What is the wave packet size of neutrinos?

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To appear soon (I hope)….

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Neutrinos oscillations in a nutshell

- The oscillation formula is usually derived similarly a two level QM oscillator.
- So easy that it fits in here:

$$e^{-iHt}|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} e^{-iE_i t}|\nu_i\rangle$$

$$\downarrow$$

$$\mathscr{A}_{\alpha\to\beta}(L) = \langle \nu_\beta | \nu_\alpha(L)\rangle = \sum_i U^*_{\alpha i} U_{\beta i} e^{-iE_i L}$$

$$\downarrow$$

$$\mathscr{P}_{\alpha\to\beta}(L) = |\mathscr{A}_{\alpha\to\beta}(L)|^2 = \langle \nu_\beta | \nu_\alpha(L)\rangle = \sum_{ij} U^*_{\alpha i} U_{\beta j} U_{\alpha i} U^*_{\beta j} e^{-i(E_i - E_j)L}$$

The Devil is in the details

- ~~The oscillation formula is usually derived similarly a two level QM oscillator.~~
- ~~So easy that it fits in here:~~

Position propagation?

$$e^{-iHt}|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} e^{-iE_i t}|\nu_i\rangle$$

How to move from t to L?

Plane waves x localization?

$$\mathscr{A}_{\alpha\to\beta}(L) = \langle \nu_\beta | \nu_\alpha(L)\rangle = \sum_i U^*_{\alpha i} U_{\beta i} e^{-iE_i L}$$

Relativistic approximation equal energy or equal momentum?

$$\mathscr{P}_{\alpha\to\beta}(L) = |\mathscr{A}_{\alpha\to\beta}(L)|^2 = \langle \nu_\beta | \nu_\alpha(L)\rangle = \sum_{ij} U^*_{\alpha i} U_{\beta j} U_{\alpha i} U^*_{\beta j} e^{-i(E_i - E_j)L}$$

What should we do? Wave packets as a resort

- The neutrino should have localization properties.
- Gaussian wave packets have been extensively used as an ansatz. Probability formula is modified to

Necessary input: The wave packet width

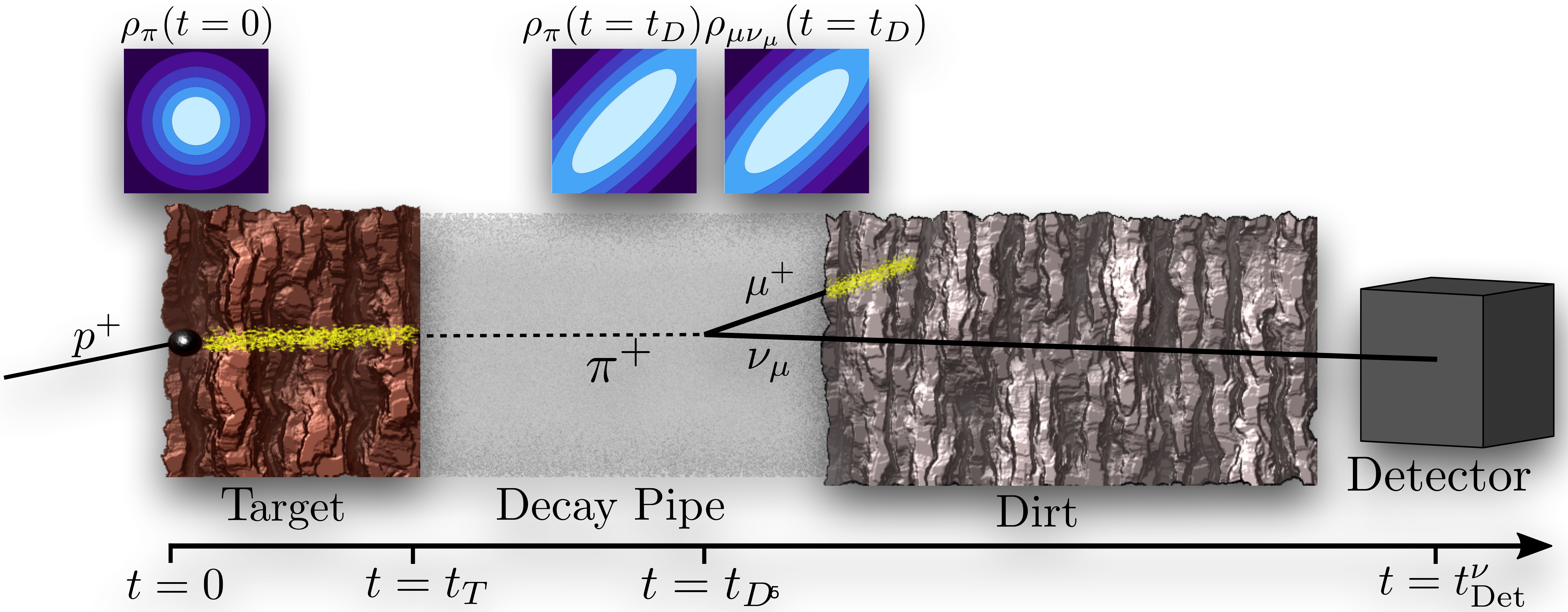
$$\mathscr{P}_{\alpha\to\beta}(L) = \mathscr{P}^{\text{p.w.}}_{\alpha\to\beta}(L) \times \exp\left[-\left(\frac{L}{L^{\text{coh}}}\right)^2 - \left(\frac{\sigma_x}{L^{\text{osc}}}\right)^2 \right]$$

$$L^{\text{coh}} = \frac{4\sqrt{2}E^2\sigma_x}{|\Delta m^2|} \qquad L^{\text{osc}} = \frac{4\pi E}{\Delta m^2}$$

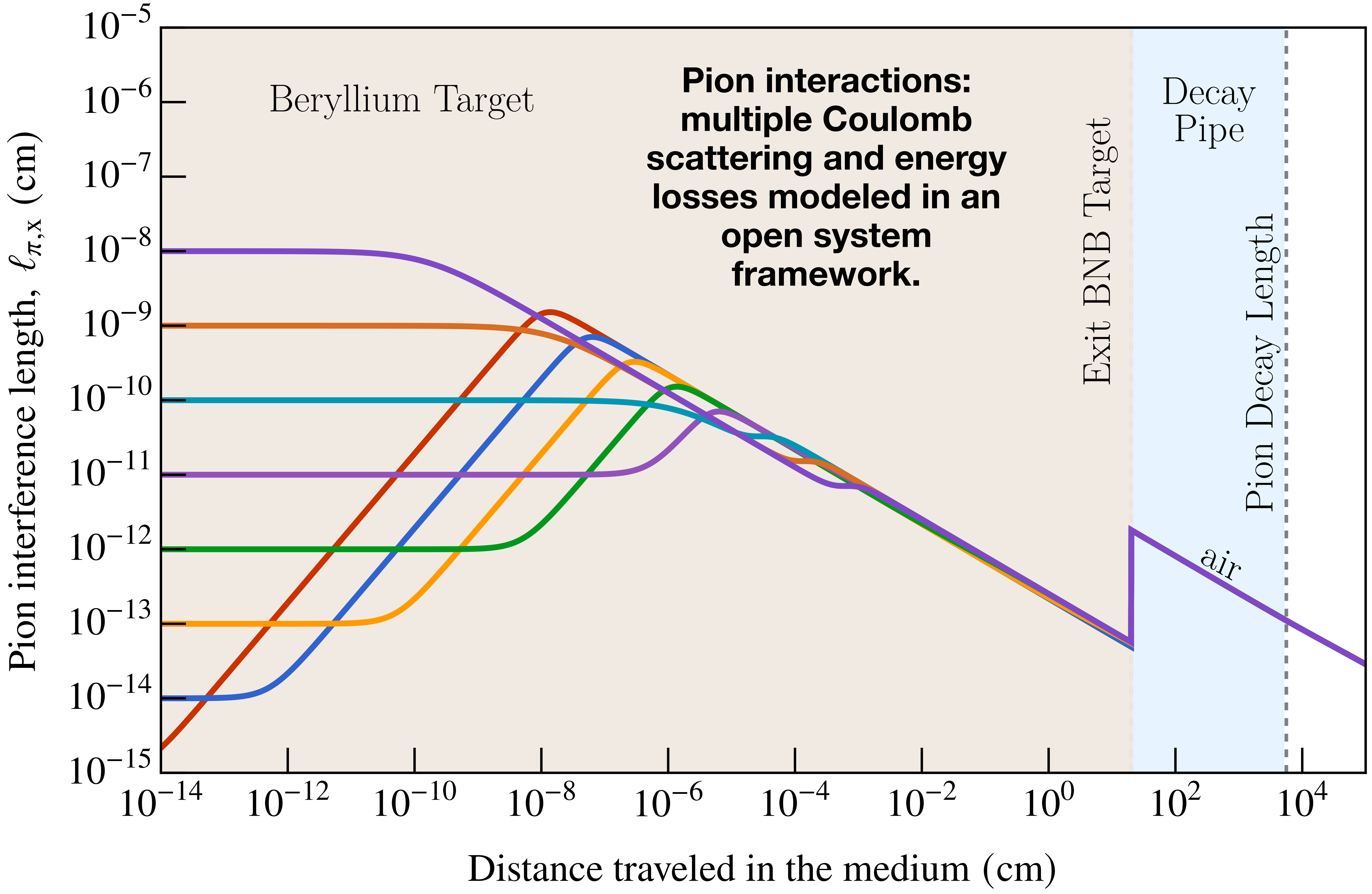
Are there alternatives to estimates?

Decoherence Models

The idea:



Stage 1: Pion interactions fix its state coherence properties.



A taste of the physics at play:

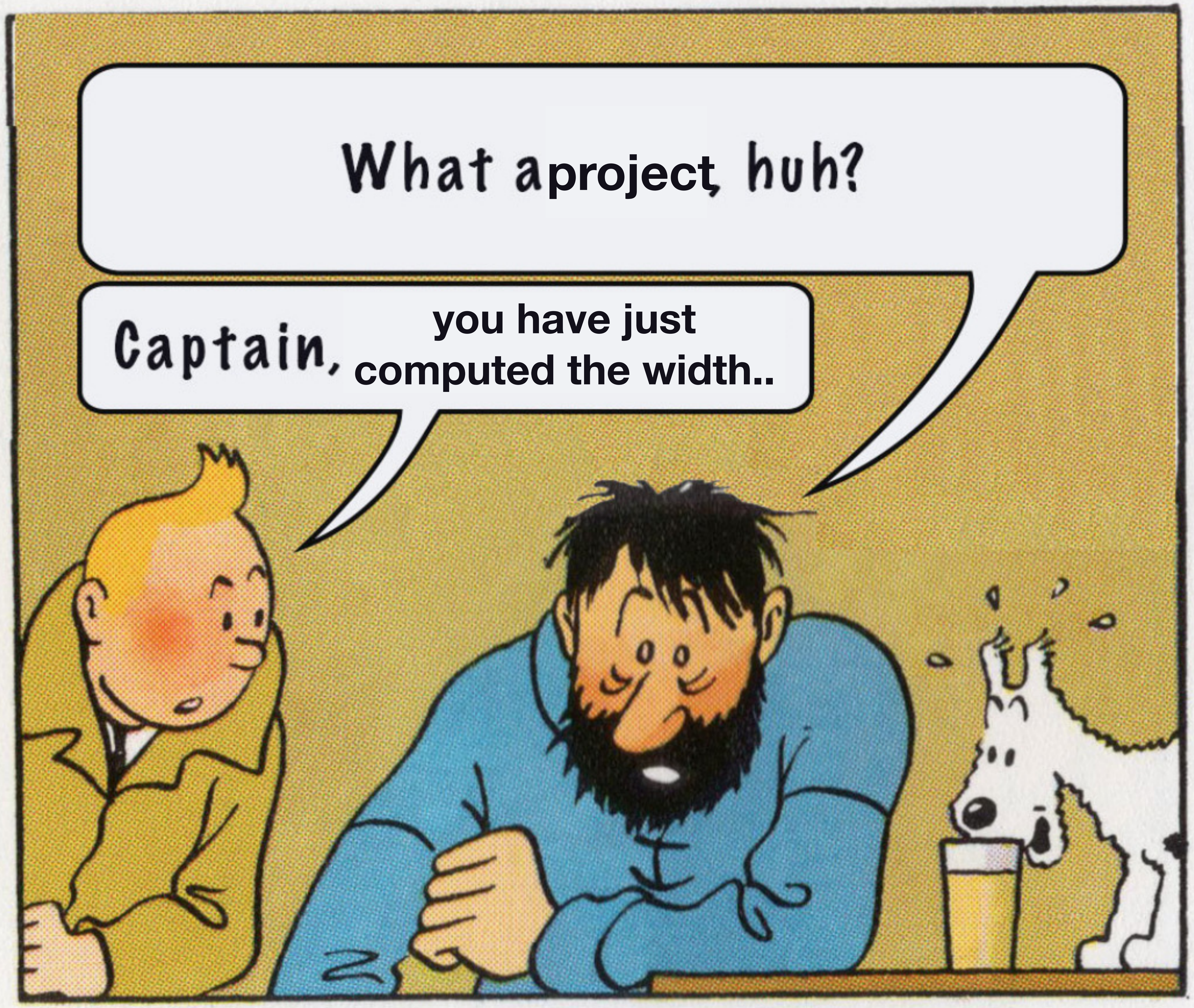
Scattering governed by parameter Λ that affects the state as:

$$\rho(x, x', t) = e^{-\Lambda(x-x')^2 t}\rho(x, x', 0)$$

Hence superpositions get dampened.

What did we learn?

1. **Pion interference properties is fixed from interactions.**
2. **Its interference width at decay is $\ell_{\pi,x} = 10^{-13}$ cm**


What aproject, huh?
Captain, you have just computed the width..


Wave packet width
Production conditions
Localization properties
Detection conditions
Probabilities
Number of events

Come to my poster to hear about other details

Take home message:

1. We could compute the wave packet size from the pion interactions.
2. We could not find any scenario where we can probe the wave packet separation (yet).
3. Detection/production provides the biggest challenge to be overcomed.