delta_{21}^\phi) P_\odot \\ + J_0(\Delta_{\text{atm}}^\phi) \frac{\sin^2(2\theta_{13})}{2} \sqrt{1 - \sin^2(2\theta_{12}) \sin^2 \Delta_{21}} \cos(2|\Delta_{ee}| \pm \Phi_\odot)$$

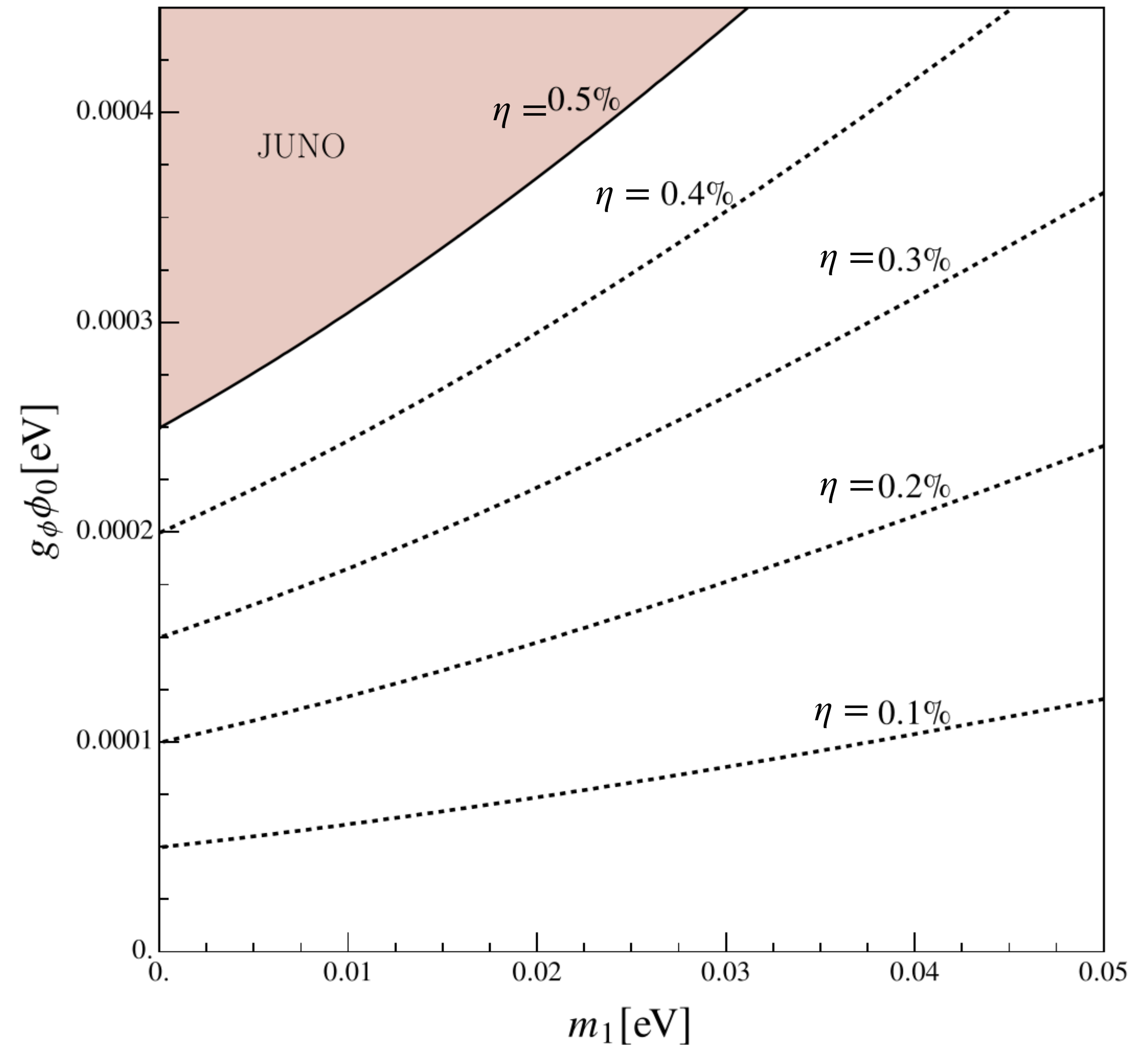
To leading order they coincide

$$J_0(x) = 1 - \frac{x^2}{4} + \mathcal{O}(x^4)$$

$$\exp\left(-\frac{x^2}{4}\right) = 1 - \frac{x^2}{4} + \mathcal{O}(x^4)$$

Open system can be mapped to model parameters

$$g_\phi \phi_0 (m_i - m_j) = \Delta m_{ij}^2 (\eta_\phi)_{ij}$$



How does the probability change?

Scaling

Ultralight background

$$\exp \left[-\frac{(\eta \Delta m^2)^2 L^2}{E^2} \right]$$

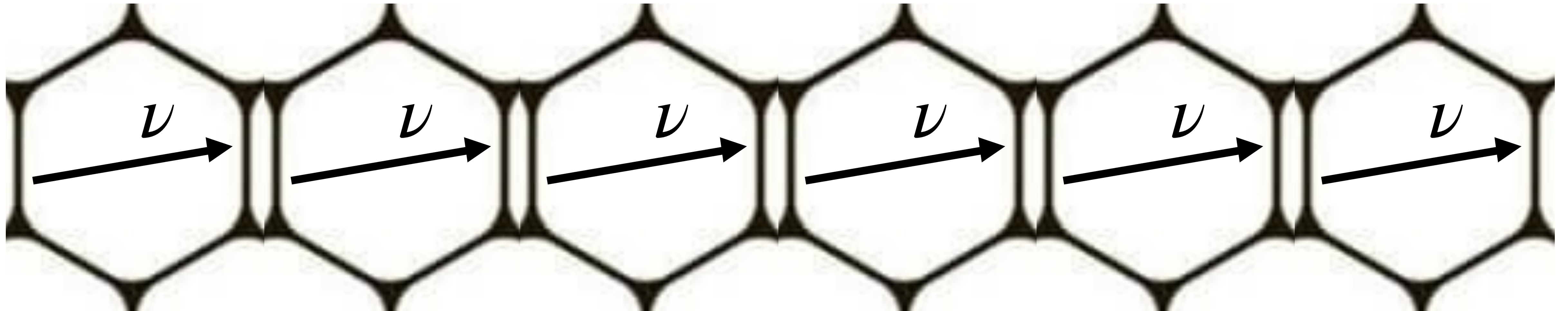
Wave packet

$$\exp \left[-\frac{|\Delta m^2| L^2}{32 \sigma_x^2 E^4} \right]$$

General approach

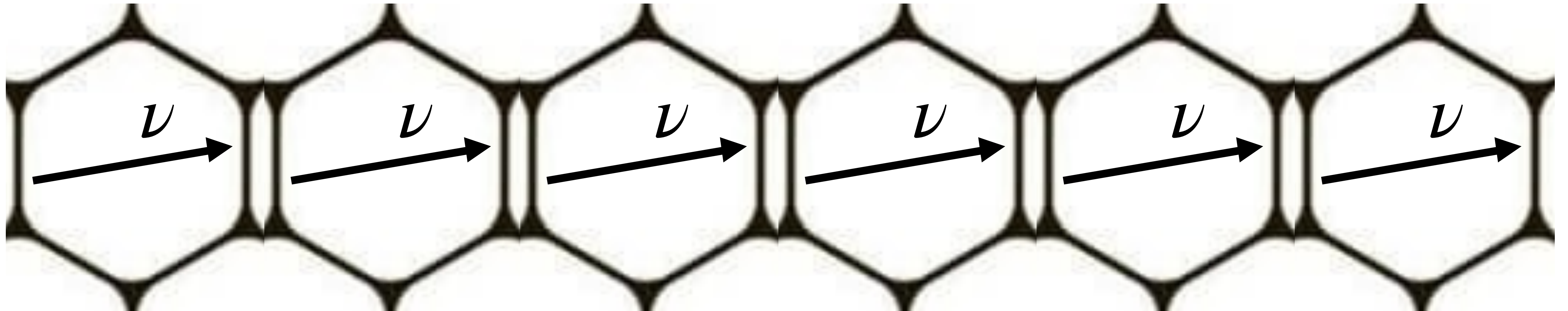
$$\exp \left[-\Gamma_0 \left(\frac{E}{E_0} \right)^n L \right]$$

Statistical decoherence, not quantum



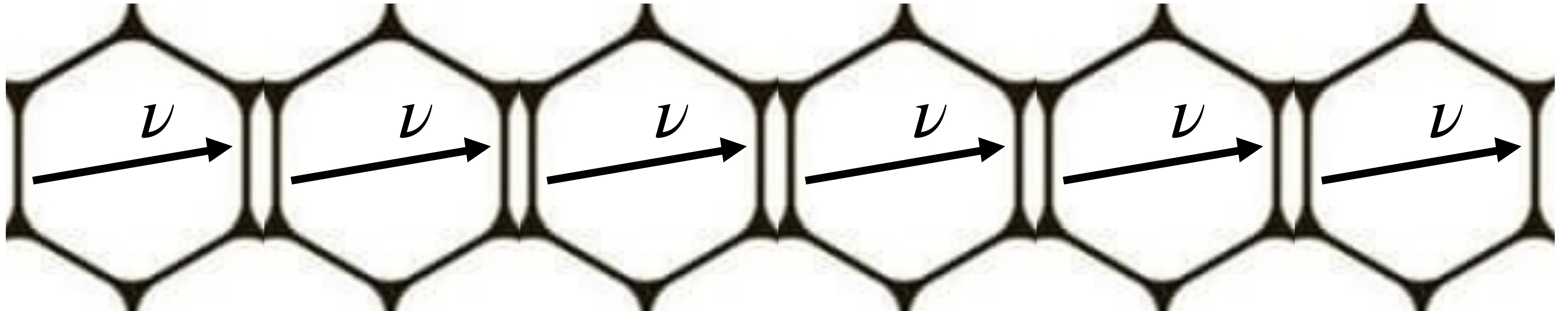
Statistical decoherence, not quantum

$$\rho(t, \xi_1)$$



Statistical decoherence, not quantum

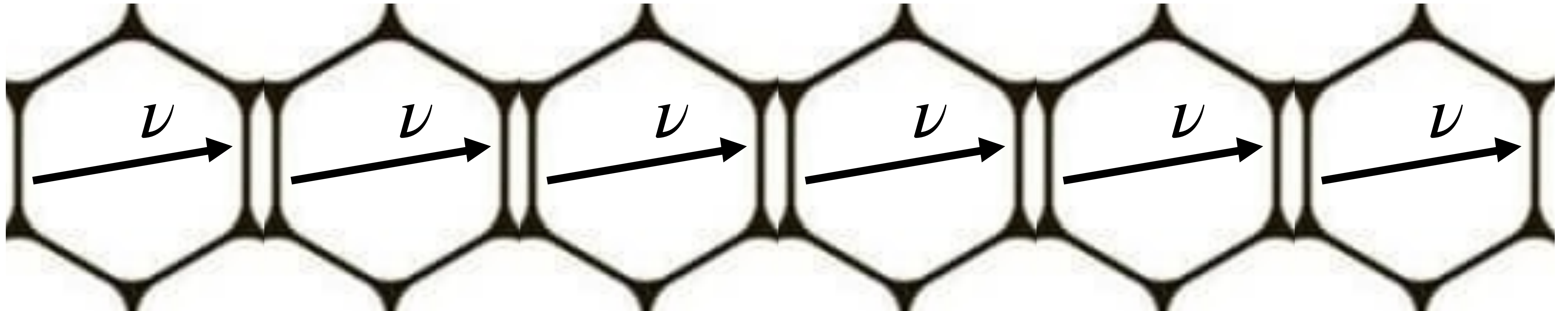
$$\rho(t, \xi_1) \quad \rho(t, \xi_2) \quad \rho(t, \xi_3) \quad \rho(t, \xi_4) \quad \rho(t, \xi_5) \quad \rho(t, \xi_6)$$



$$\text{Tr}(\rho^2) = 1$$

Statistical decoherence, not quantum

$$\rho(t, \xi_1) \quad \rho(t, \xi_2) \quad \rho(t, \xi_3) \quad \rho(t, \xi_4) \quad \rho(t, \xi_5) \quad \rho(t, \xi_6)$$



$$\bar{\rho}(t) = \frac{1}{2\pi} \int_0^{2\pi} d\xi U(t, \xi) \rho(0) U^\dagger(t, \xi)$$

$$\text{Tr}(\bar{\rho}^2) < 1$$

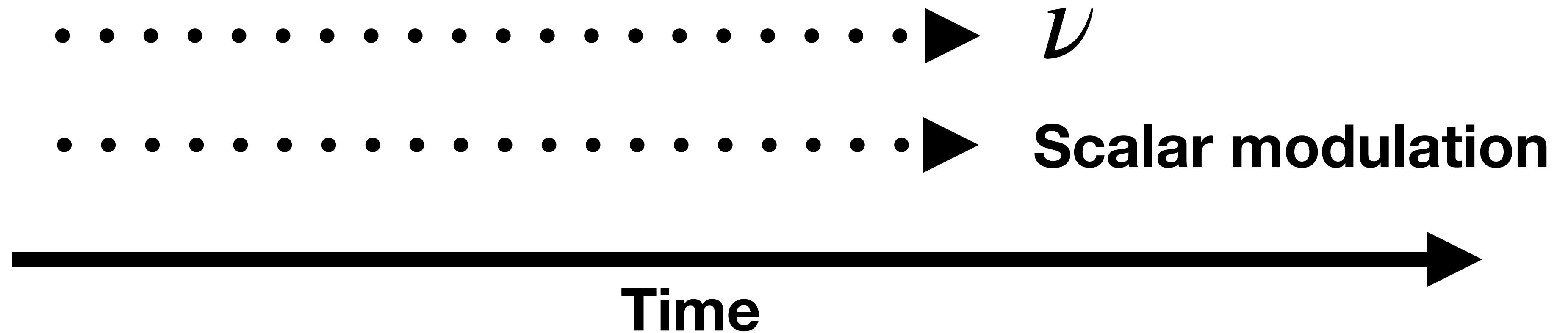
Routes to pursue

“The end of a story is like the end of a good meal. We’re sated but it doesn’t mean that we will never be hungry again.”

— Kate Tellers

Can we have an observable quantum decoherence effect?

Dynamical field regime



How the master equation should look

$$\partial_t \rho(t) = -i[H_0 + H_\phi(t), \rho(t)] - \int d\tau \int d\tau' \frac{\text{Tr}(\Delta\phi(\tau)\Delta\phi(\tau')\rho_\phi)}{2E^2} (\hat{m}_\nu^2 \rho(t) + \rho(t) \hat{m}_\nu^2 - 2\hat{m}_\nu \rho(t) \hat{m}_\nu)$$

$$\Delta\phi = \phi - \text{Tr}(\phi\rho_\phi) \quad \rho_\phi = \int d\alpha P(\alpha) |\alpha\rangle\langle\alpha|$$

Requirements

- 1. We must fix the scalar state ρ_ϕ .**

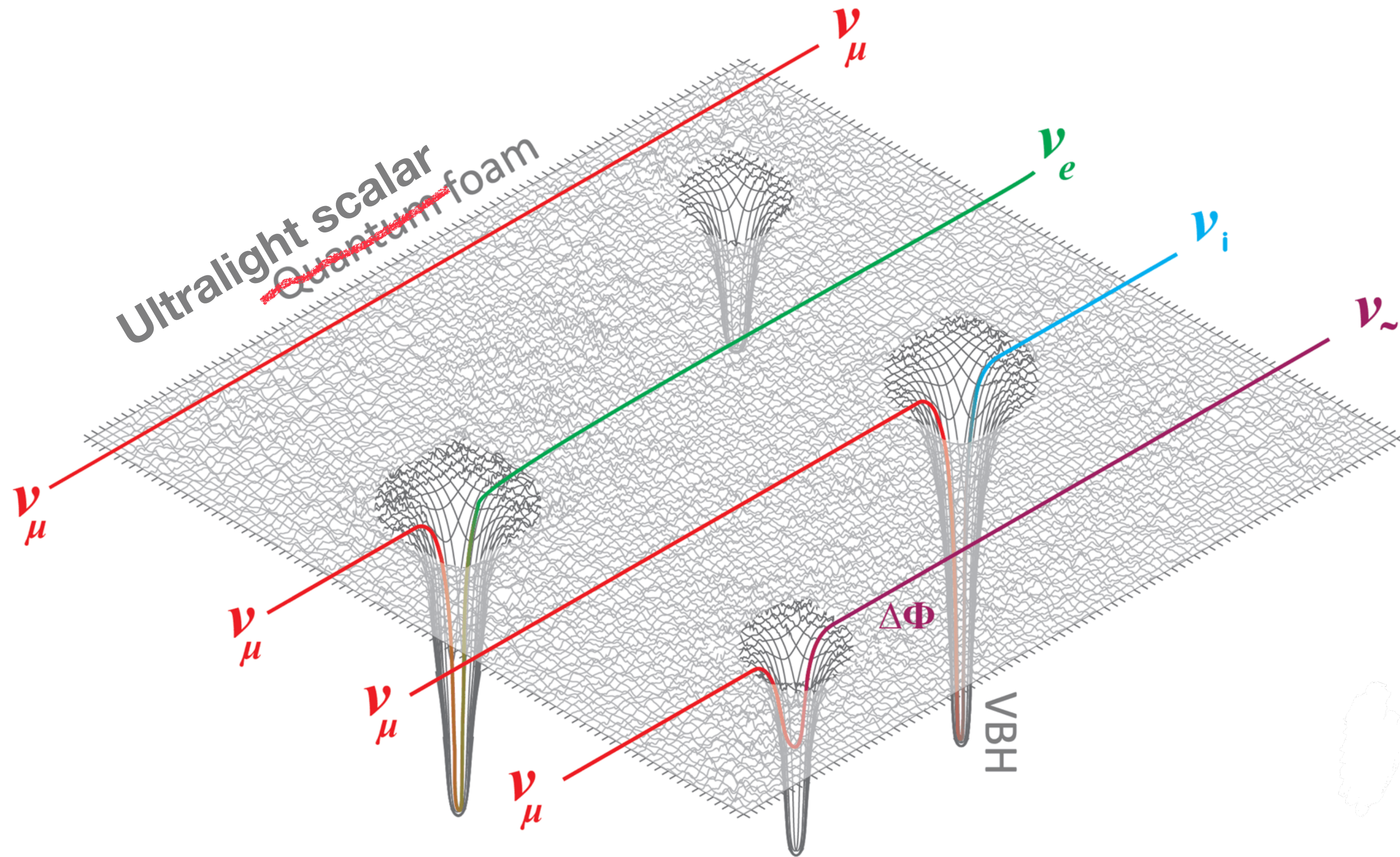
Requirements

1. We must fix the scalar state ρ_ϕ .
2. The field must have a nontrivial variance $\langle \Delta\phi\Delta\phi \rangle$

Requirements

- 1. We must fix the scalar state ρ_ϕ .**
- 2. The field must have a nontrivial variance $\langle \Delta\phi \Delta\phi \rangle$**
- 3. The field must not oscillate too fast**

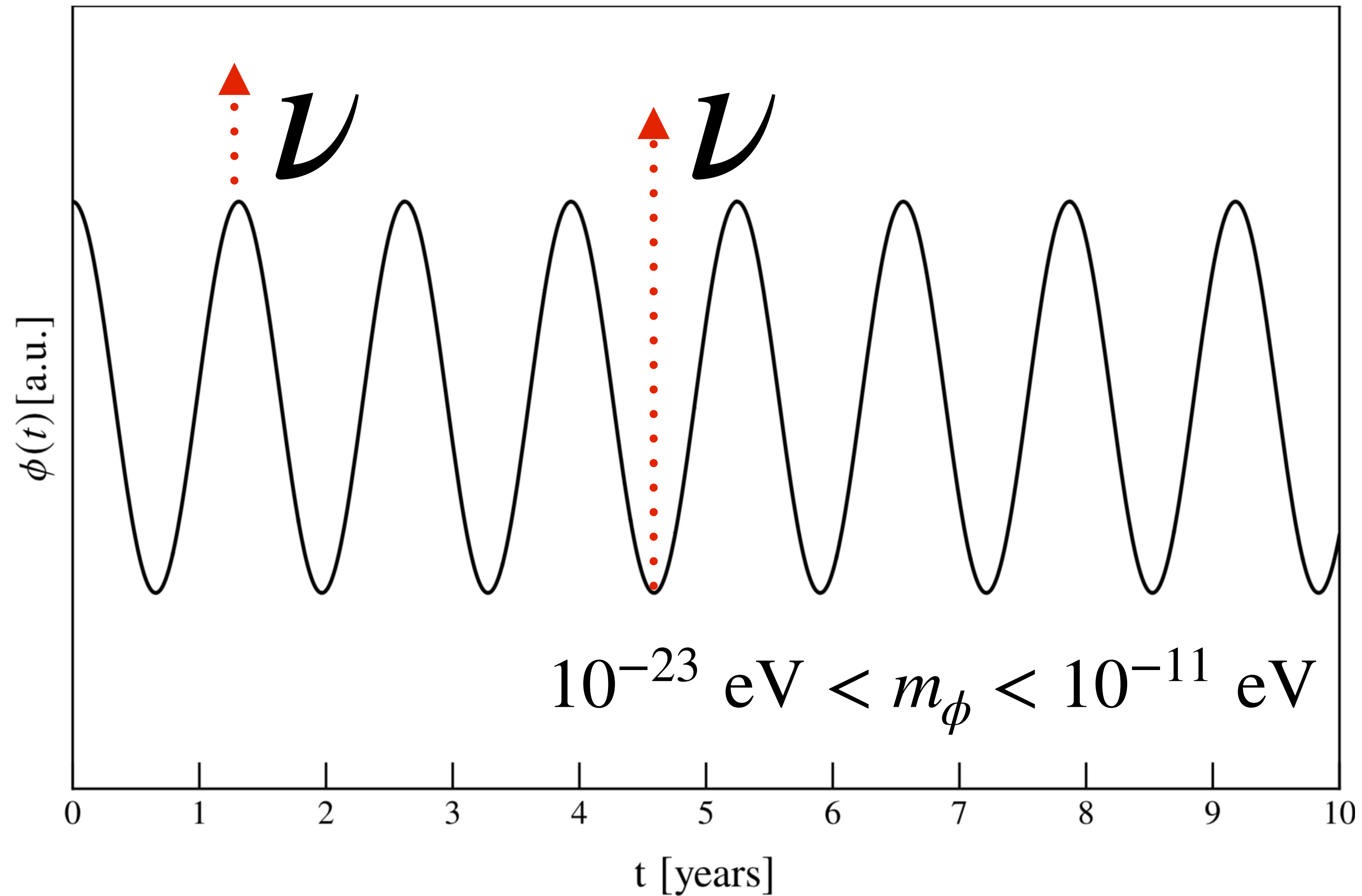
Routes to pursue



Can we probe true quantum decoherence instead of statistical decoherence?

Summary

Quick recap



**If neutrino masses are clocks,
we can hear them tick.**

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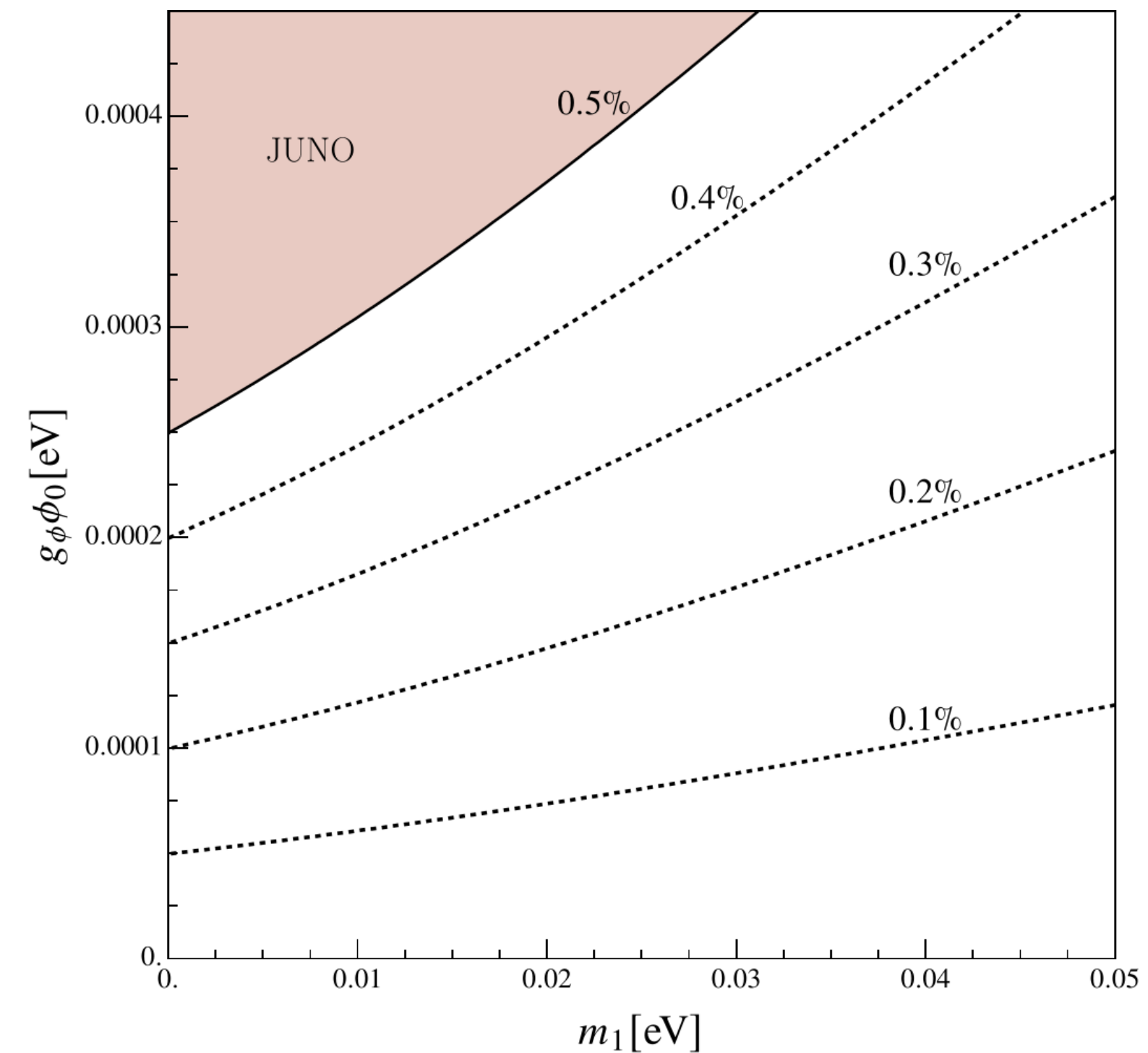
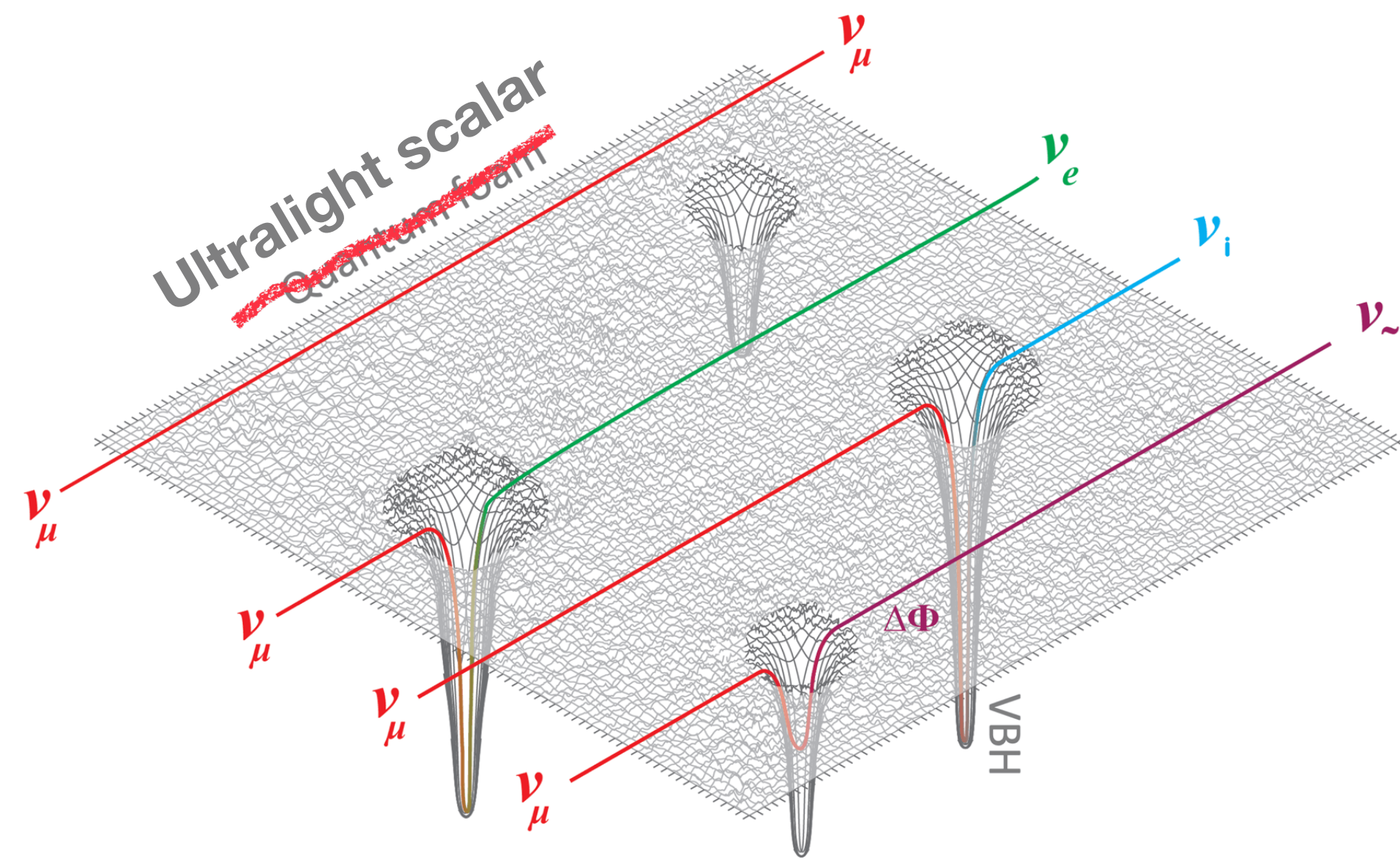
**The fact that the clocks
start out of sync can be
modeled as an open system**



**And even if we listen only to the
sound of silence...**

And even if we listen only to the sound of silence...

We can still learn something new!



BACKUP

Ultralight scalars have interesting properties

$$\frac{N}{V} = \frac{\rho_\phi}{m_\phi}$$

For ultralight scalar dark matter

$$[a, a^\dagger] = 1 \rightarrow aa^\dagger = \underbrace{a^\dagger a}_N + 1 \approx N$$

It's convenient to describe the scalar state as a coherent state

Quick review of coherent states

**A coherent state is a
superposition of states of the
harmonic oscillator**

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

Quick review of coherent states

**It is an eigenstate of the
annihilation operator**

$$a|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \sqrt{n} |n-1\rangle$$

Quick review of coherent states

It is an eigenstate of the annihilation operator

$$\begin{aligned} a|\alpha\rangle &= e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \sqrt{n} |n-1\rangle \\ &= e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{(n-1)!}} \sqrt{n} |n-1\rangle = \alpha|\alpha\rangle \end{aligned}$$

Quick review of coherent states

Summary

$$a |\alpha\rangle = \alpha |\alpha\rangle$$

$$\alpha = |\alpha| e^{i\theta}$$

Quick review of coherent states

Summary

$$a |\alpha\rangle = \alpha |\alpha\rangle$$

$$\alpha = |\alpha| e^{i\theta}$$

Effectively

$$a \rightarrow \alpha \quad a^\dagger \rightarrow \alpha^*$$

“Vectorizing” the time evolution

$$\rho(t, \xi) = \sum_{\mu=0}^8 \rho^\mu \lambda_\mu = \begin{pmatrix} \frac{\rho_0}{\sqrt{3}} + \frac{\rho_3}{\sqrt{2}} + \frac{\rho_8}{\sqrt{6}} & \frac{\rho_1}{\sqrt{2}} - \frac{i\rho_2}{\sqrt{2}} & \frac{\rho_4}{\sqrt{2}} - \frac{i\rho_5}{\sqrt{2}} \\ \frac{\rho_1}{\sqrt{2}} + \frac{i\rho_2}{\sqrt{2}} & \frac{\rho_0}{\sqrt{3}} - \frac{\rho_3}{\sqrt{2}} + \frac{\rho_8}{\sqrt{6}} & \frac{\rho_6}{\sqrt{2}} - \frac{i\rho_7}{\sqrt{2}} \\ \frac{\rho_4}{\sqrt{2}} + \frac{i\rho_5}{\sqrt{2}} & \frac{\rho_6}{\sqrt{2}} + \frac{i\rho_7}{\sqrt{2}} & \frac{\rho_0}{\sqrt{3}} - \sqrt{\frac{2}{3}} \rho_8 \end{pmatrix}$$

$$H(\xi) = \sum_{\mu=0}^8 h^\mu \lambda_\mu, \quad \lambda_\mu = \left\{ \frac{\mathbf{1}_{3 \times 3}}{\sqrt{3}}, \lambda_i \right\}, \quad \text{Tr}(\lambda^\mu \lambda^\nu) = \delta_{\mu\nu}$$

“Vectorizing” the time evolution

Defining

$$|\rho(t, \xi)\rangle = \begin{pmatrix} \rho_0 \\ \rho_1 \\ \vdots \\ \rho_8 \end{pmatrix}$$

**Expanding
both sides**

$$\partial_t \rho_\mu \lambda^\mu = -i h_\nu \rho_\theta [\lambda^\nu, \lambda^\theta]$$

**Collect
coefficients**

$$\partial_t |\rho(t, \xi)\rangle = \mathcal{H}(\xi) |\rho(t)\rangle$$

This can be treated analytically

$$\mathcal{U}(t, \xi) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos \Delta_{21}^{\text{eff}} & \sin \Delta_{21}^{\text{eff}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\sin \Delta_{21}^{\text{eff}} & \cos \Delta_{21}^{\text{eff}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos \Delta_{31}^{\text{eff}} & \sin \Delta_{31}^{\text{eff}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sin \Delta_{31}^{\text{eff}} & \cos \Delta_{31}^{\text{eff}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cos \Delta_{32}^{\text{eff}} & \sin \Delta_{32}^{\text{eff}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\sin \Delta_{32}^{\text{eff}} & \cos \Delta_{32}^{\text{eff}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Delta_{ij}^{\text{eff}} = \frac{\Delta m_{ij}^2}{2E_\nu} L + g_\phi \phi_0 \frac{(m_i - m_j)}{E_\nu} L \equiv \Delta_{ij} + \Delta_{ij}^\phi$$

The averaged evolution

$$\overline{\cos(\Delta_{ij}^{\text{eff}})} = J_0(\Delta_{ij}^{\phi})\cos(\Delta_{ij})$$

Oscillation probability

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P^{\text{SM}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) + \frac{\cos^4 \theta_{13} \sin^2(2\theta_{12})}{2} \left(J_0(\Delta_{21}^{\phi}) - 1 \right) \cos(\Delta_{21}) \\ + \frac{\sin^2(2\theta_{13})}{2} \left(\cos^2 \theta_{12} \left(J_0(\Delta_{31}^{\phi}) - 1 \right) \cos(\Delta_{31}) + \sin^2 \theta_{12} \left(J_0(\Delta_{32}^{\phi}) - 1 \right) \cos(\Delta_{32}) \right)$$