Open system approach to neutrinos propagating in an ultralight scalar background

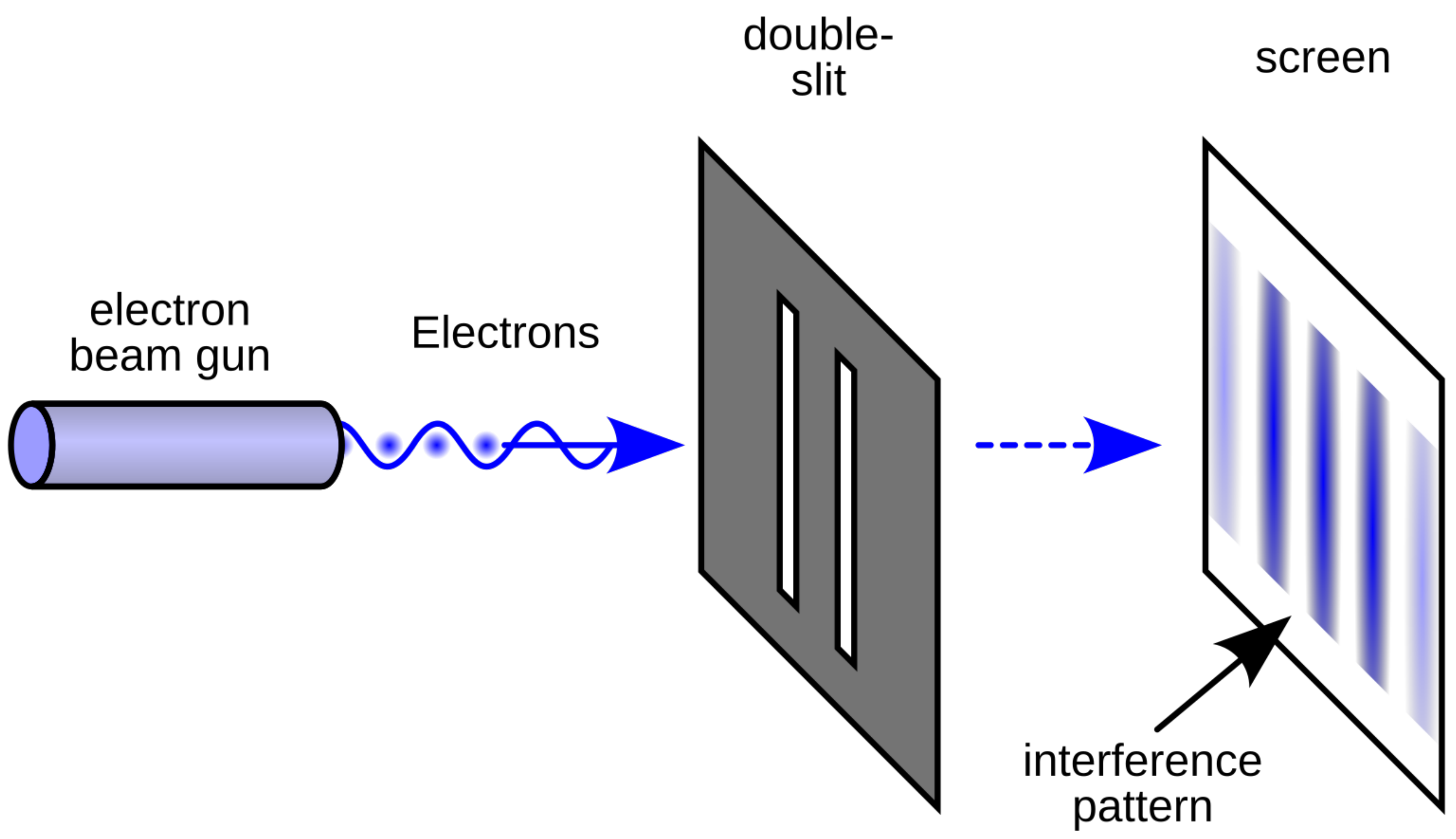
Gustavo F. S. Alves

arXiv:2603.02382

In Collaboration with:

Lua F. T. Airoldi
Pedro Machado
Peter Vander Griend

We live in a quantum world…



But we often don't see its quantumness

$\frac{1}{\sqrt{2}}$  $+ \frac{1}{\sqrt{2}}$ 

But we often don't see its quantumness

$$\frac{1}{\sqrt{2}}$$

$$+\ \frac{1}{\sqrt{2}}$$

$$\neq$$

Superpositions tend to decohere

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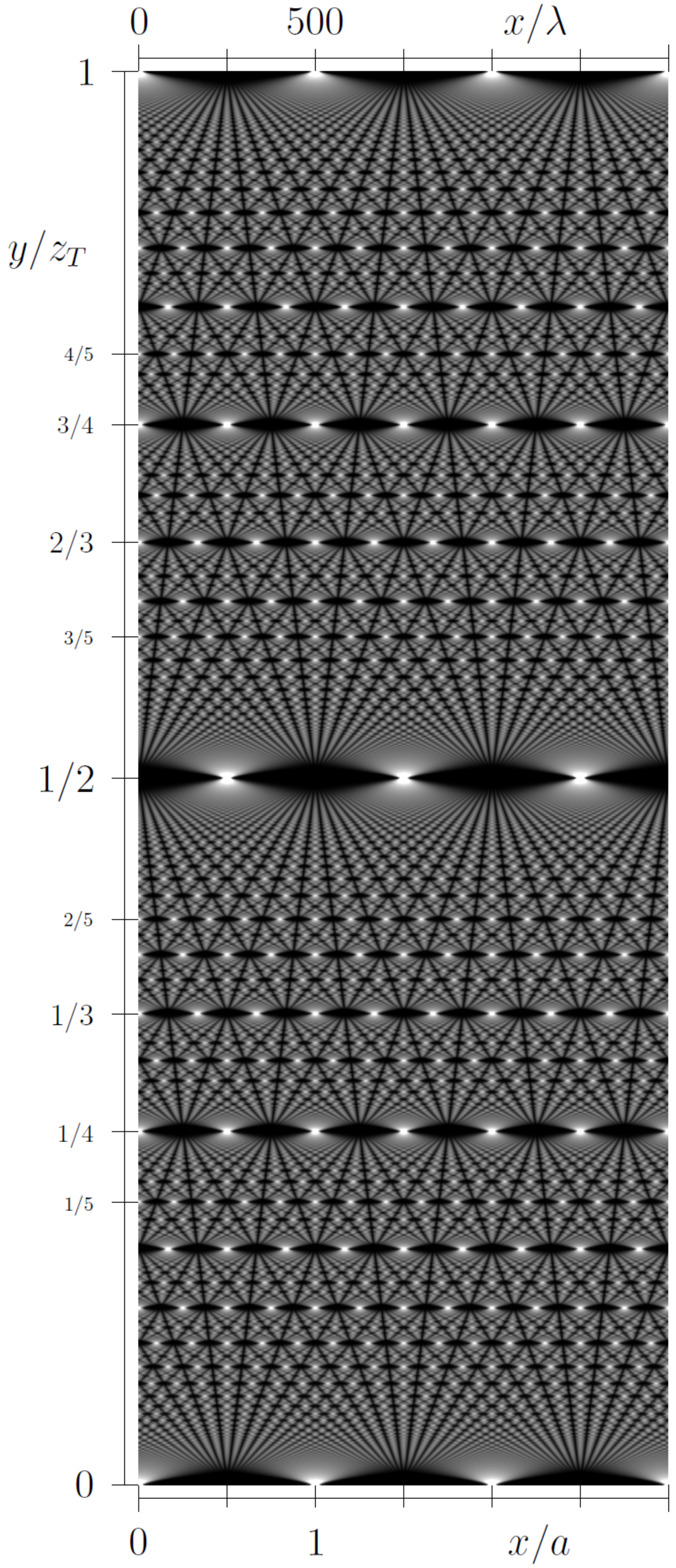
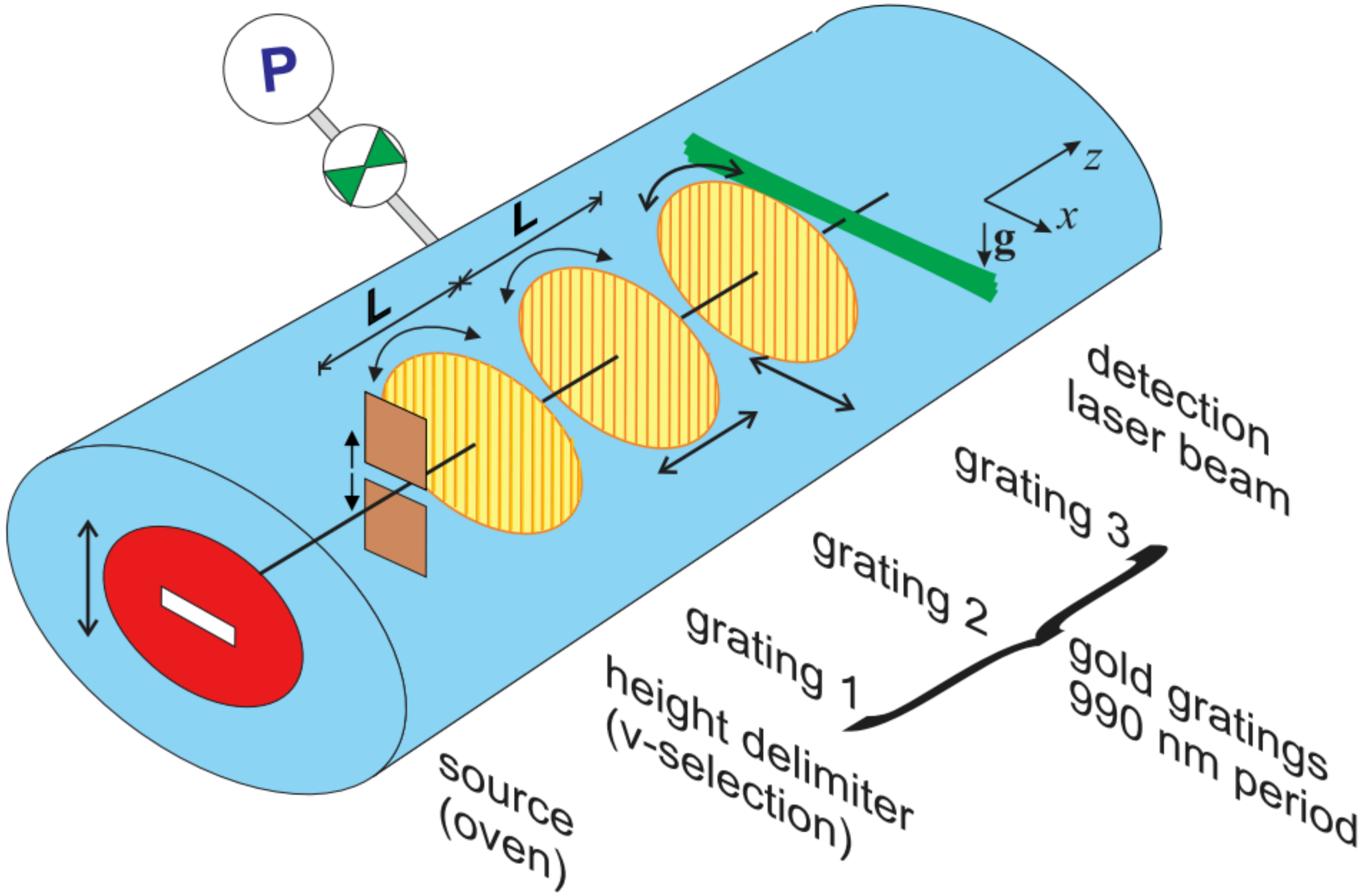
"The loss of visibility in the interference pattern due to the coupling of a quantum system to its environment."

Appl. Phys. B 77 (2003) 781-787, Zeilinger, et al.

Superpositions tend to decohere

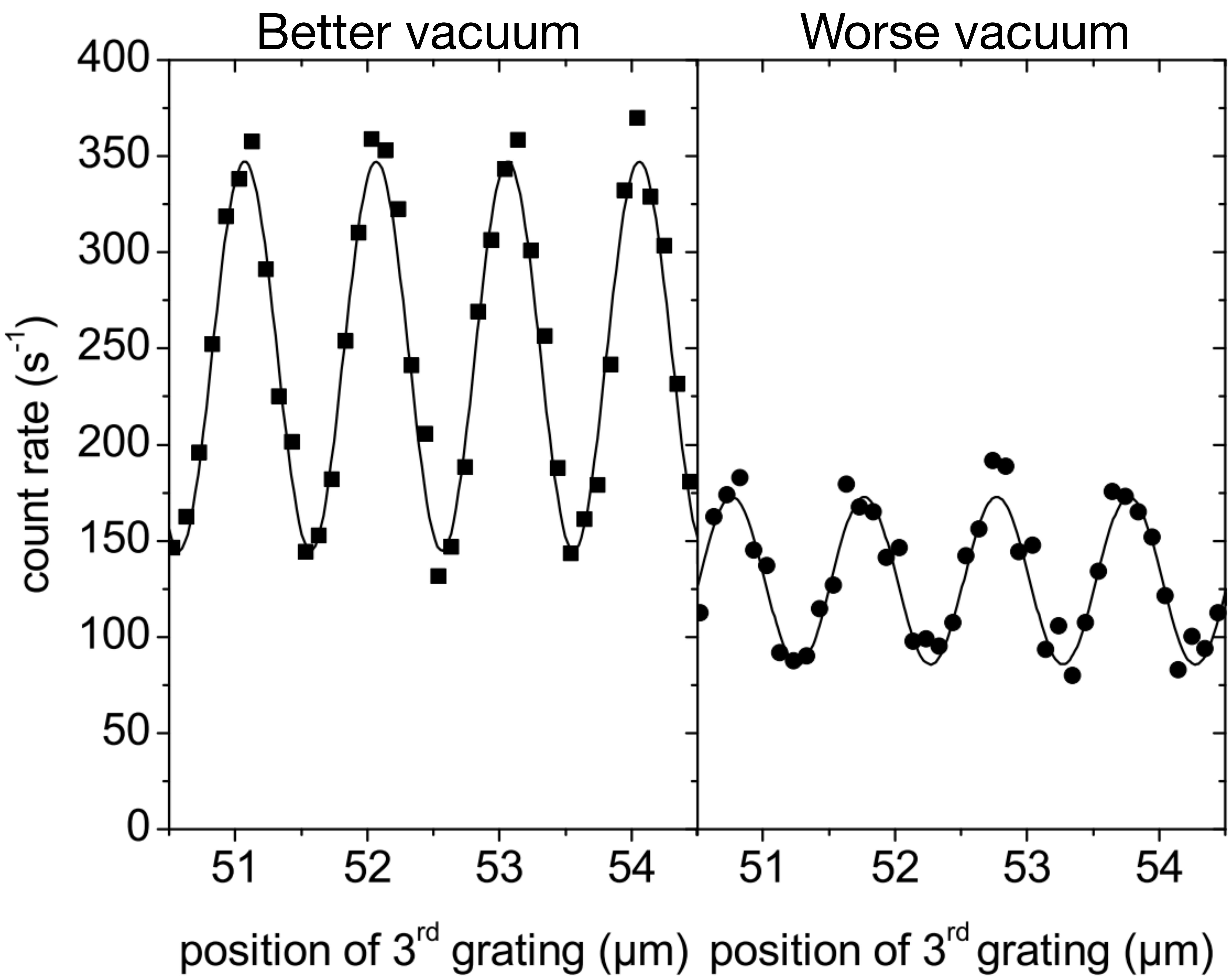


Wikipedia, The Talbot effect

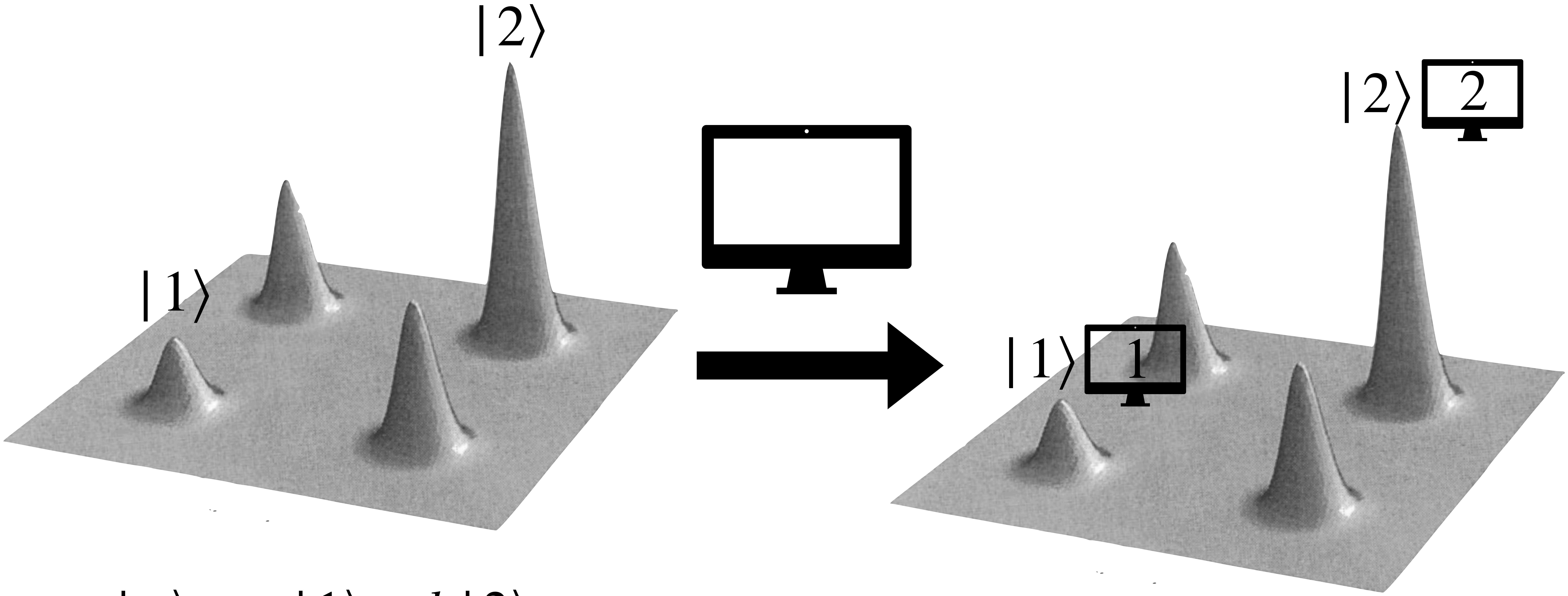


Appl. Phys. B 77 (2003) 781-787, Zeilinger, et al.

Superpositions tend to decohere

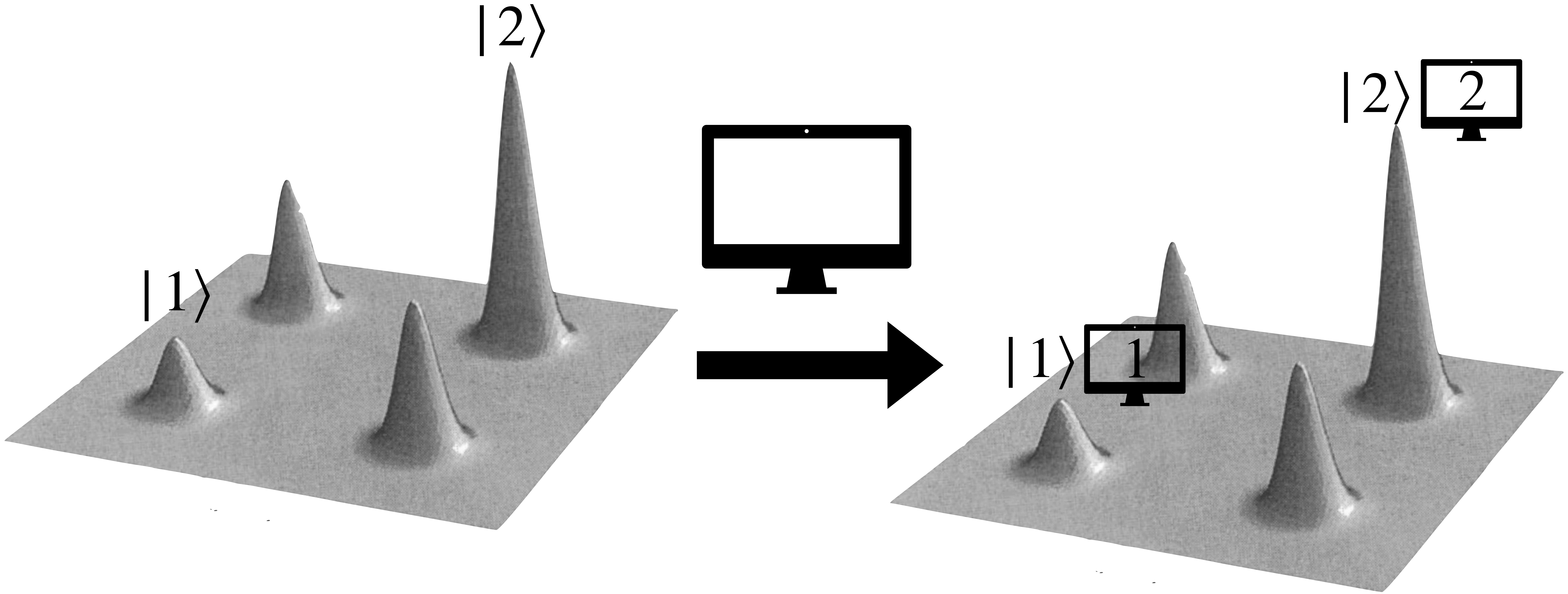


Decoherence mechanism



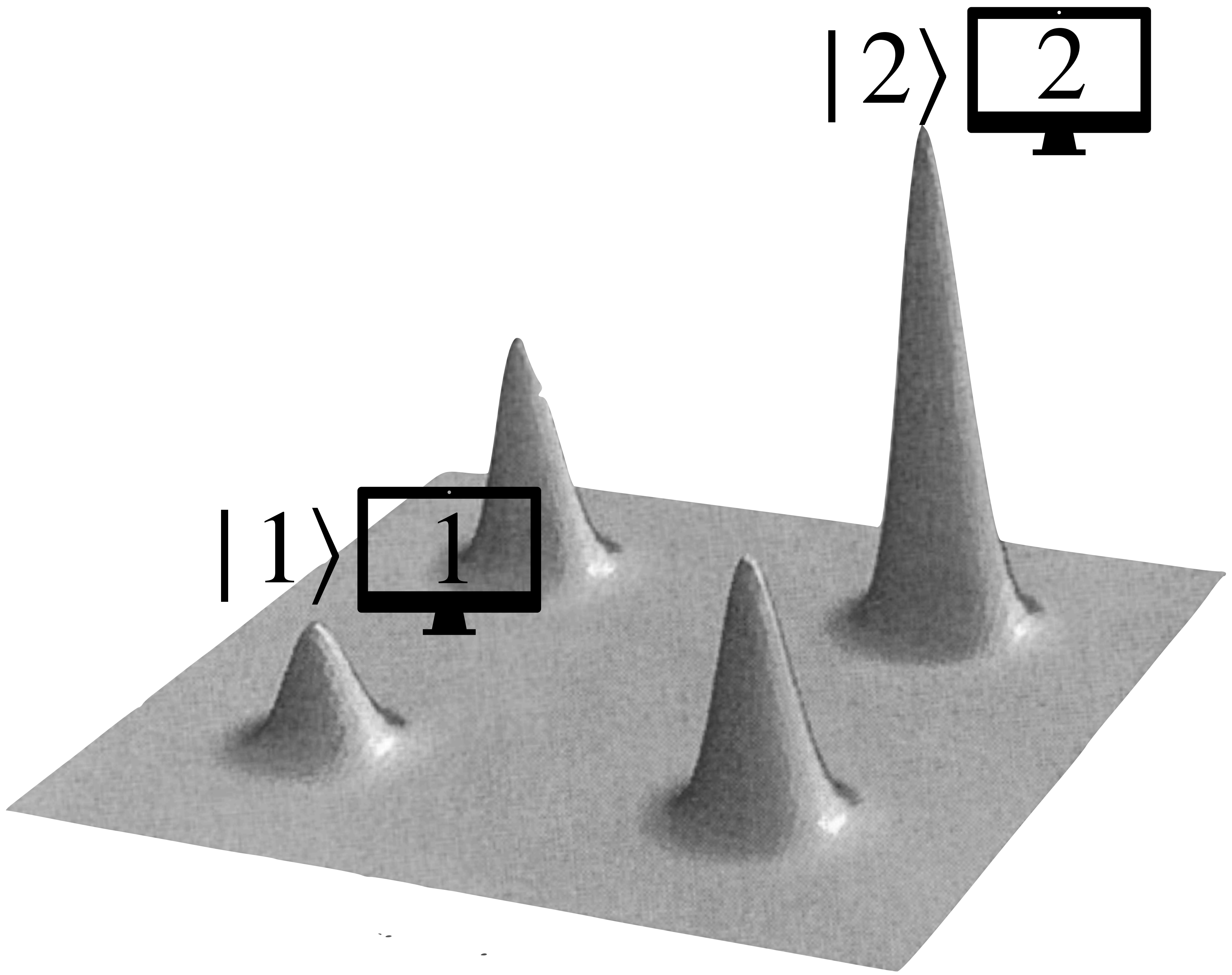
$$|\psi\rangle = a|1\rangle + b|2\rangle$$

Decoherence mechanism



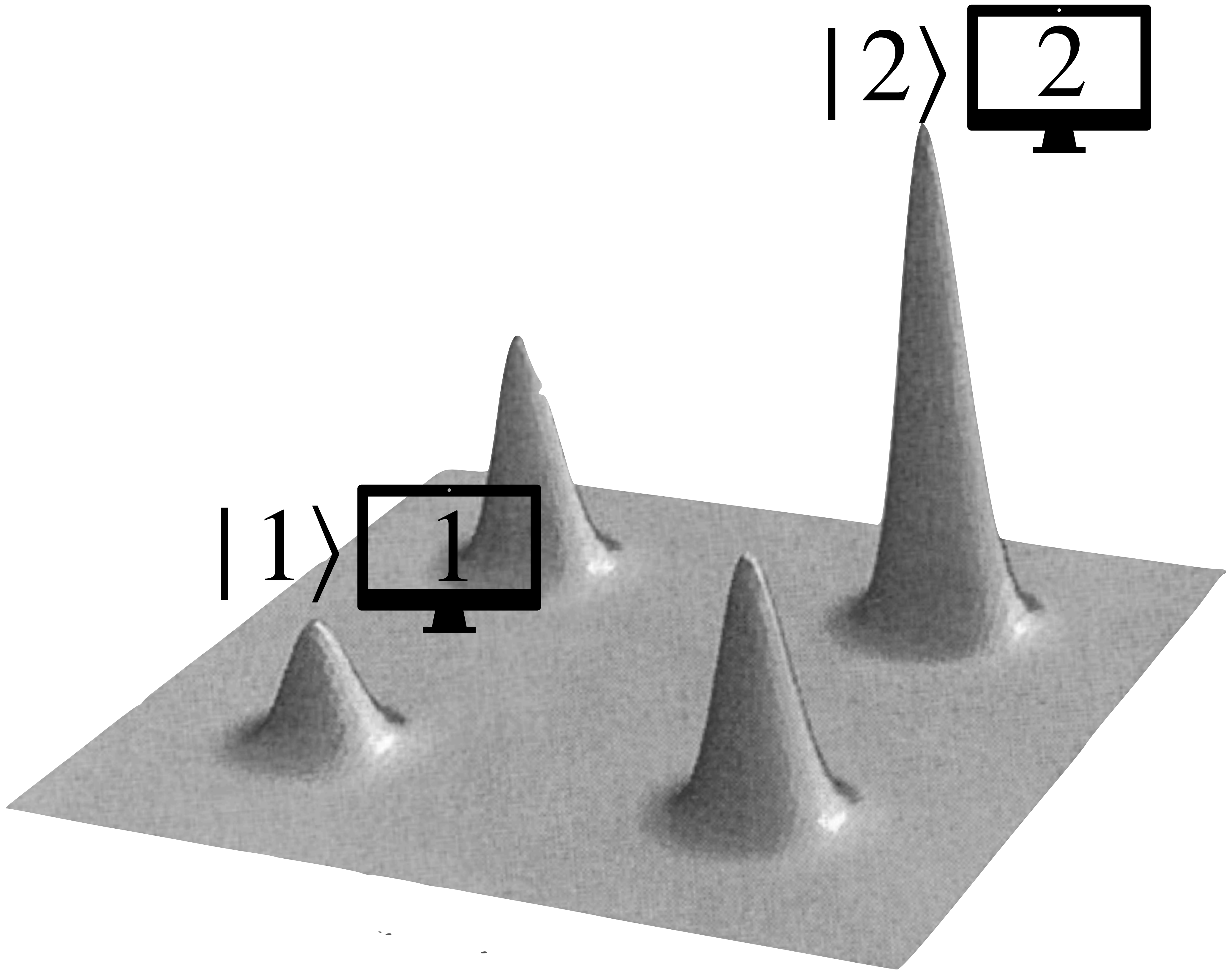
$$|\psi\rangle \otimes |B_0\rangle \rightarrow a\,|1\rangle\,|B_1\rangle + b\,|2\rangle\,|B_2\rangle$$

Decoherence is everywhere



$$|\langle\phi|\psi\rangle|^2 = |a|^2 |\langle\phi|1\rangle|^2$$

Decoherence is everywhere



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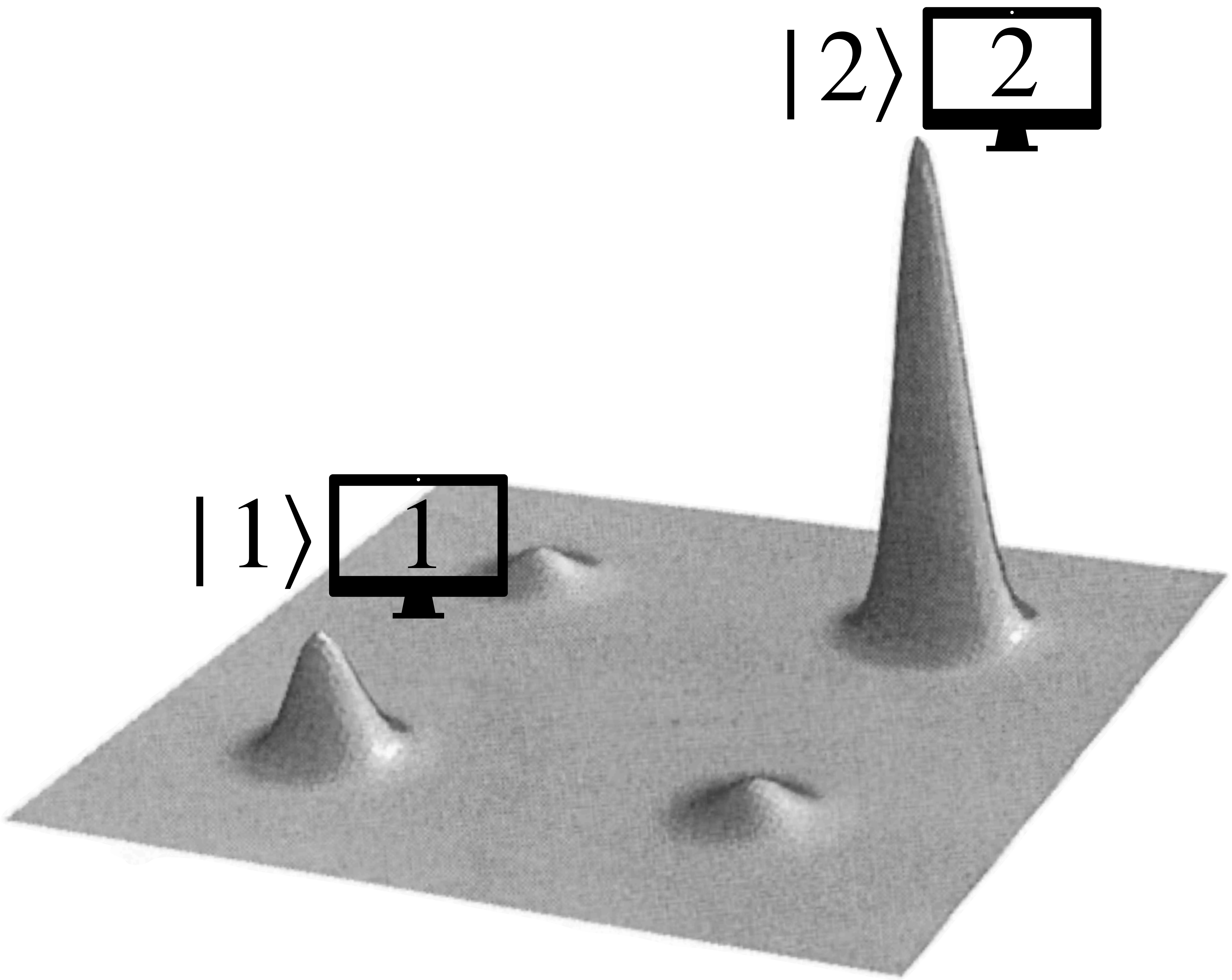
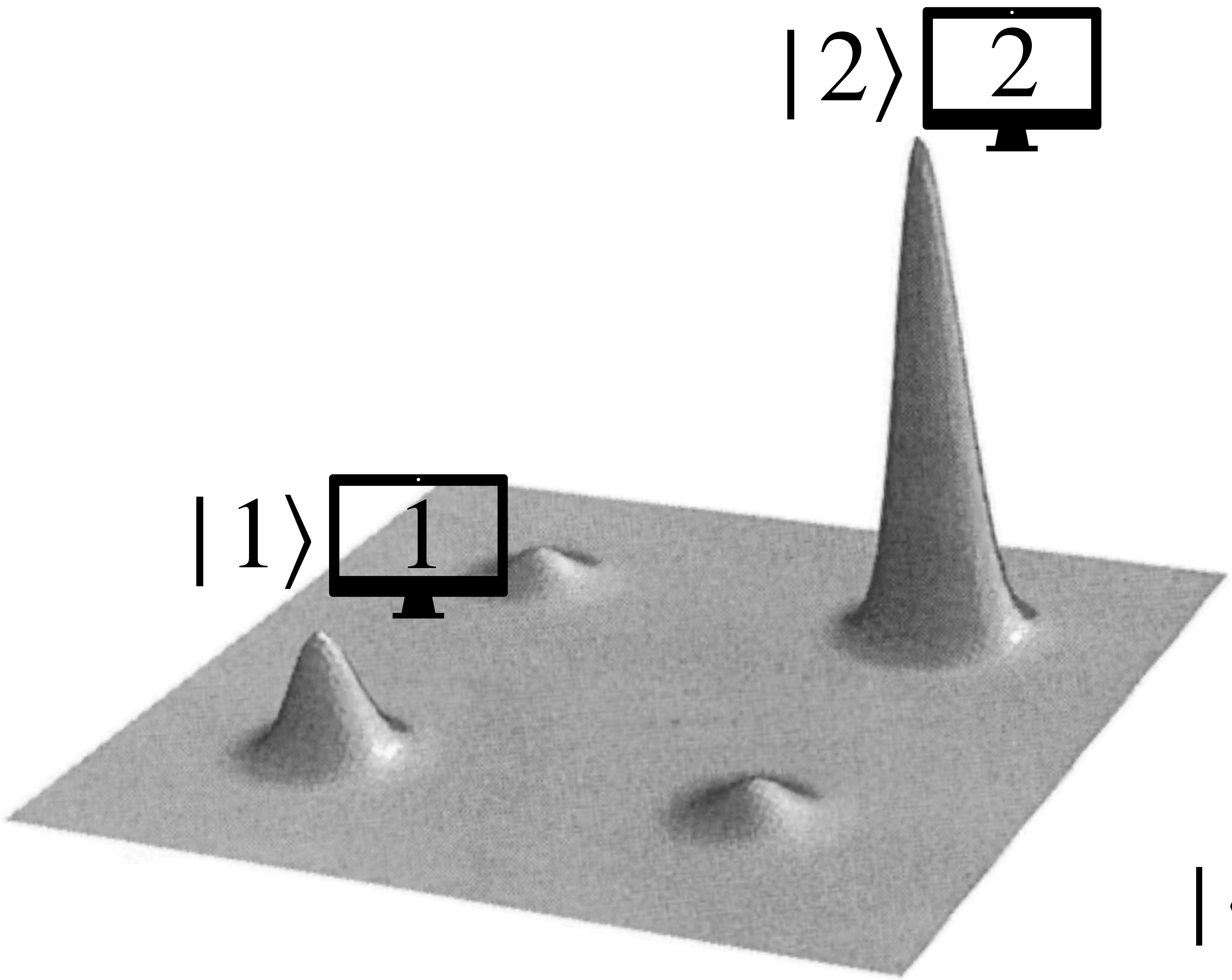
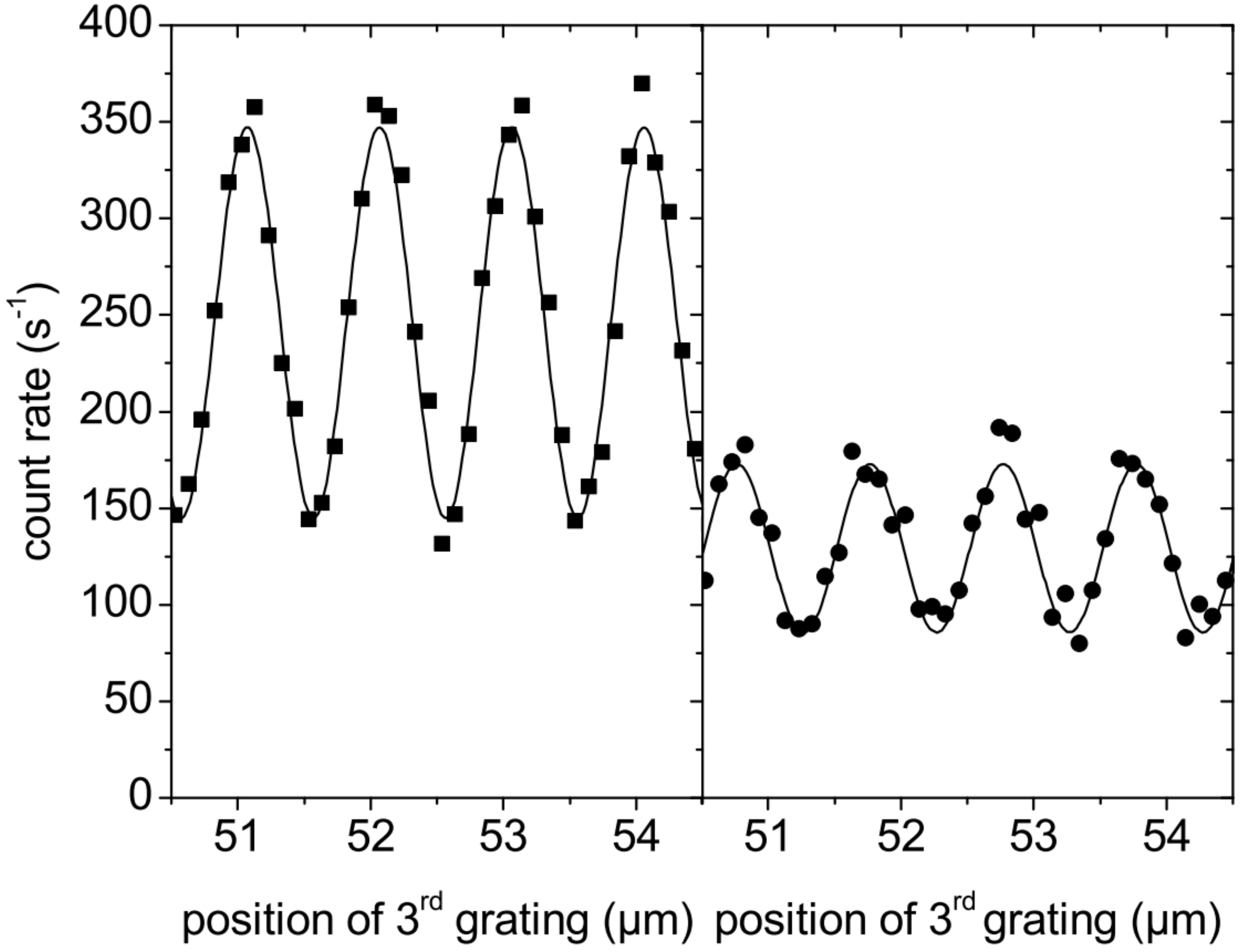


$$|\langle \phi | \psi \rangle|^2 = |a|^2 |\langle \phi | 1 \rangle|^2 + |b|^2 |\langle \phi | 2 \rangle|^2 + 2\mathfrak{R}\left[ab^* \langle \phi | 1 \rangle (\langle \phi | 2 \rangle)^* \langle B_2 | B_1 \rangle\right]$$

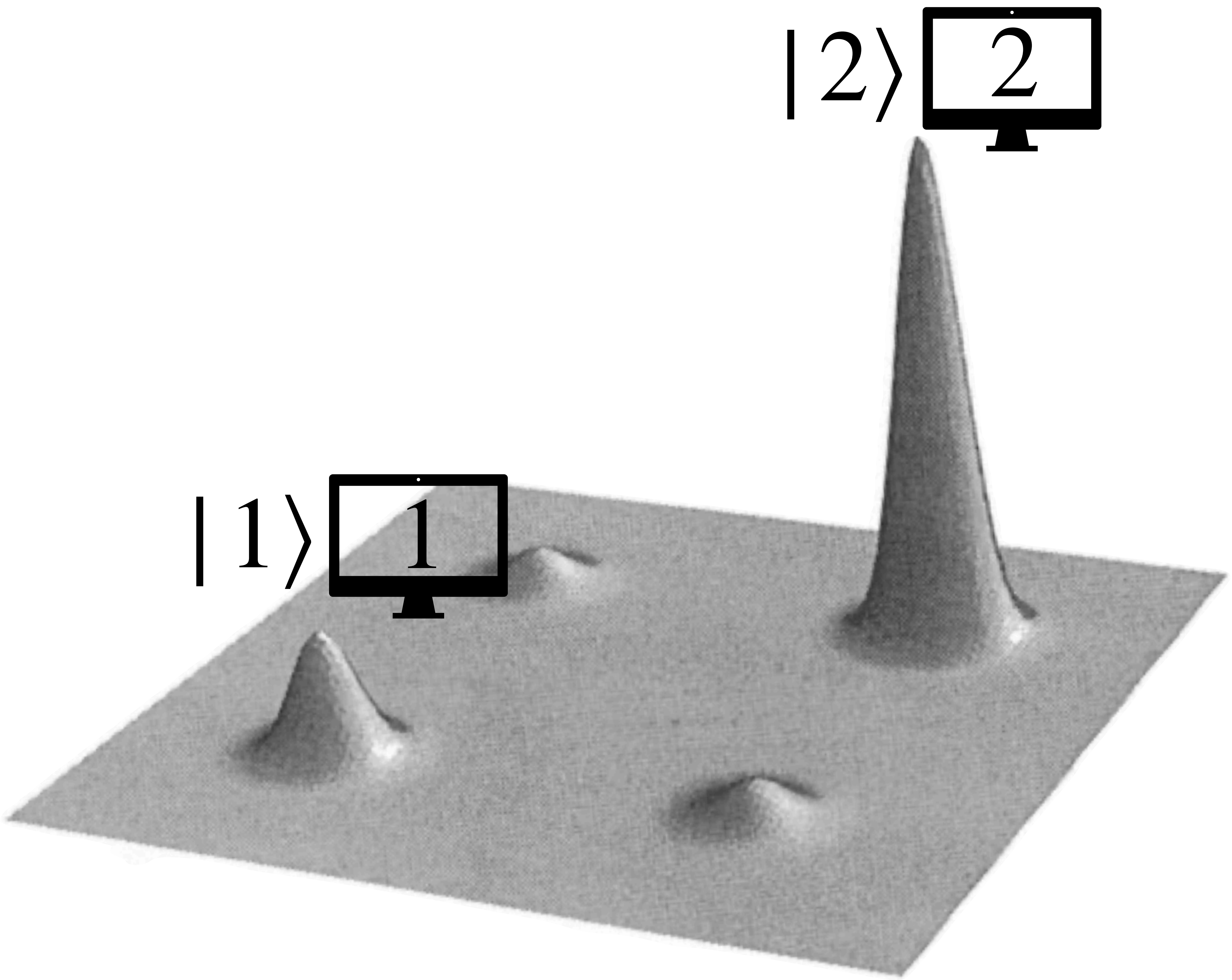
Decoherence is everywhere

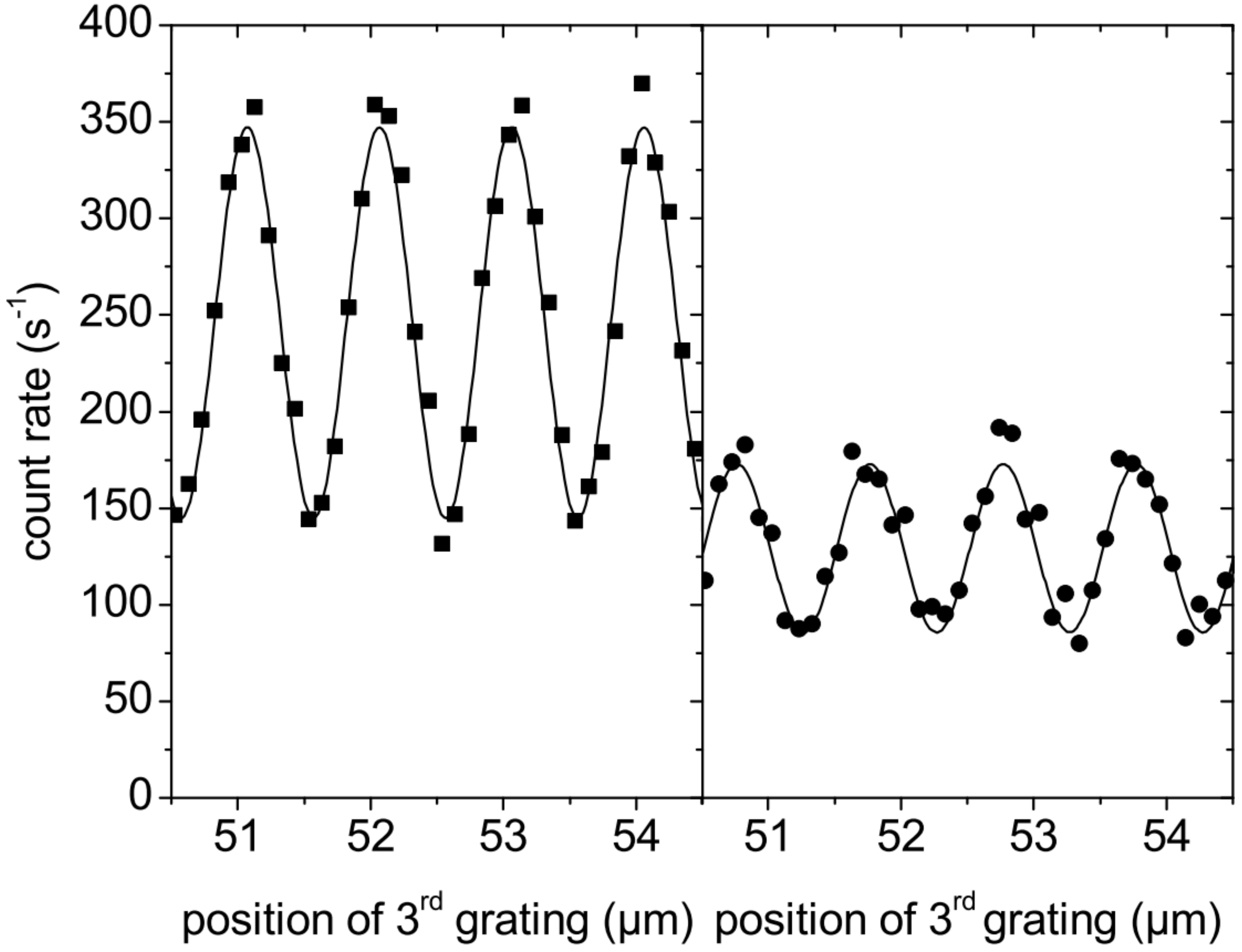




$$|\langle\phi|\psi\rangle|^2 = |a|^2|\langle\phi|1\rangle|^2$$
$$+|b|^2|\langle\phi|2\rangle|^2$$
$$+2\Re\left[ab^*\langle\phi|1\rangle(\langle\phi|2\rangle)^*\langle B_2|B_1\rangle\right]$$

Decoherence is everywhere





- To keep superpositions coherent in general require very careful preparation and controlled conditions.
- It is hard to maintain quantum superpositions

Nature knows better

Imagine preparing a quantum state
in a superposition in Chicago and
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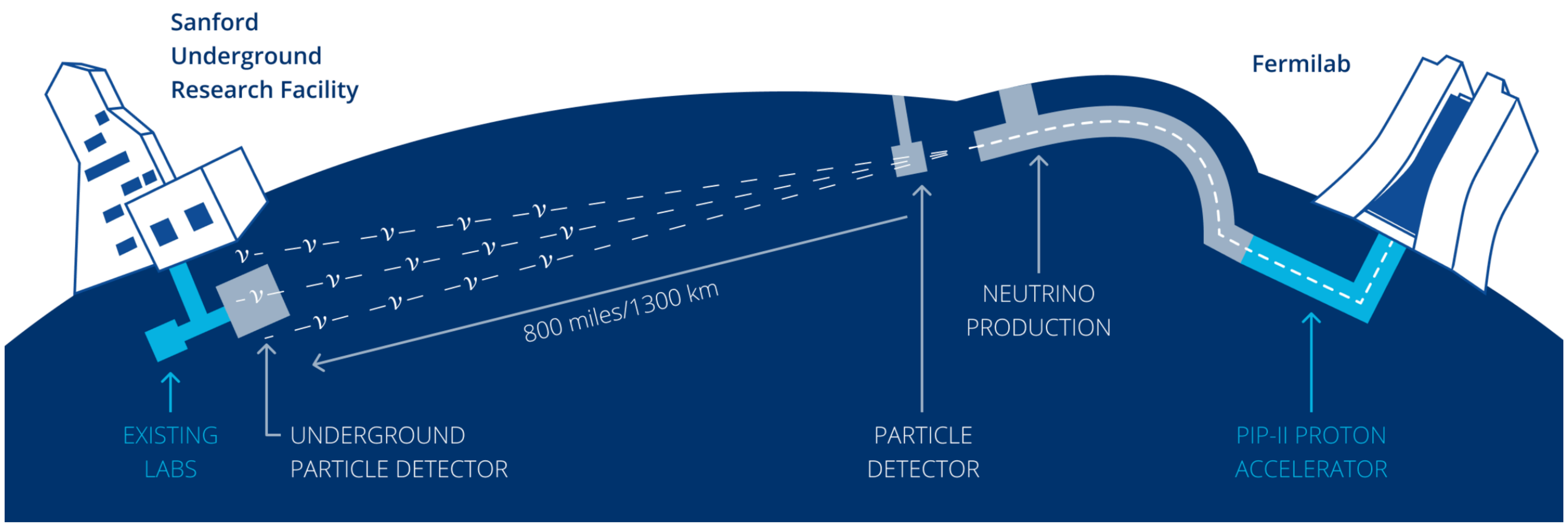
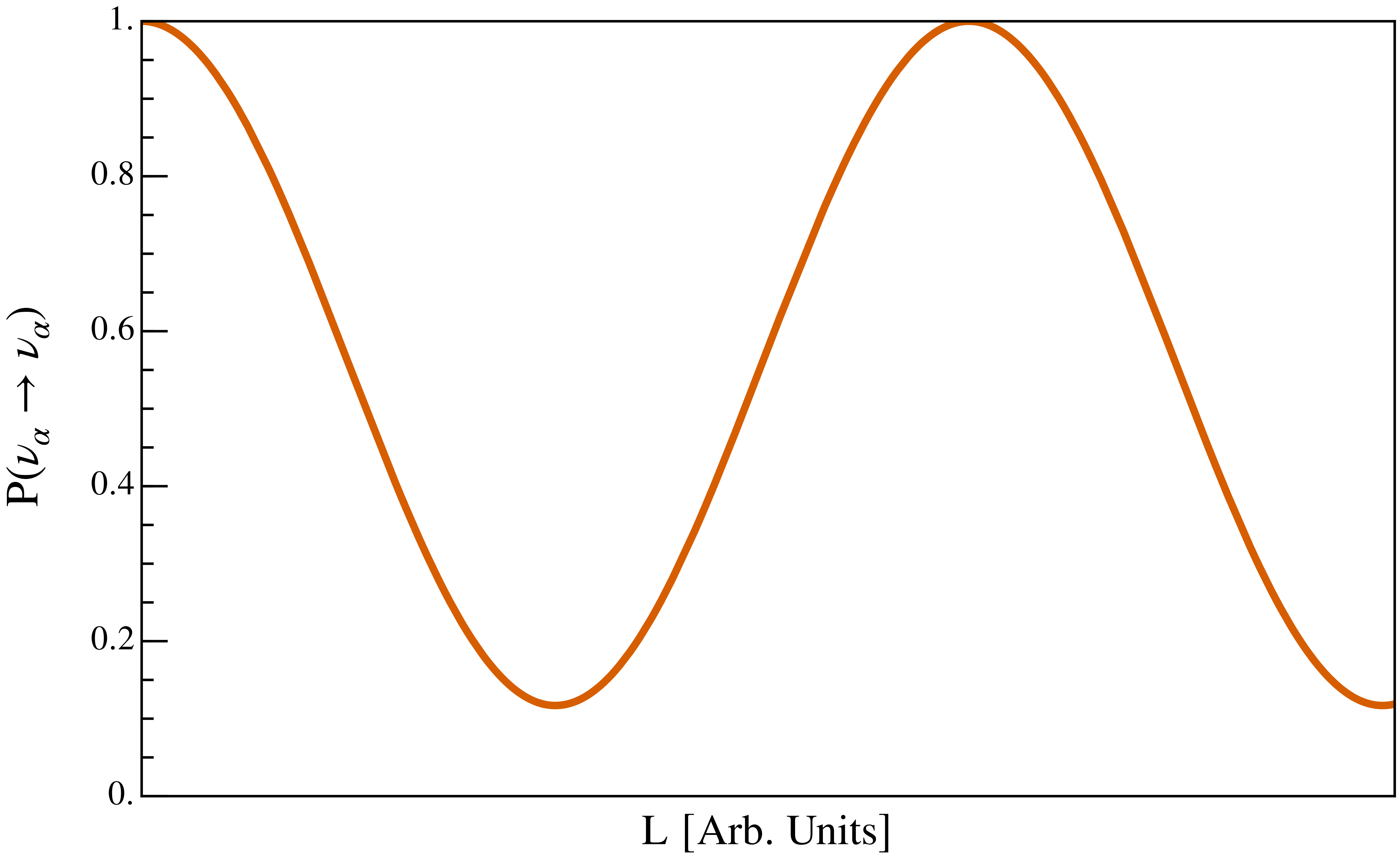
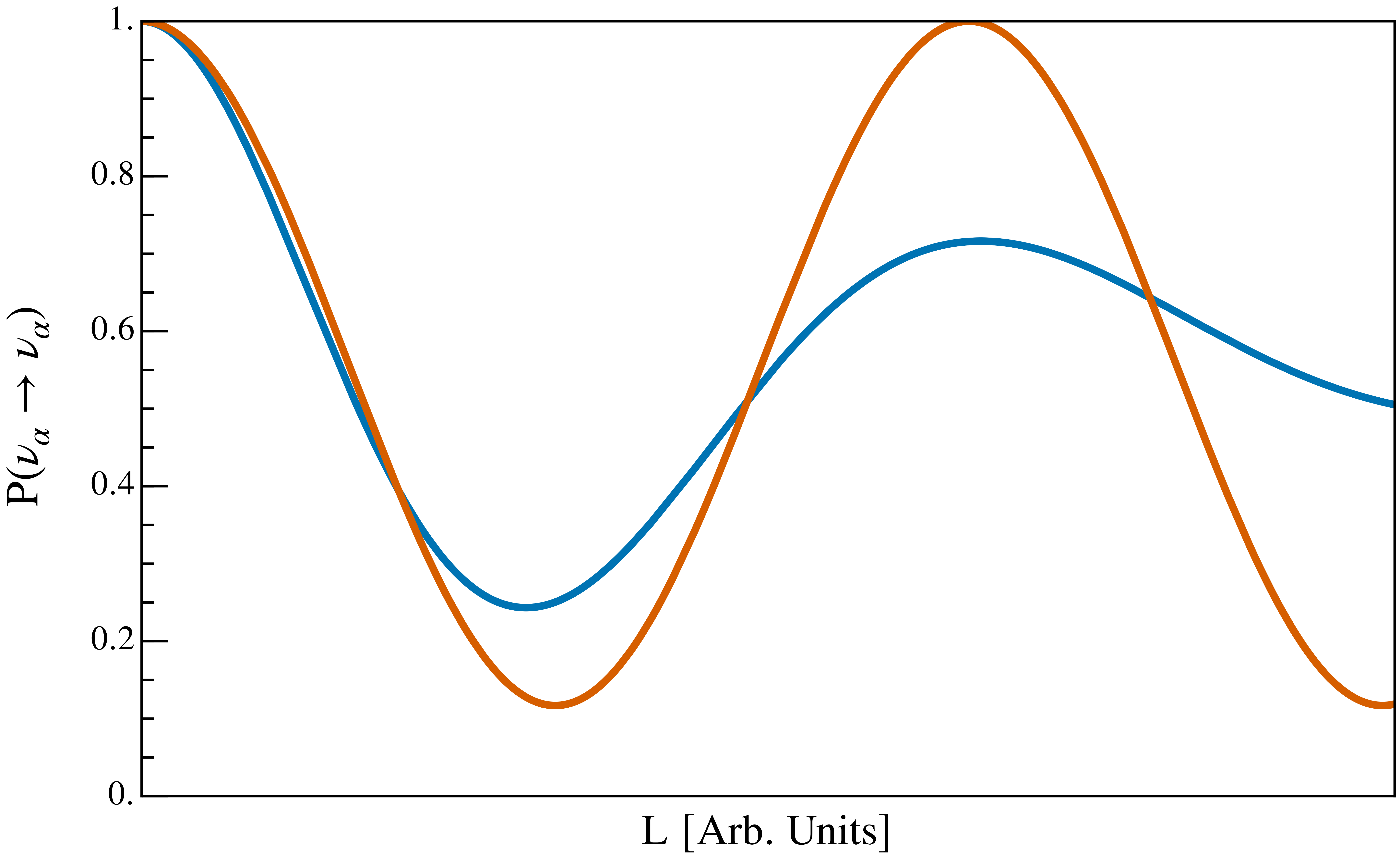


Neutrinos oscillations are the “free lunch” of quantum interference

"Visibility" = oscillation pattern

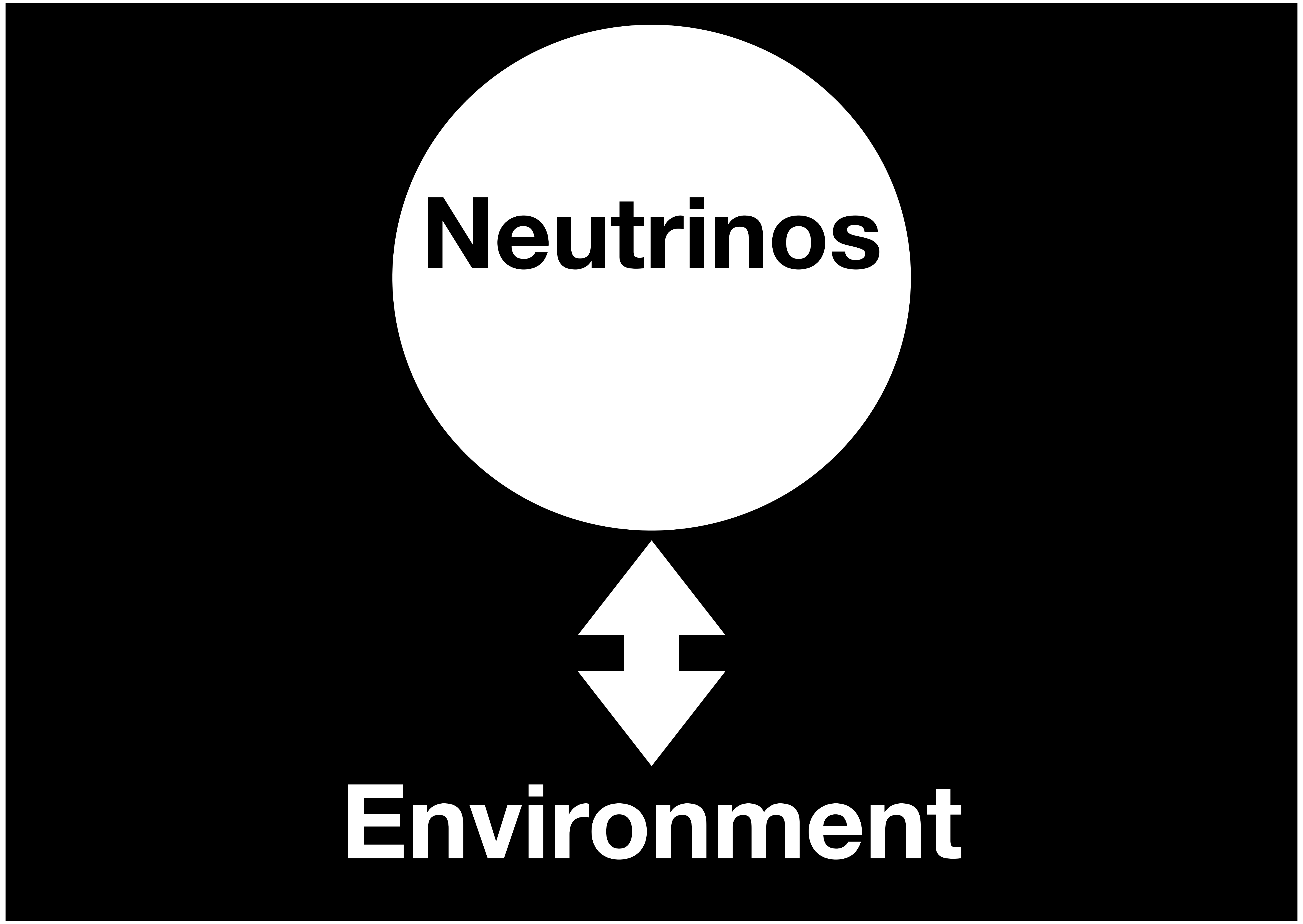


…but what if?



Decoherence in neutrino oscillations?

The framework



"Agnostic way to look for new effects"

Outline

Neutrino oscillations are a quantum interference phenomenon

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Decoherence dampens interference patterns

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Neutrino oscillations are a quantum interference phenomenon

Decoherence dampens interference patterns

What can we learn from decoherence in neutrino oscillations, and how?

Open systems

Review of density operators

Review of density operators

The density operator satisfies

$$\rho = \rho^\dagger \quad \mathrm{Tr}(\rho) = 1 \quad \rho \geq 0$$

Given a ket $|\psi\rangle$ the corresponding density operator is

$$\rho = |\psi\rangle\langle\psi|$$

Review of density operators

Time evolution

$$\partial_t \rho(t) = -i[H, \rho(t)]$$

$$\rho(t) = U(t)\rho(0)U^\dagger(t)$$

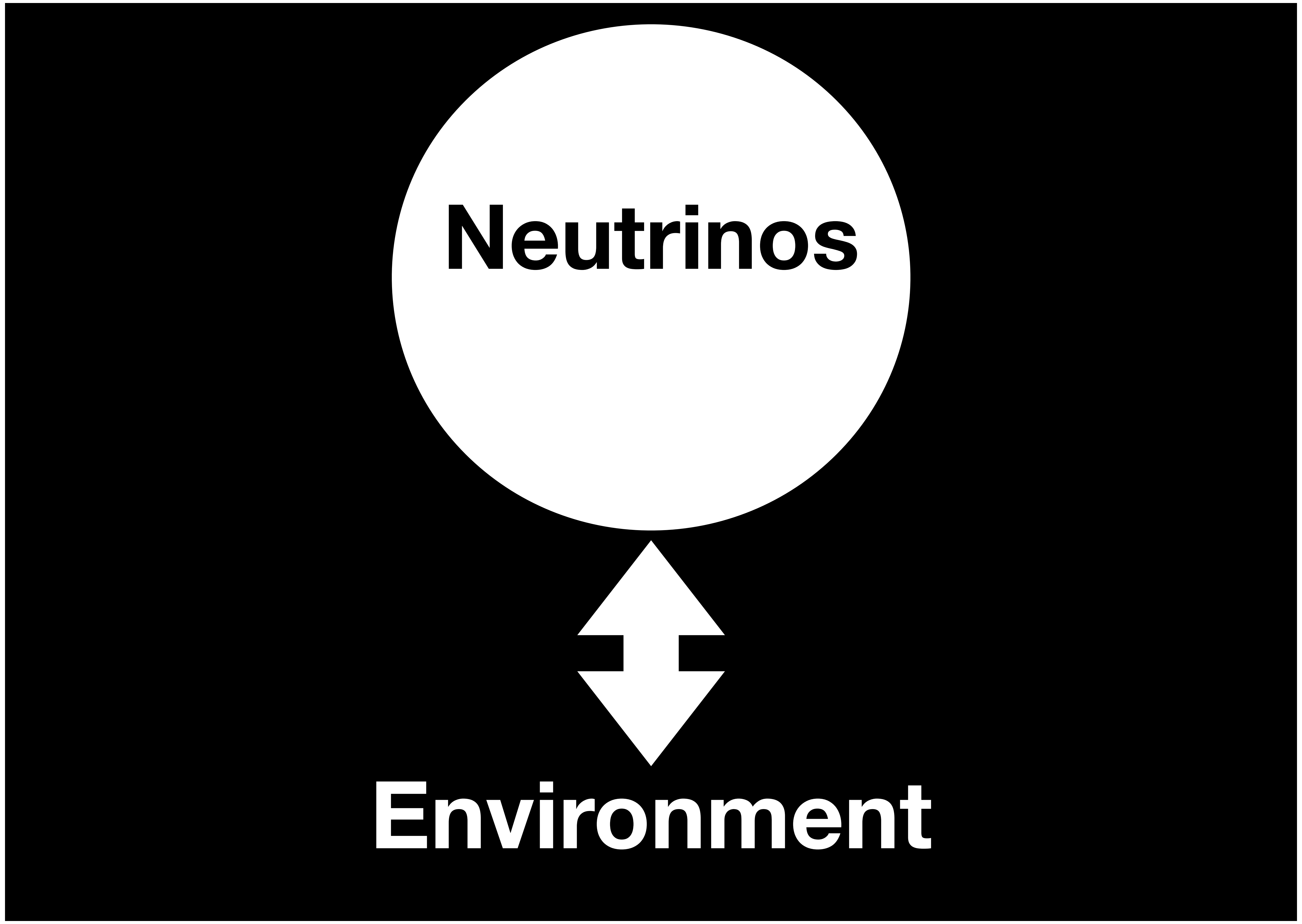
Review of density operators

Time evolved neutrino state

$$|\nu_e(t)\rangle = \cos\theta e^{-iE_1 t}|\nu_1\rangle + \sin\theta e^{-iE_2 t}|\nu_2\rangle$$

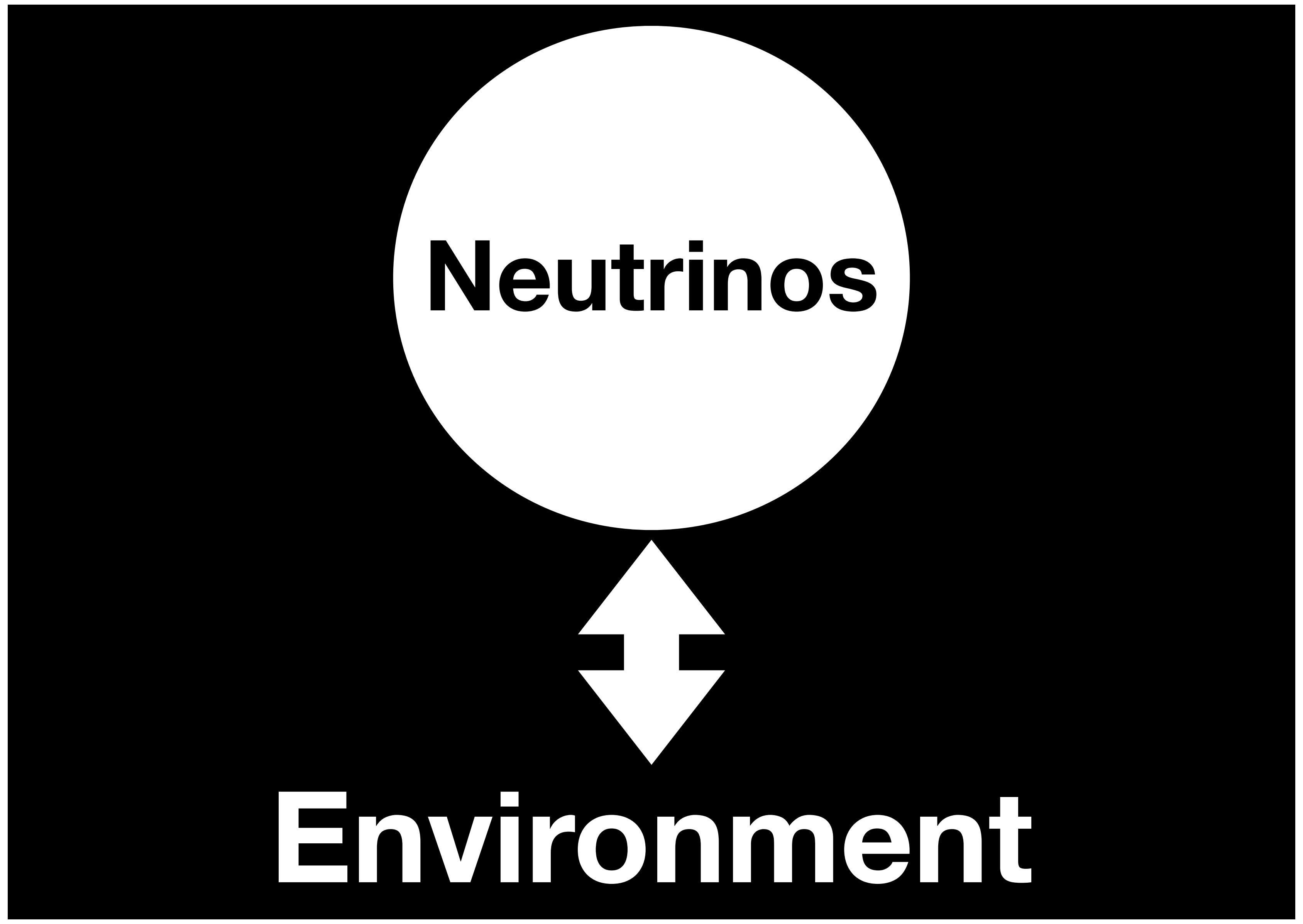
$$\rho_e(t) = \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta e^{-i\Delta E_{12}t} \\ \cos\theta\sin\theta e^{i\Delta E_{12}t} & \sin^2\theta \end{pmatrix}$$

Open system time evolution



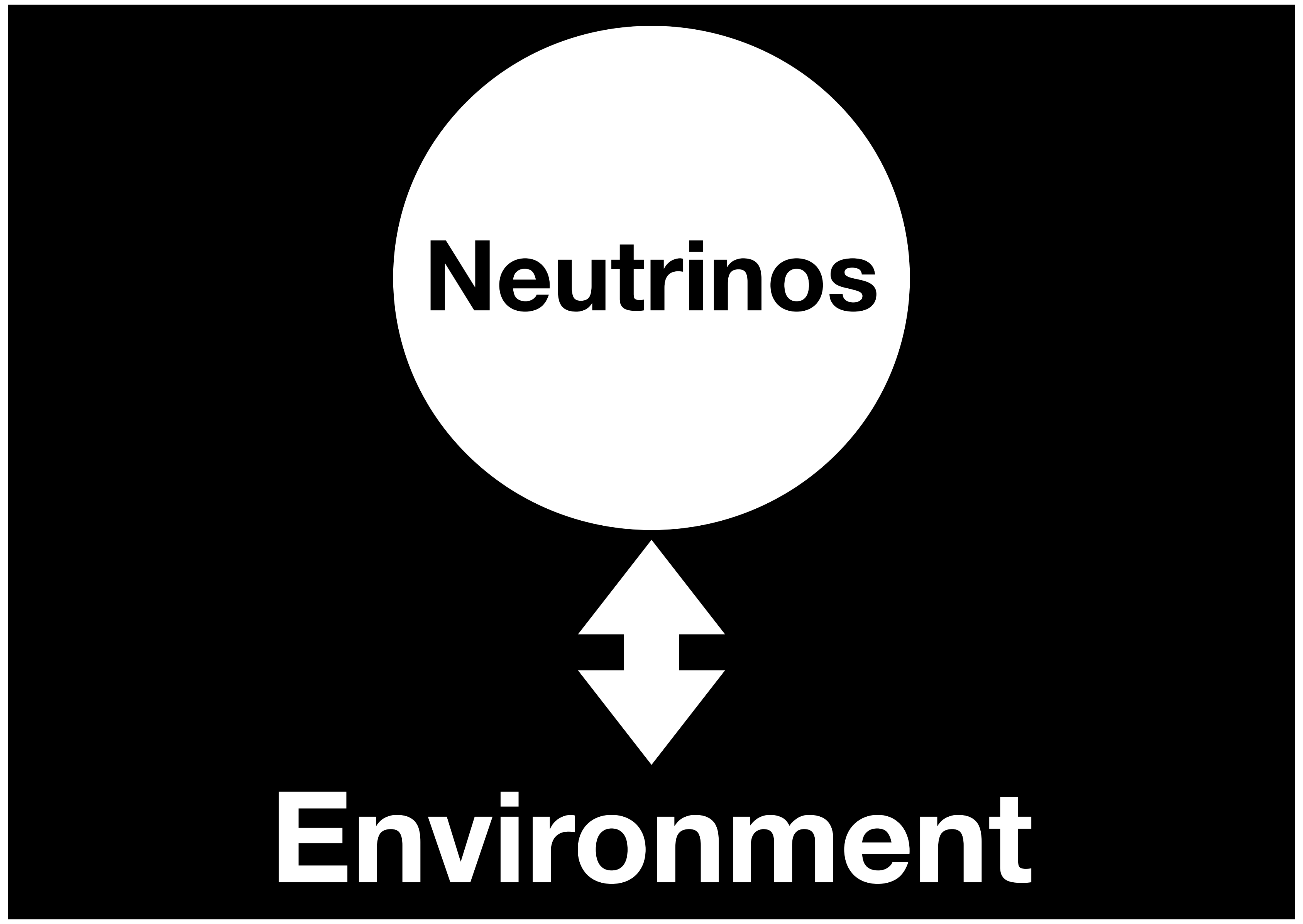
The framework

$$\rho_{\text{tot}}(t_0) = \rho_\nu(t_0) \otimes \rho_B(t_0)$$



The framework

$$\rho_{\rm tot}(t_0) = \rho_\nu(t_0) \otimes \rho_B(t_0) \xrightarrow{\text{Unitary evolution}} \rho_{\rm tot}(t) = U(t,t_0)[\rho_\nu(t_0) \otimes \rho_B(t_0)]U^\dagger(t,t_0)$$

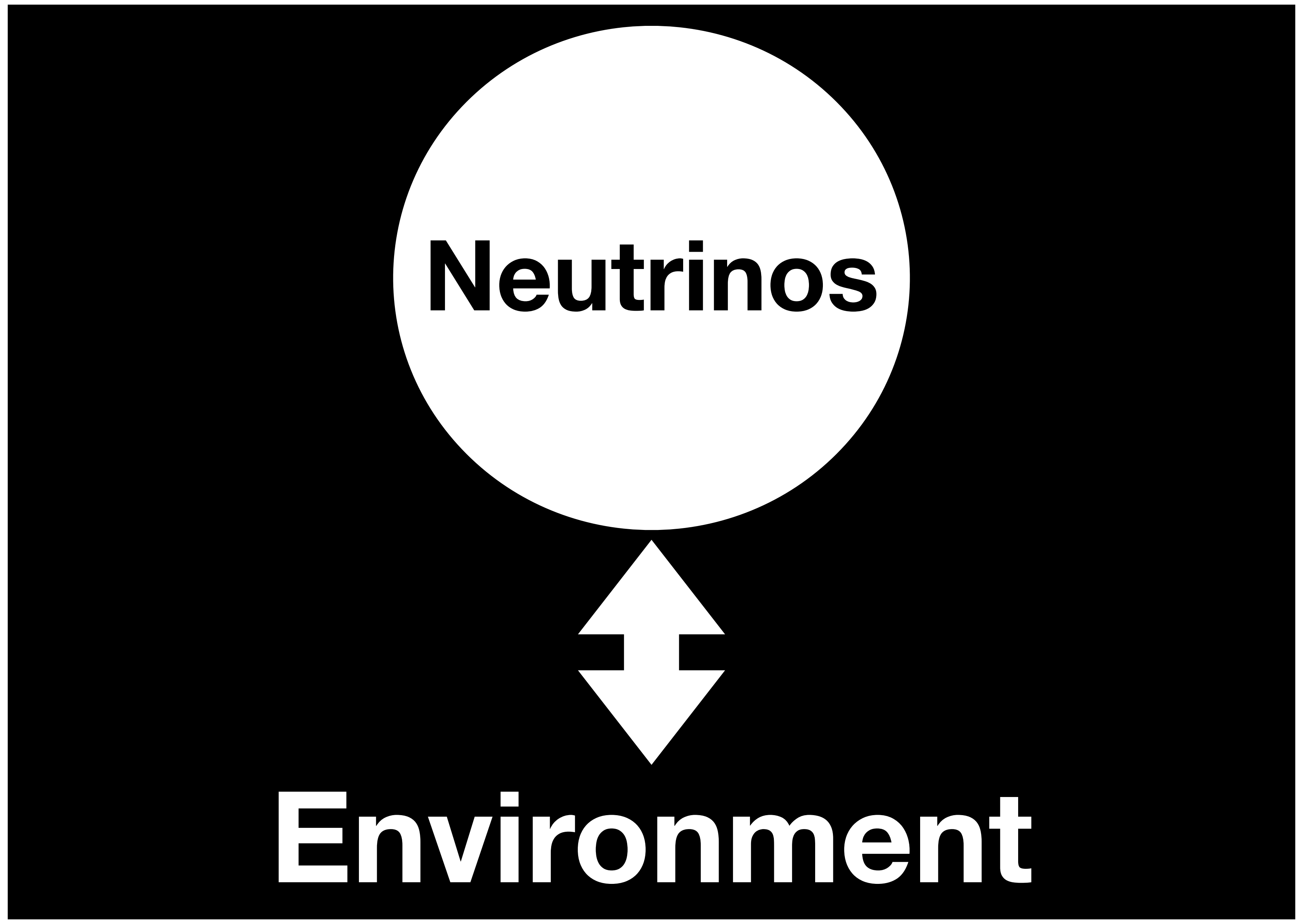


The framework

$$\rho_{\text{tot}}(t_0) = \rho_\nu(t_0) \otimes \rho_B(t_0) \longrightarrow \rho_{\text{tot}}(t) = U(t, t_0)[\rho_\nu(t_0) \otimes \rho_B(t_0)]U^\dagger(t, t_0)$$

Unitary evolution

Tr_B ↓



$$\rho_\nu(t_0)$$

The framework

$$\rho_{\mathrm{tot}}(t_0) = \rho_\nu(t_0) \otimes \rho_B(t_0)$$

Unitary evolution

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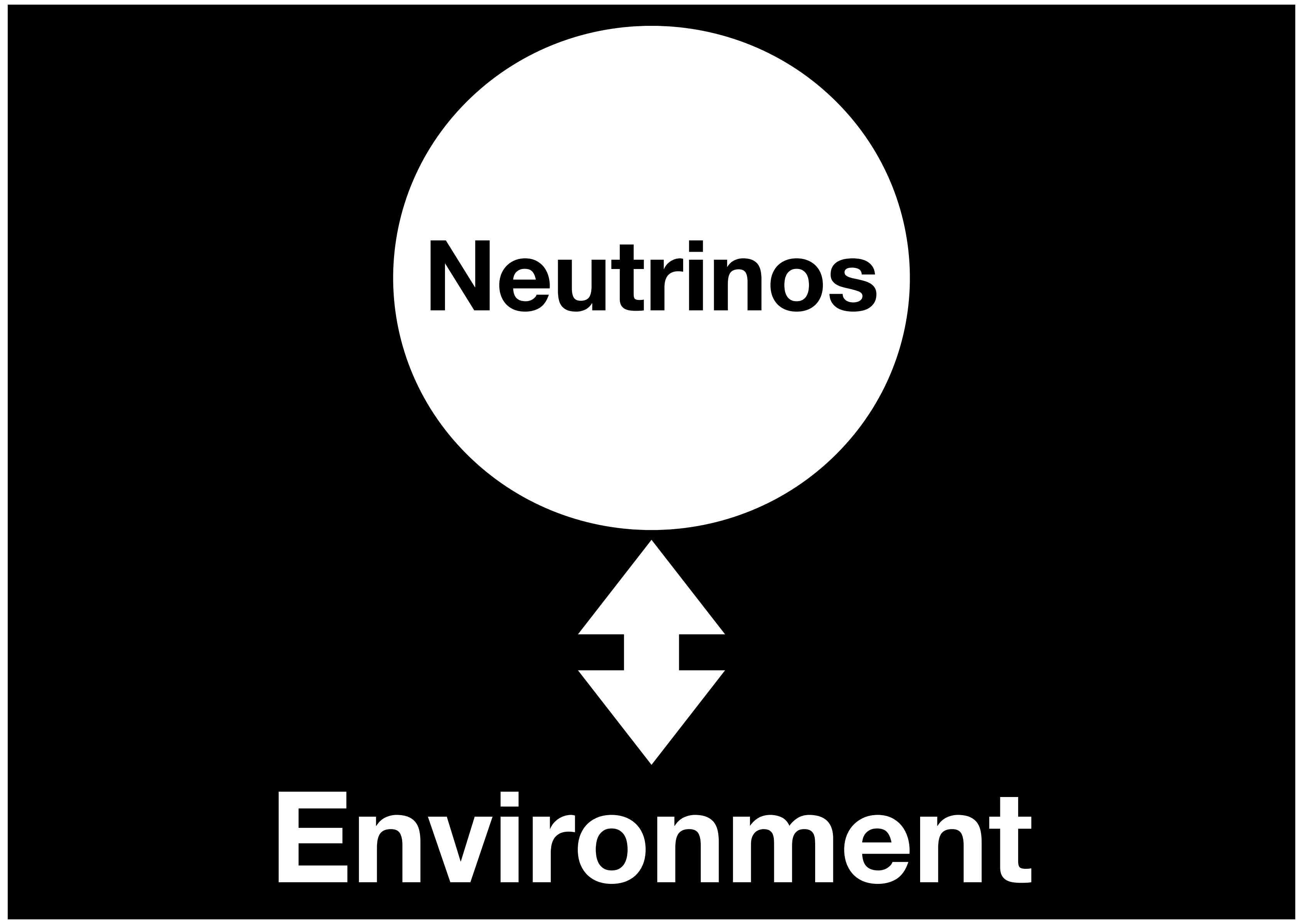
Tr_{B}



Tr_{B}

$$\rho_\nu(t_0)$$

Dynamical Map

$$\rho_\nu(t)$$

Open system time evolution

Final result: Lindblad equation

$$\partial_t \rho(t) = -i[H, \rho(t)]$$
$$-L^\dagger L \rho(t) - \rho(t) L^\dagger L + 2L\rho(t)L^\dagger$$

Open system time evolution

Final result: Lindblad equation

$$\partial_t \rho(t) = -i[H, \rho(t)] \\ -L^\dagger L \rho(t) - \rho(t) L^\dagger L + 2L\rho(t)L^\dagger$$

Conserves the trace and keeps eigenvalues positive

How has the framework been applied to neutrino physics?

Open system open possibilities

Article | Published: 26 March 2024

Search for decoherence from quantum gravity with atmospheric neutrinos

The IceCube Collaboration

Nature Physics **20**, 913–920 (2024) | Cite this article

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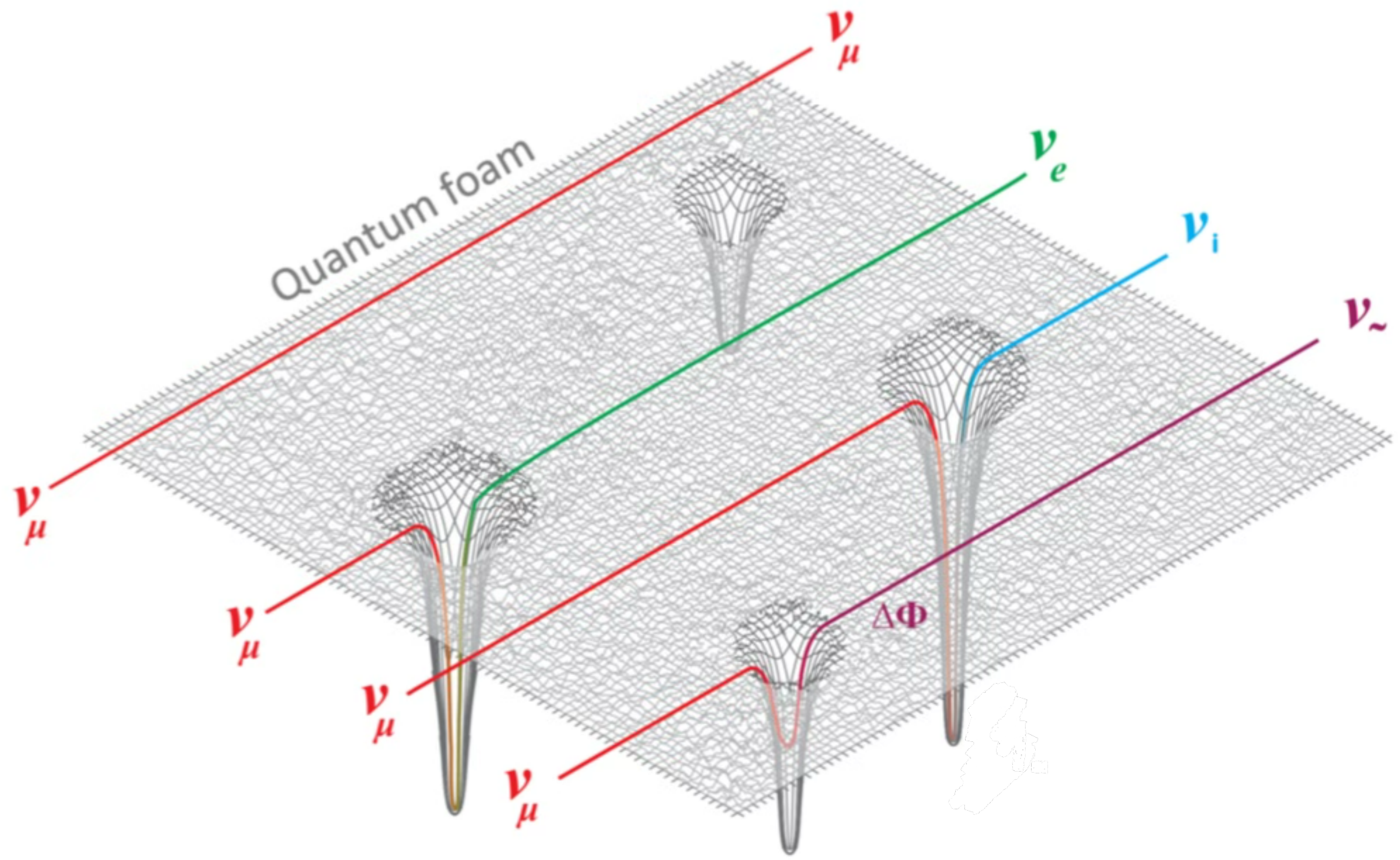
How the framework is applied to neutrino physics

Motivation:



From: The IceCube collaboration, Nature Physics 20, 913-920

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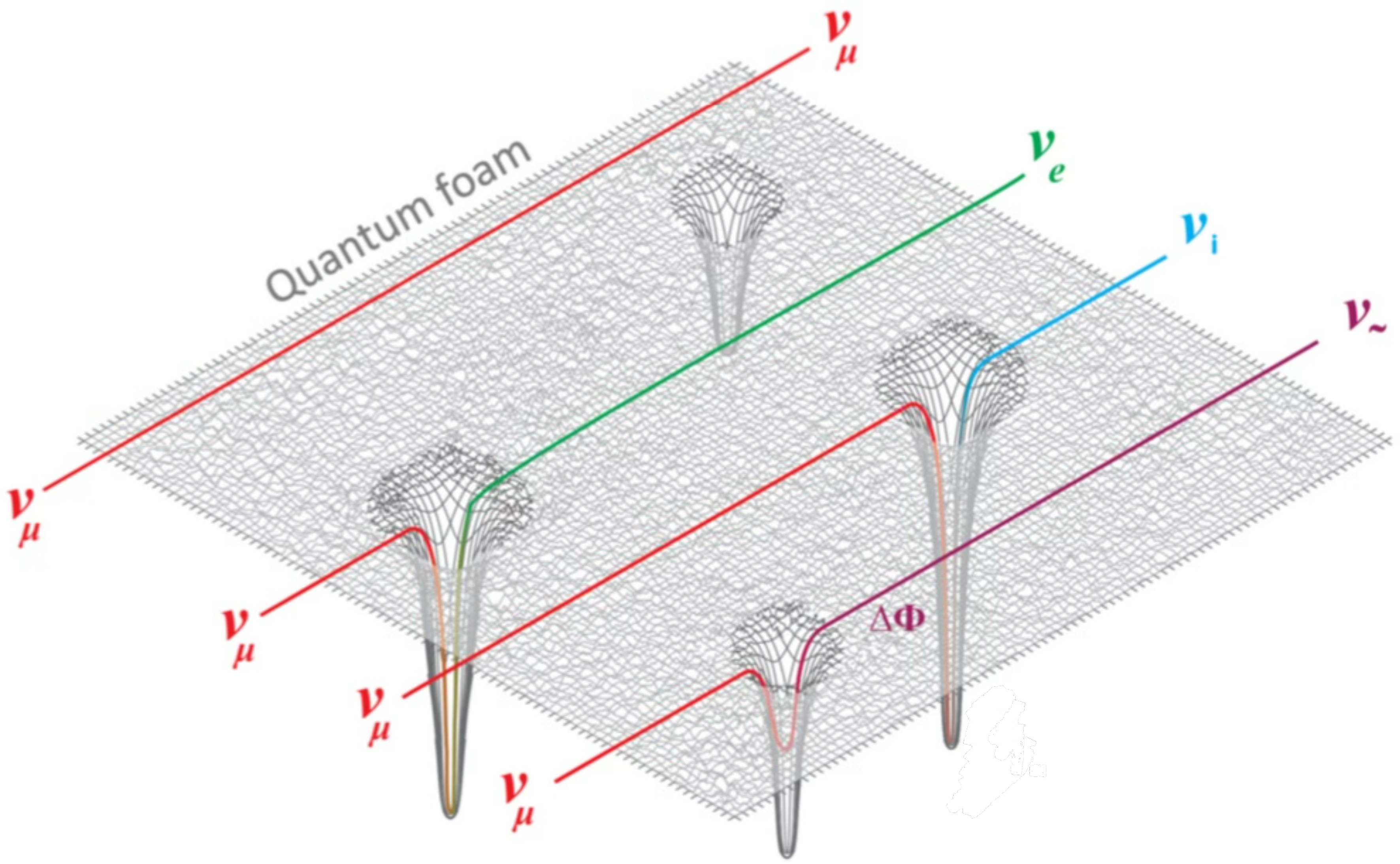

From: The IceCube collaboration, Nature Physics 20, 913-920

Method:

$$\dot{\rho} = -i[H, \rho] - \mathcal{D}[\rho]$$

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

How the framework is applied to neutrino physics

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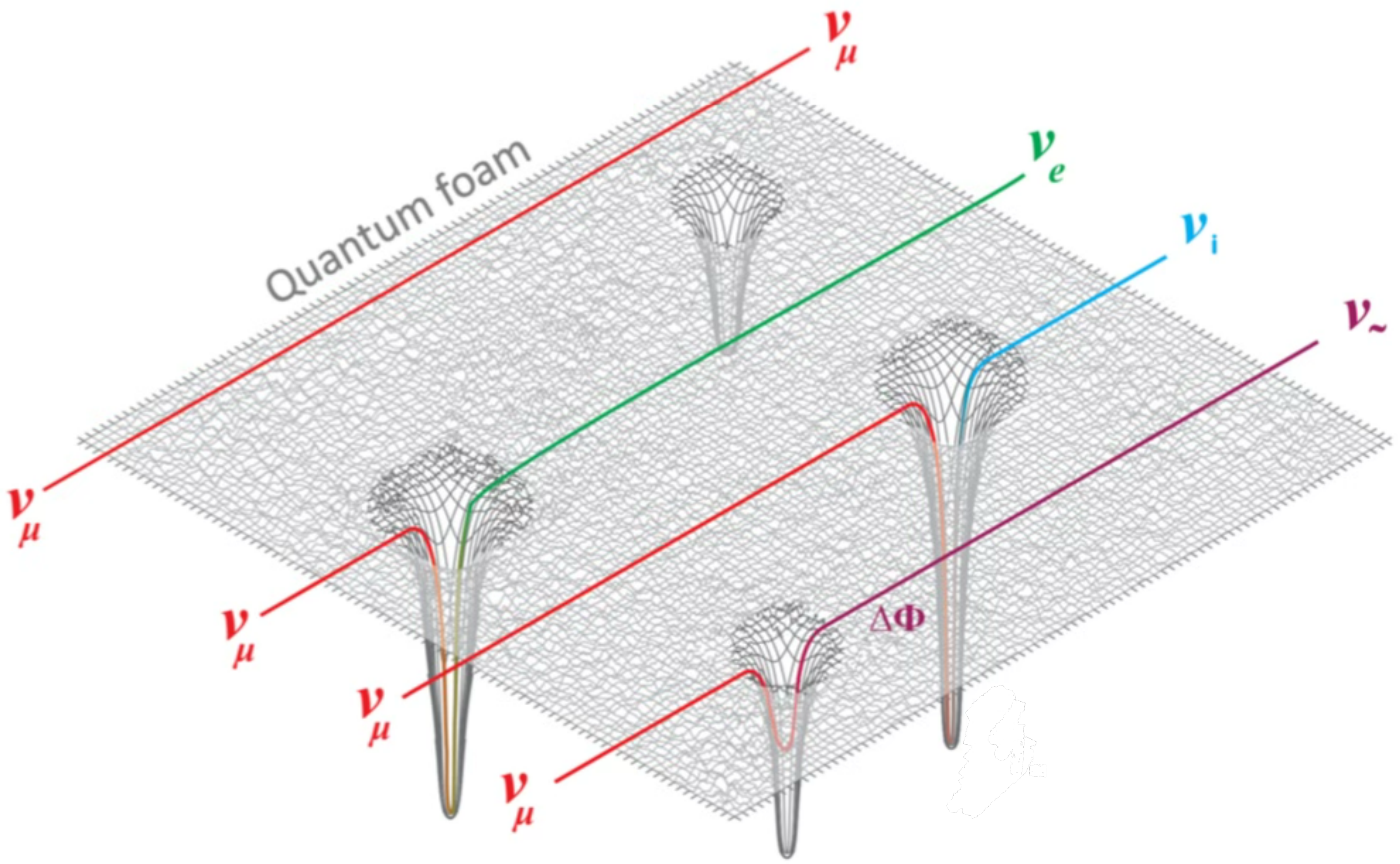

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Pheno:

$$\Gamma(E_\nu) = \Gamma_0 \left(\frac{E_\nu}{E_0} \right)^n$$

How the framework is applied to neutrino physics

Pheno:

$$\Gamma(E_\nu) = \Gamma_0 \left(\frac{E_\nu}{E_0}\right)^n$$

Scaling:

$$\rho_e(t) = \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta e^{-i\Delta E_{12}t - \Gamma L} \\ \cos\theta\sin\theta e^{i\Delta E_{12}t - \Gamma L} & \sin^2\theta \end{pmatrix}$$

2018

		$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$
Normal Ordering	**IceCube (this work)**					
	Atmospheric ($\gamma_{31}=\gamma_{32}$)	$2.8\cdot10^{-18}$	$4.2\cdot10^{-21}$	$\mathbf{4.0\cdot10^{-24}}$	$\mathbf{1.0\cdot10^{-27}}$	$\mathbf{1.0\cdot10^{-31}}$
	Solar I ($\gamma_{31}=\gamma_{21}$)	$6.8\cdot10^{-19}$	$1.2\cdot10^{-21}$	$\mathbf{1.3\cdot10^{-24}}$	$\mathbf{3.5\cdot10^{-28}}$	$\mathbf{1.9\cdot10^{-32}}$
	Solar II ($\gamma_{32}=\gamma_{21}$)	$5.2\cdot10^{-19}$	$9.2\cdot10^{-22}$	$9.7\cdot10^{-25}$	$2.4\cdot10^{-28}$	$9.0\cdot10^{-33}$
	DeepCore (this work)					
	Atmospheric ($\gamma_{31}=\gamma_{32}$)	$\mathbf{4.3\cdot10^{-20}}$	$2.0\cdot10^{-21}$	$8.2\cdot10^{-23}$	$3.0\cdot10^{-24}$	$1.1\cdot10^{-25}$
	Solar I ($\gamma_{31}=\gamma_{21}$)	$\mathbf{1.2\cdot10^{-20}}$	$5.4\cdot10^{-22}$	$2.1\cdot10^{-23}$	$6.6\cdot10^{-25}$	$2.0\cdot10^{-26}$
	Solar II ($\gamma_{32}=\gamma_{21}$)	$7.5\cdot10^{-21}$	$3.5\cdot10^{-22}$	$1.4\cdot10^{-23}$	$4.2\cdot10^{-25}$	$1.1\cdot10^{-26}$

GeV*

Coloma, et al., Eur.Phys.J.C 78 (2018) 8, 614

n	Phase Perturbation Γ_{90}	State Selection Γ_{90}
0	$1.18\cdot10^{-15}$ eV	$1.17\cdot10^{-15}$ eV
1	$6.89\cdot10^{-16}$ eV	$6.67\cdot10^{-16}$ eV
2	$9.80\cdot10^{-18}$ eV	$9.48\cdot10^{-18}$ eV
3	$1.58\cdot10^{-19}$ eV	$1.77\cdot10^{-19}$ eV

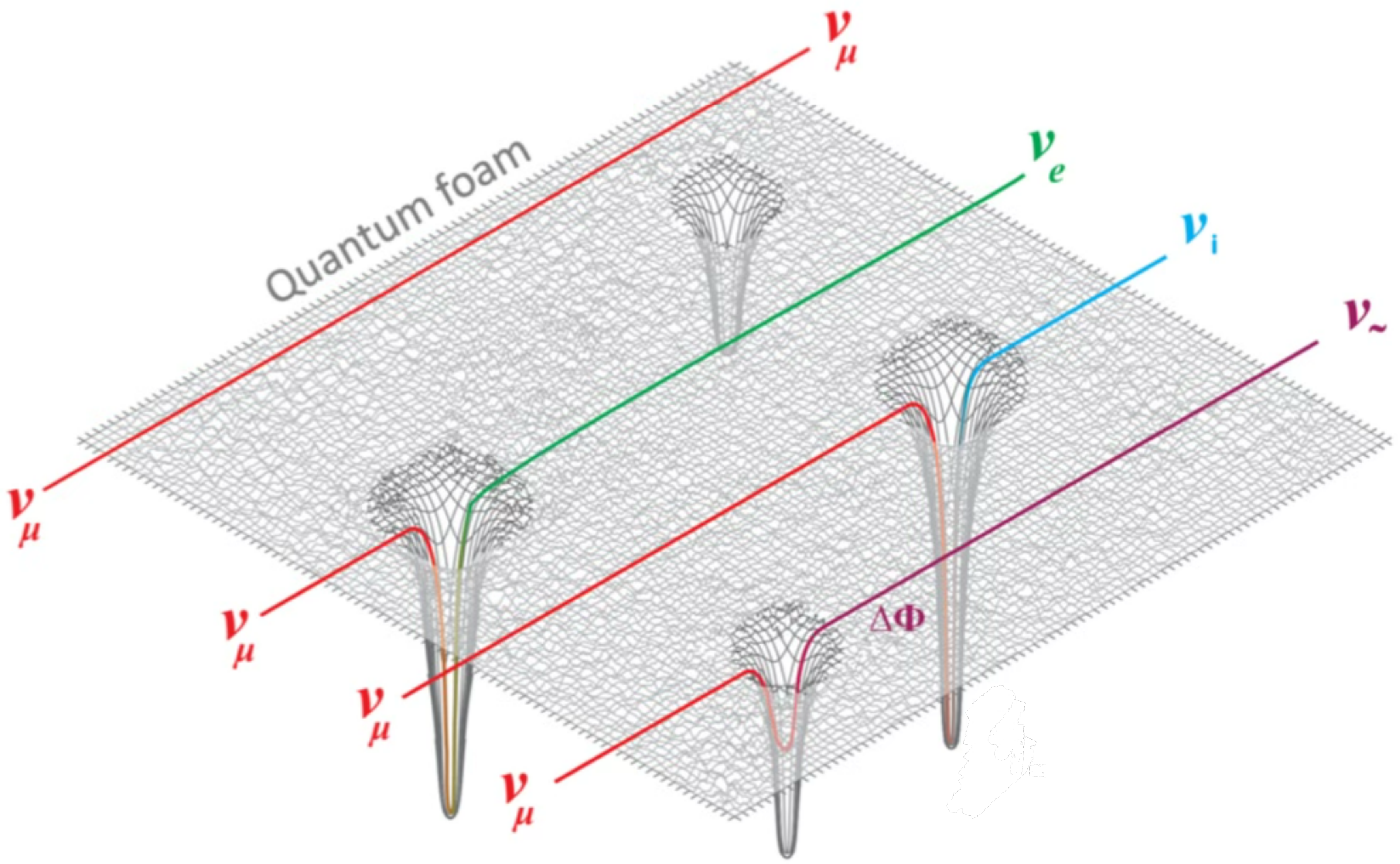
IceCube collaboration, Nature Physics 20, 913-920

2024

TABLE I. The 90% CL upper limits on Γ_{90} for each n in E_0=1 TeV in the state selection (SS) and phase perturbation (PS) models.

How the framework is applied to neutrino physics

Motivation:



From: The IceCube collaboration, Nature Physics 20, 913-920

Method:

$$\dot{\rho} = -i[H,\rho] - \mathcal{D}[\rho]$$

$$D_{\text{phase perturbation}} = \mathrm{diag}(0,\Gamma,\Gamma,0,\Gamma,\Gamma,\Gamma,\Gamma,0)$$

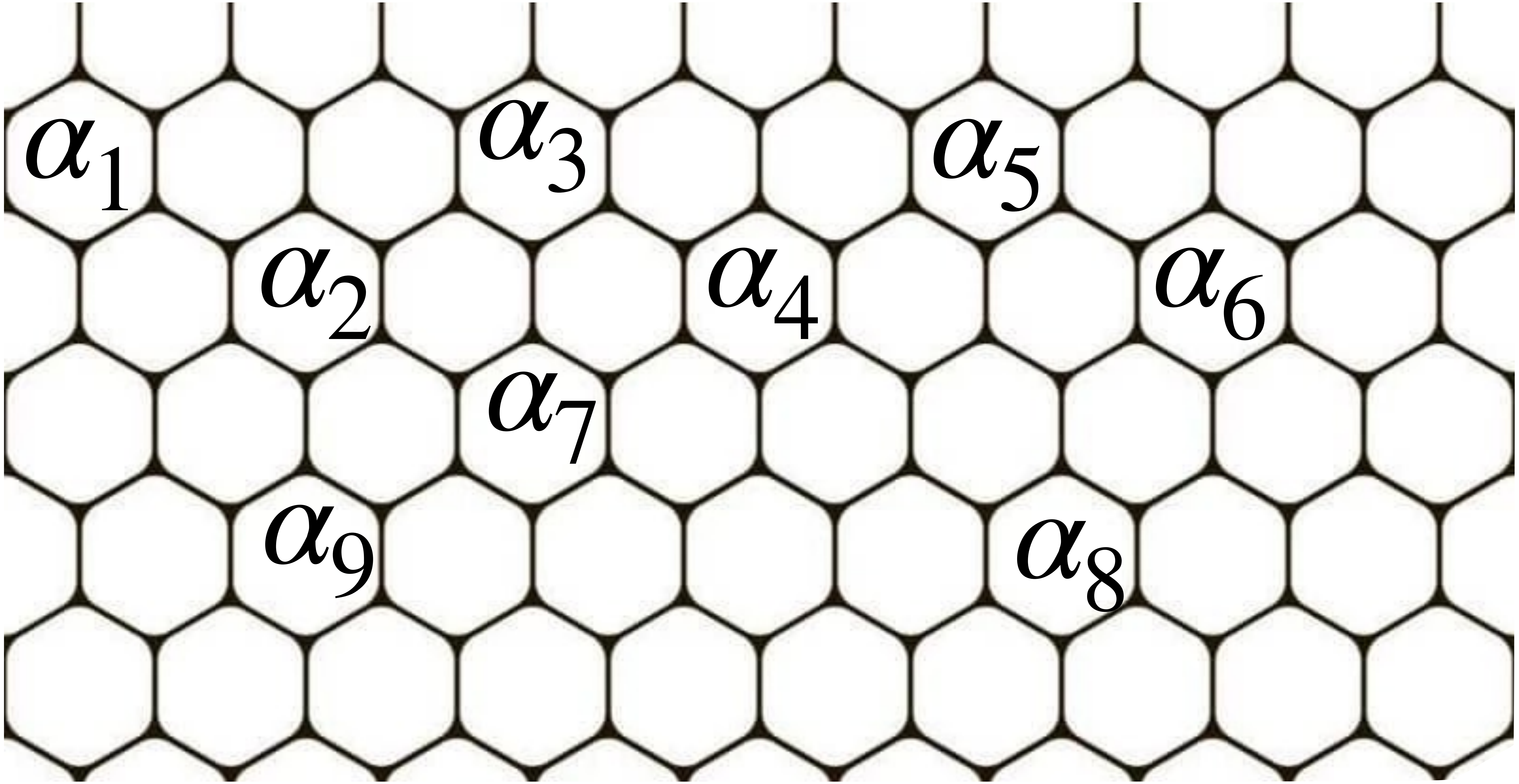
Pheno:

$$\Gamma(E_\nu) = \Gamma_0\left(\frac{E_\nu}{E_0}\right)^n$$

What is the microscopic origin of Γ?

Ultralight scalar background

The scalar field can be seen as

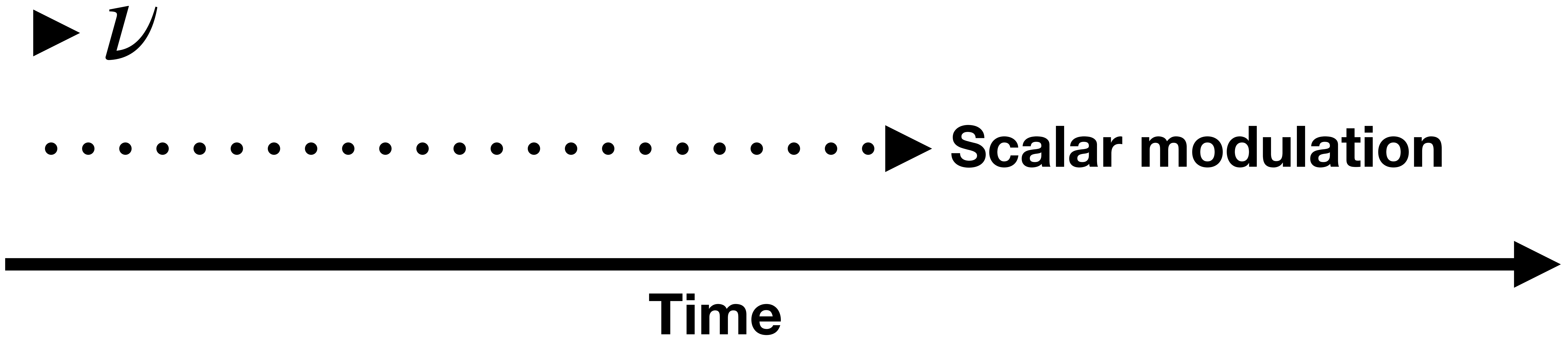


$$\alpha_i = \sqrt{n} e^{i\theta_i}$$

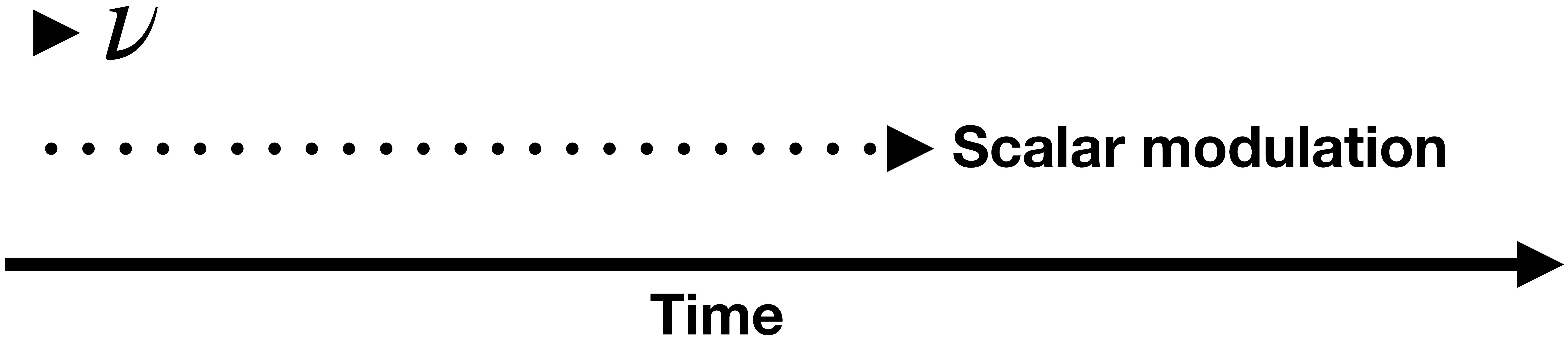
Time hierarchy

- ν

Time hierarchy

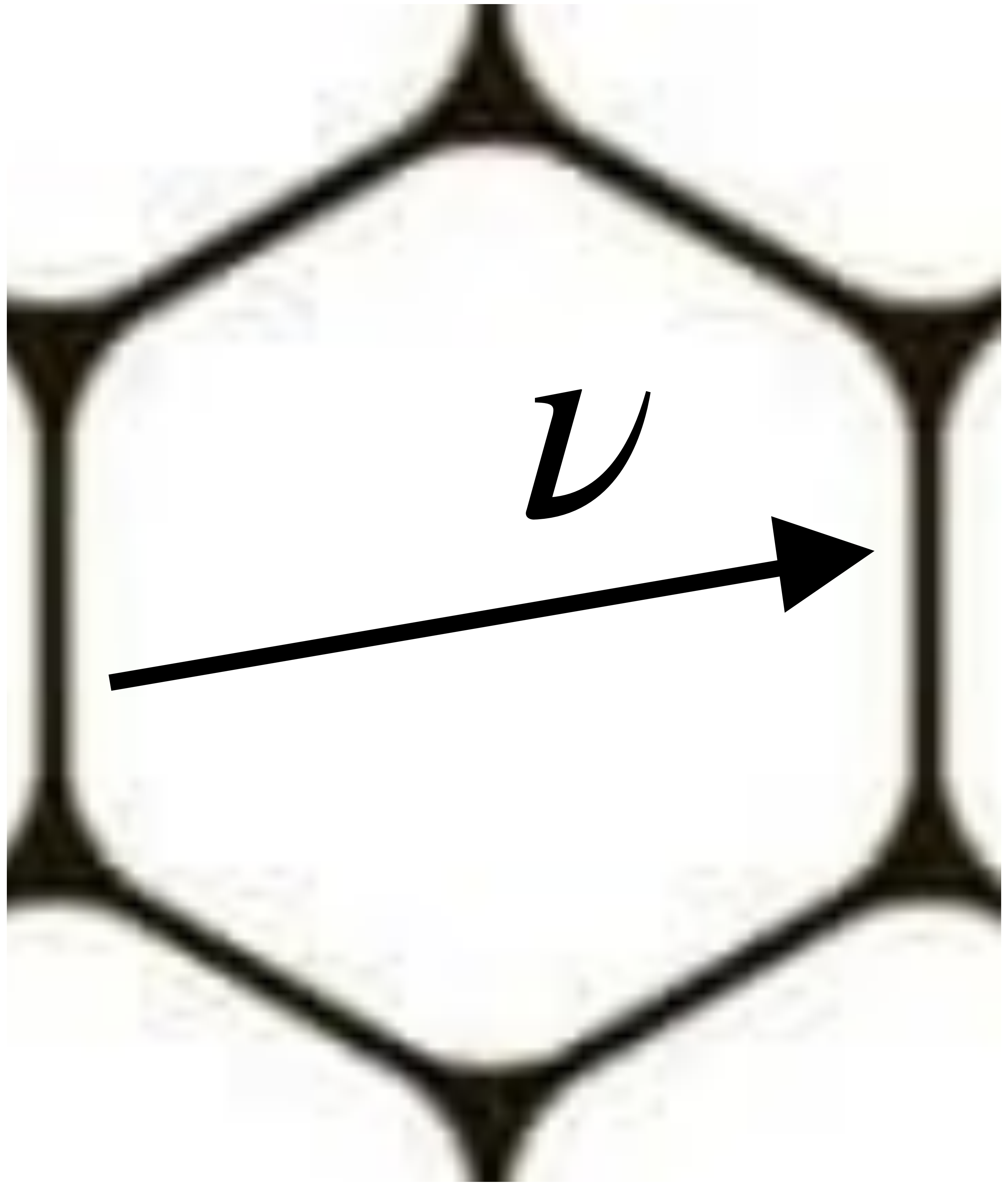


Time hierarchy



$$10^{-23}\ \mathrm{eV} < m_\phi < 10^{-11}\ \mathrm{eV}$$

The scales determine the scalar state



Field state

$$|\alpha\rangle$$

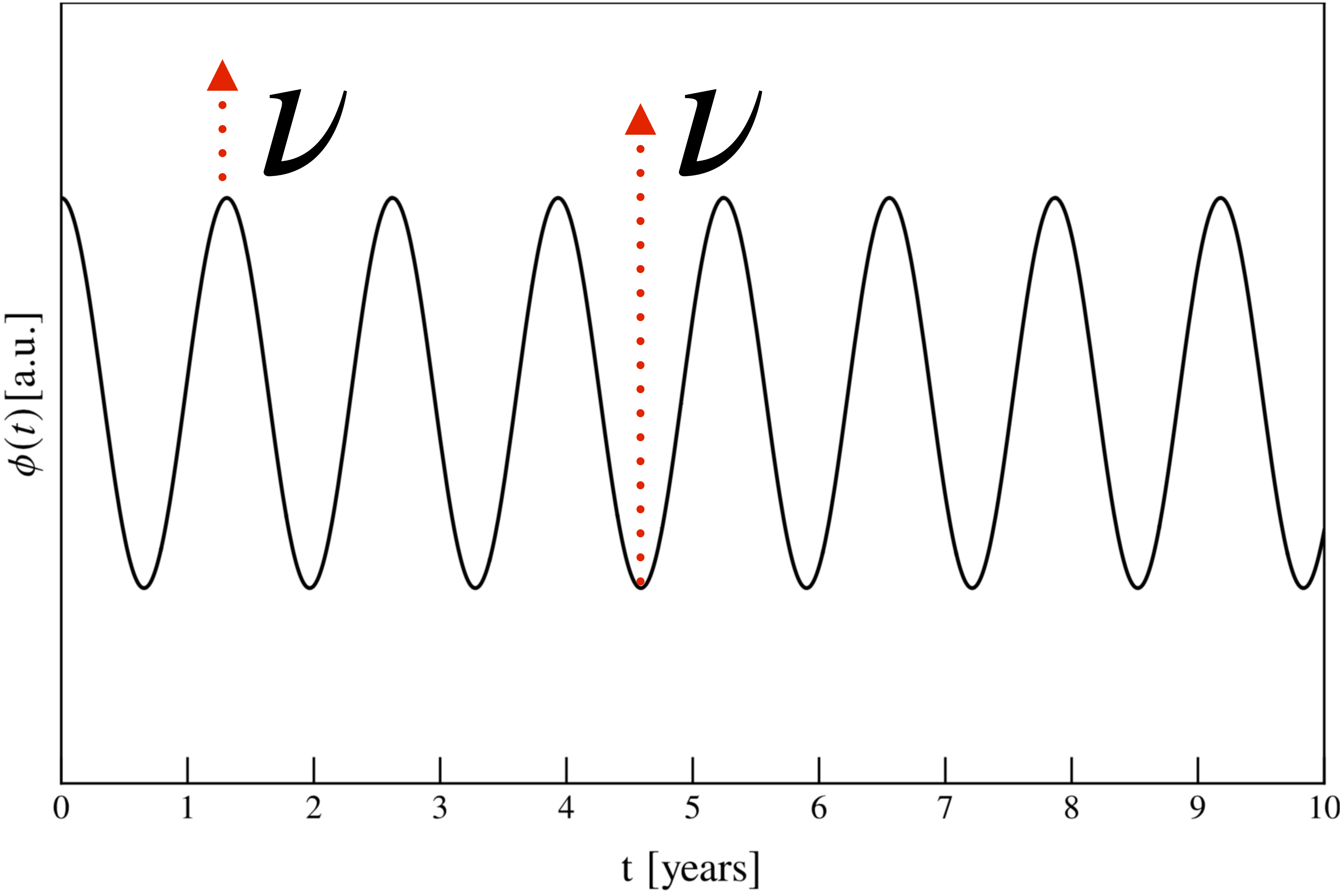
$$\phi(x,t) \approx \frac{\sqrt{2\rho_\phi}}{m_\phi}\cos(m_\phi t + \theta)$$

Neutrino oscillation frequency is modified

$$\frac{\Delta m_{ij}^2}{2E} \to \frac{\Delta m_{ij}^2}{2E} + \frac{\Delta m_{ij}^2(t)}{2E}$$

$$\Delta m_{ij}^2(t) = 2(g_i m_i - g_j m_j)\phi_0 \cos(m_\phi t + \theta)$$

Neutrinos start with different initial conditions



The field can be effectively described as

$$\phi(x,t) = \phi_0 \cos(\xi)$$

$$\xi \in [0,2\pi]$$

From an open system perspective

Recall

$$\rho(t,\xi) = U(t,\xi)\rho(0)U^{\dagger}(t,\xi)$$

From an open system perspective

Recall

$$\rho(t,\xi) = U(t,\xi)\rho(0)U^\dagger(t,\xi)$$

Average

$$\overline{\rho}(t) = \frac{1}{2\pi}\int_0^{2\pi} d\xi U(t,\xi)\rho(0)U^\dagger(t,\xi)$$

How to get the master equation?

Define

$$\Delta U = U(t,\xi) - \overline{U}(t) \qquad \overline{U}(t) = \frac{1}{2\pi}\int_0^{2\pi} d\xi U(t,\xi)$$

How to get the master equation?

To second order in perturbation theory

$$U(t,\xi) \approx \mathbf{1} - i\int_0^t d\tau H(\tau,\xi) - \int_0^t d\tau \int_0^\tau d\tau' H(\tau,\xi)H(\tau',\xi)$$

The master equation

$$\partial_t \overline{\rho}(t) = -i[H_0, \overline{\rho}(t)]$$

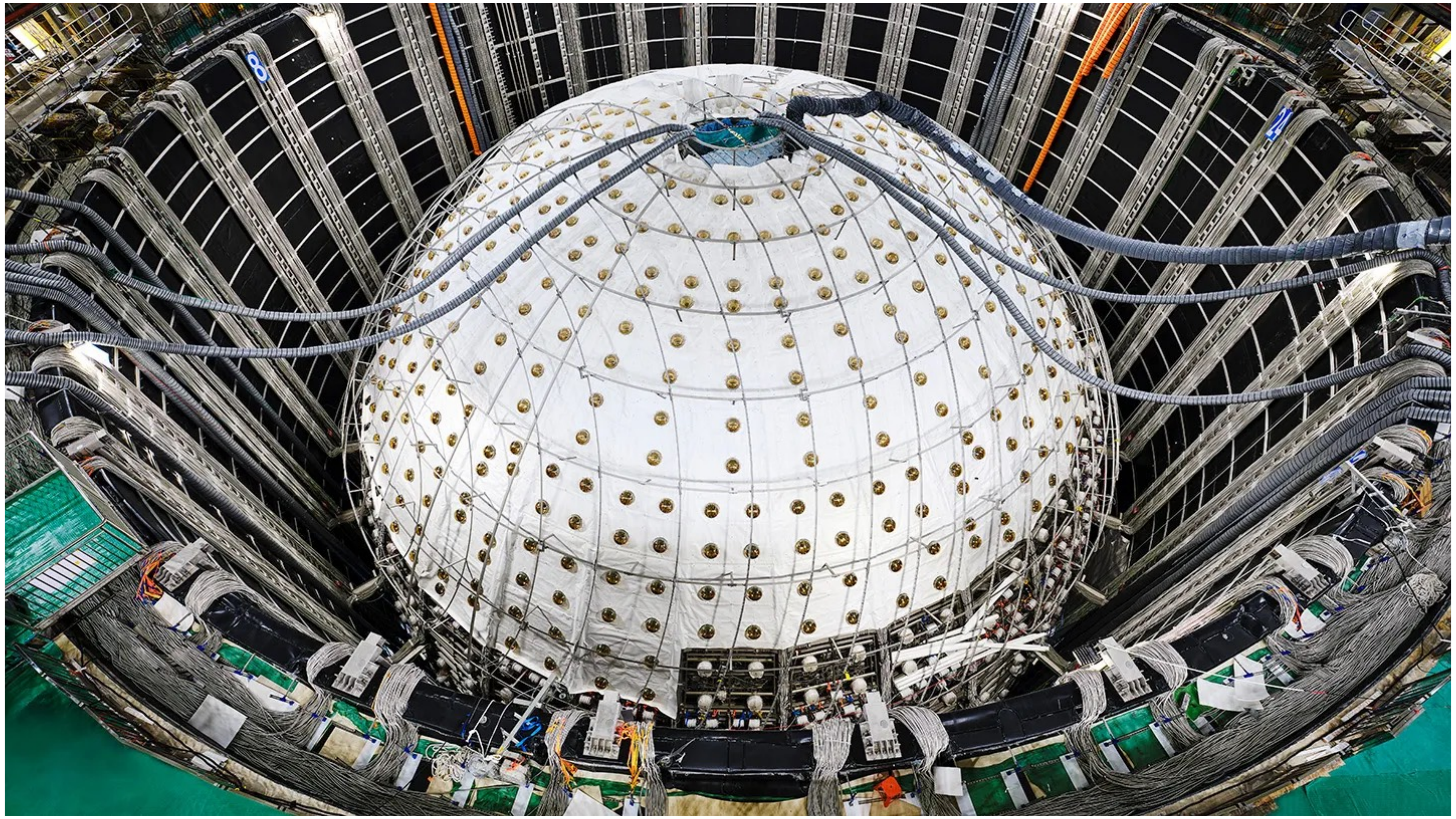
$$-\frac{\phi_0^2 t}{2E^2}((g_\phi \hat{m}_\nu)^2 \overline{\rho}(t) + \overline{\rho}(t)(g_\phi \hat{m}_\nu)^2 - 2 g_\phi \hat{m}_\nu \overline{\rho}(t) g_\phi \hat{m}_\nu))$$

Recall

$$\partial_t \rho(t) = -i[H, \rho(t)]$$

$$-L^\dagger L \rho(t) - \rho(t) L^\dagger L + 2L\rho(t)L^\dagger$$

JUNO experiment



How does the probability change?

$$P(\overline{\nu}_e \to \overline{\nu}_e) \approx 1 - \frac{1}{2}\left(\sin^2(2\theta_{13}) + \sin^2(2\theta_{12})\right)$$

$$-\exp\left(-\frac{(\Delta_{21}^{\phi})^2}{4}\right)P_{\odot}$$

$$+\exp\left(-\frac{(\Delta_{atm}^{\phi})^2}{4}\right)\frac{\sin^2(2\theta_{13})}{2}\sqrt{1 - \sin^2(2\theta_{12})\sin^2\Delta_{21}}\cos(2|\Delta_{ee}| \pm \Phi_{\odot})$$

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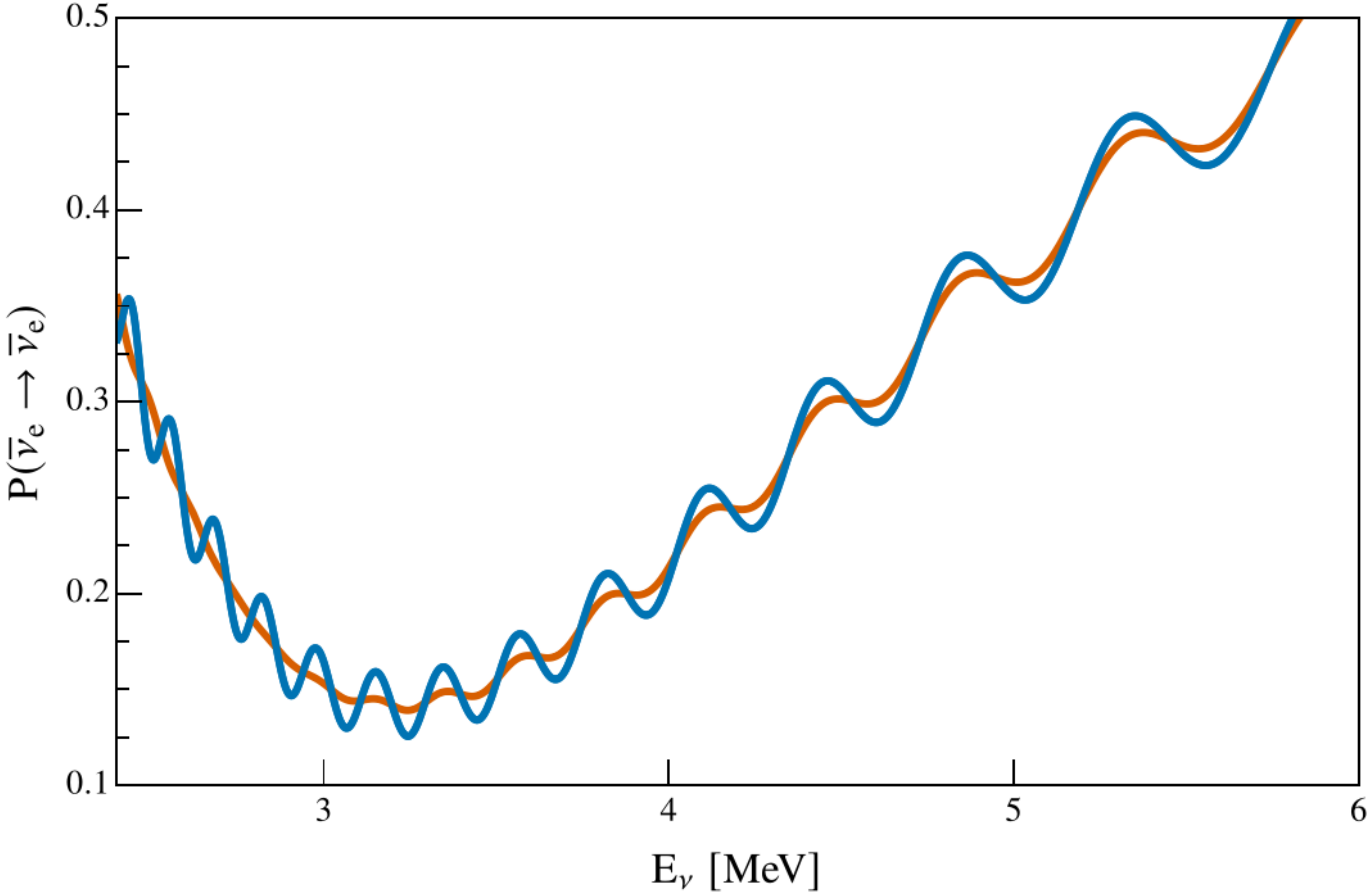
How does the probability change?

$$P(\bar{\nu}_e \to \bar{\nu}_e) \approx 1 - \frac{1}{2}\left(\sin^2(2\theta_{13}) + \sin^2(2\theta_{12})\right)$$
$$-\exp\left(-\frac{(\Delta^{\phi}_{21})^2}{4}\right) P_{\odot}$$
$$+\exp\left(-\frac{(\Delta^{\phi}_{atm})^2}{4}\right) \frac{\sin^2(2\theta_{13})}{2}\sqrt{1 - \sin^2(2\theta_{12})\sin^2\Delta_{21}}\cos(2|\Delta_{ee}| \pm \Phi_{\odot})$$

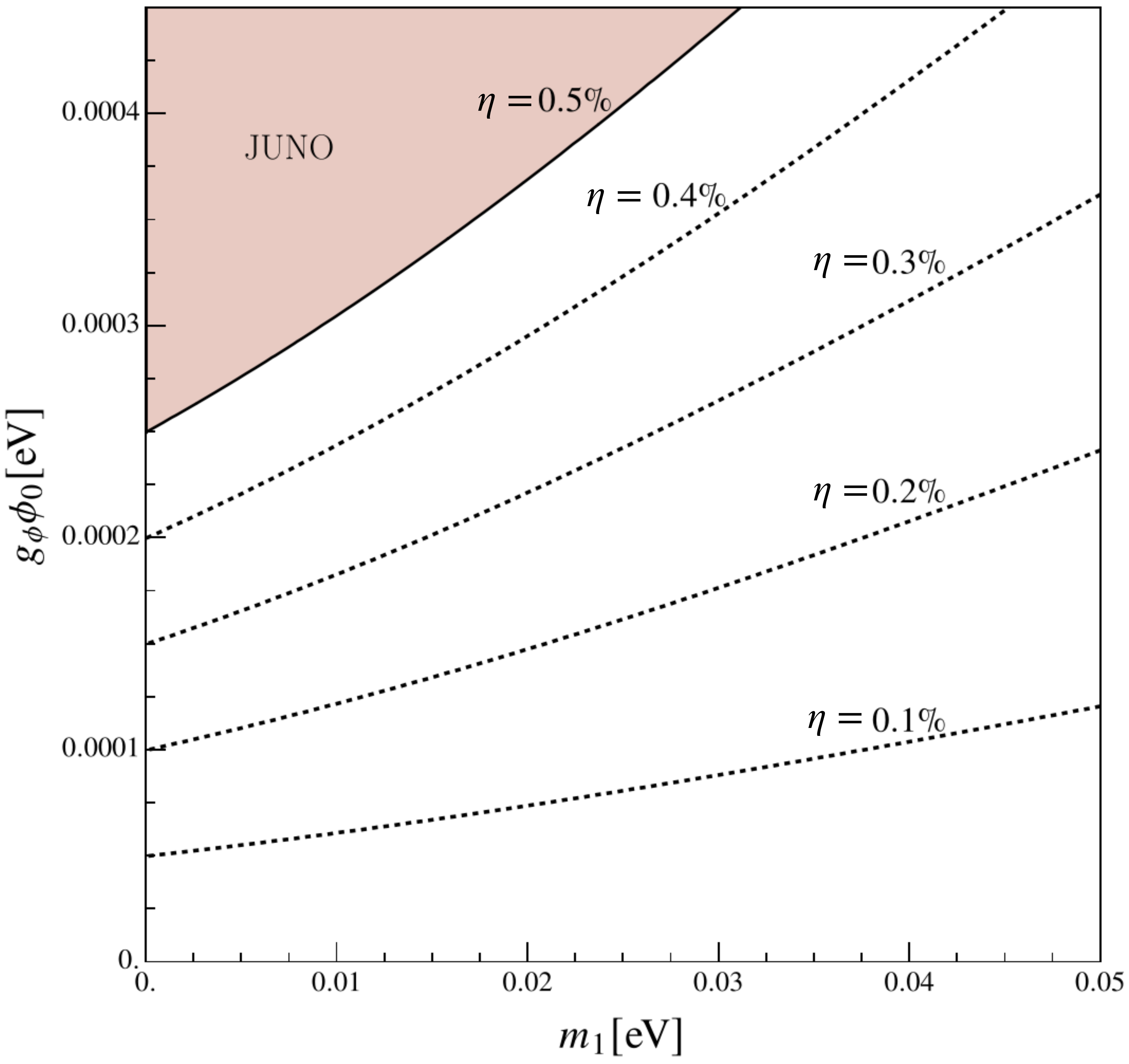
$$(\Delta^{\phi}_{ij})^2 = g^2_{\phi}\phi^2_0(m_i - m_j)^2\frac{L^2}{E^2}$$

Consequences



Open system can be mapped to model parameters

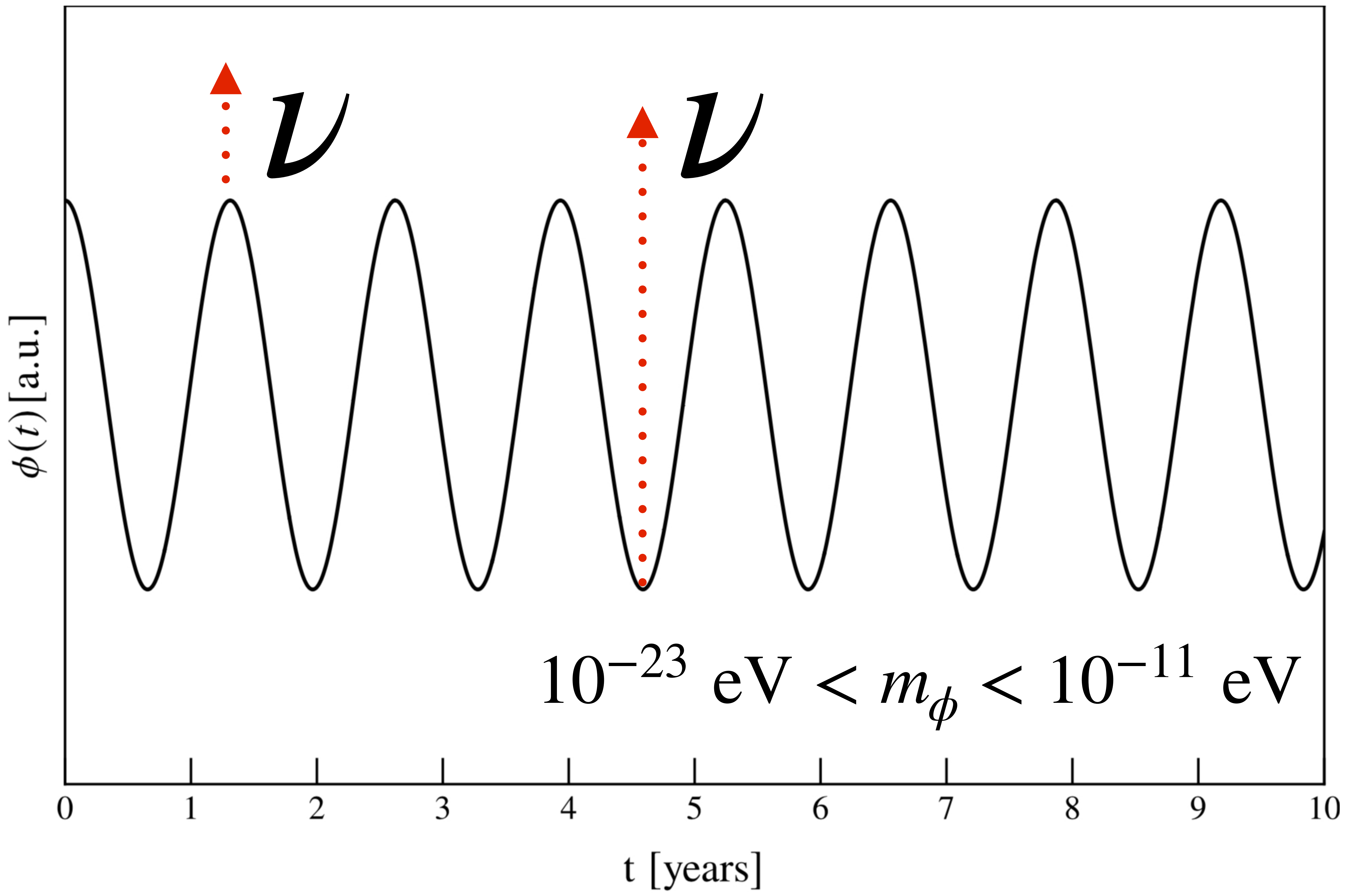
$$g_\phi \phi_0 (m_i - m_j) = \Delta m_{ij}^2 (\eta_\phi)_{ij}$$



For JUNO: Delgadillo, arXiv:2512.18186

Summary

Quick recap



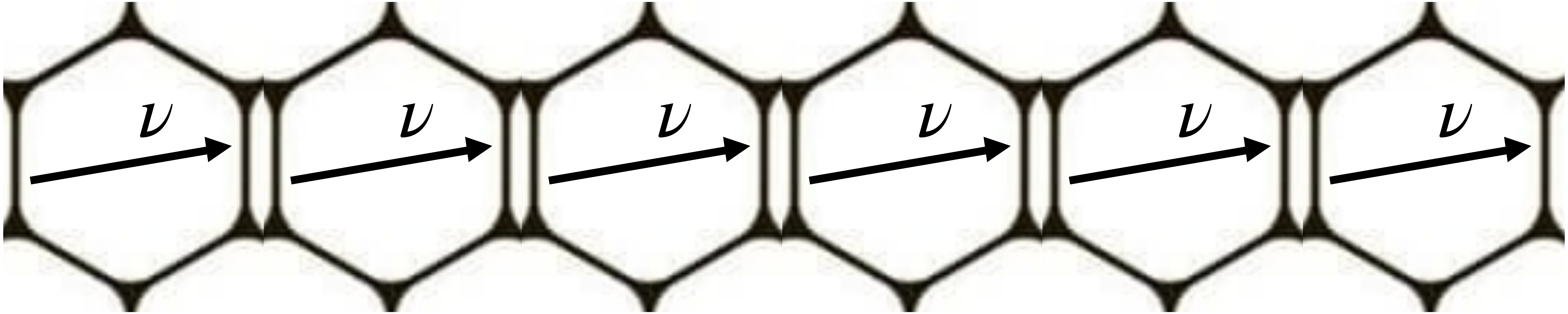
If neutrino masses are clocks, we can hear them tick.

If neutrino masses are clocks, we can hear them tick.

The fact that the clocks start out of sync can be modeled as an open system

Statistical decoherence, not quantum

$\rho(t, \xi_1)$ $\rho(t, \xi_2)$ $\rho(t, \xi_3)$ $\rho(t, \xi_4)$ $\rho(t, \xi_5)$ $\rho(t, \xi_6)$



And even if we listen only to the sound of silence…

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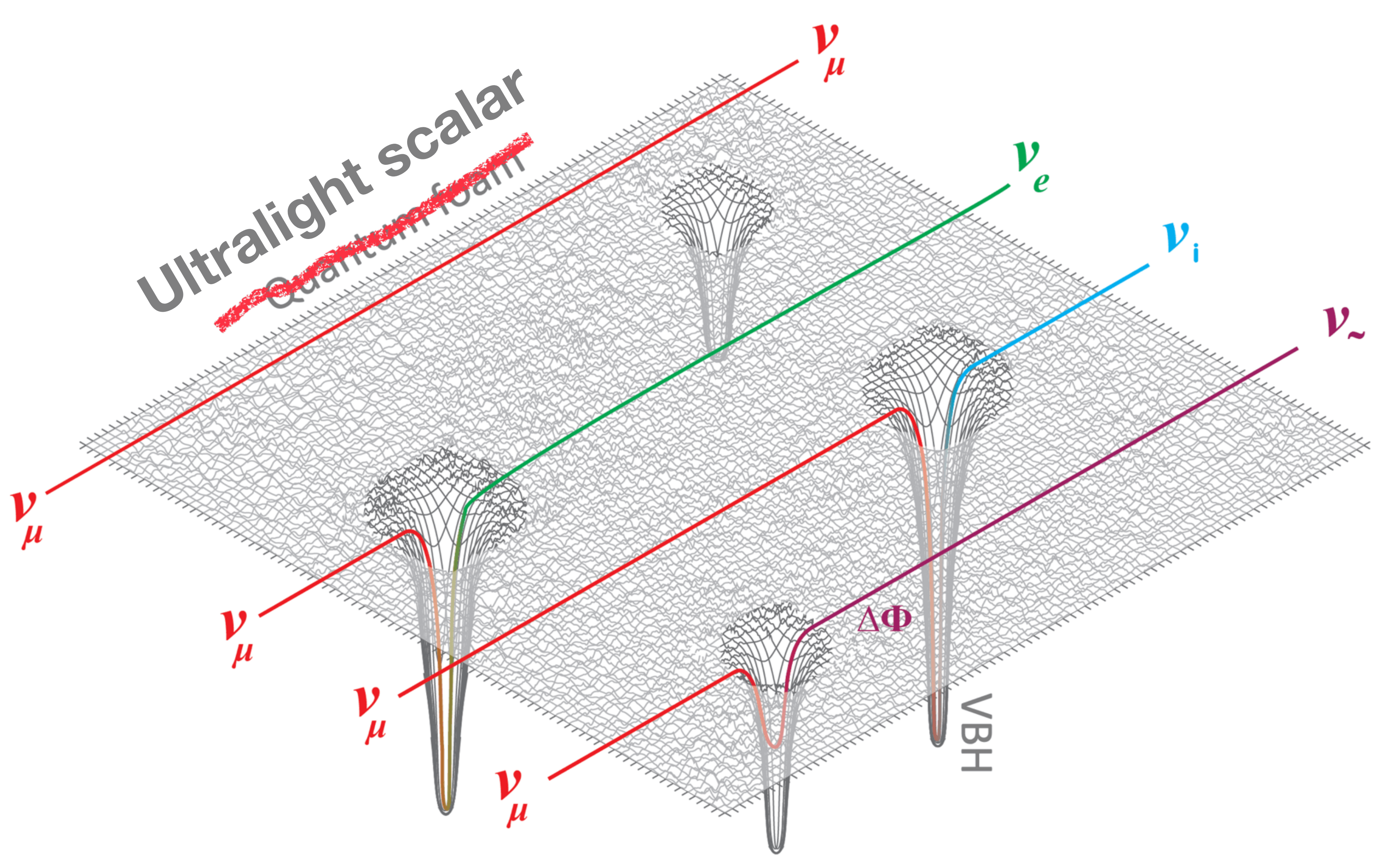
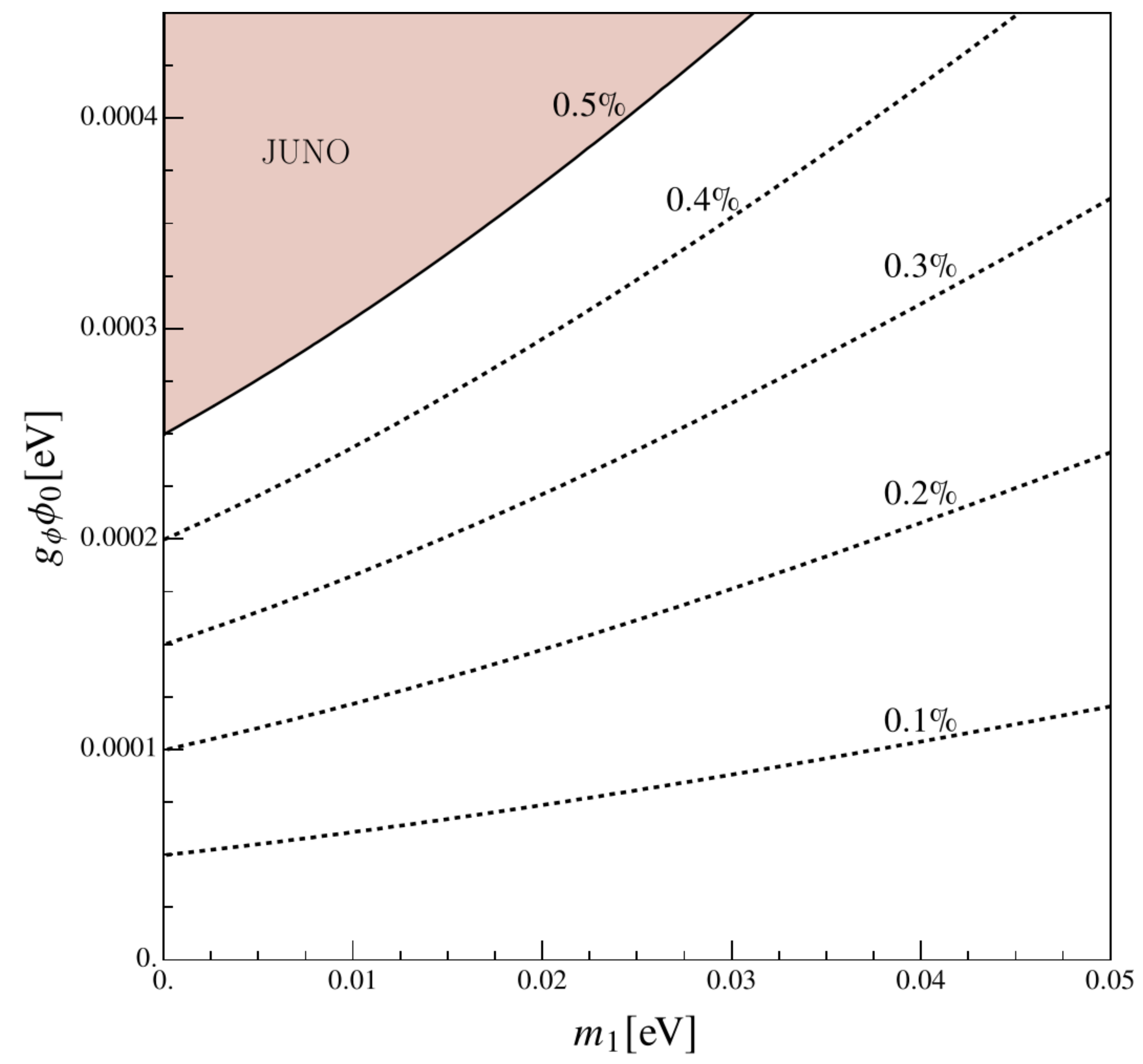
We can still learn something new!





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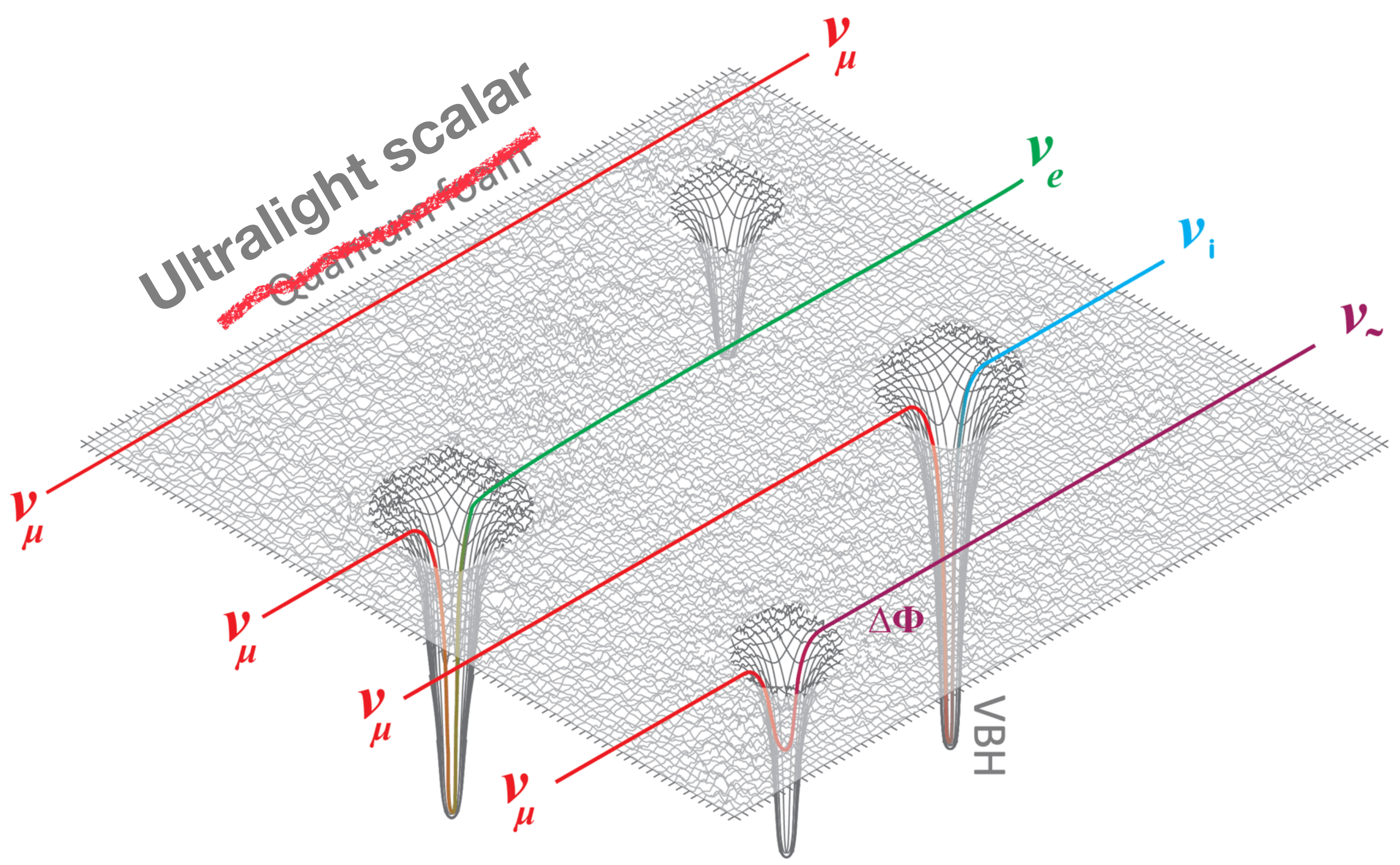

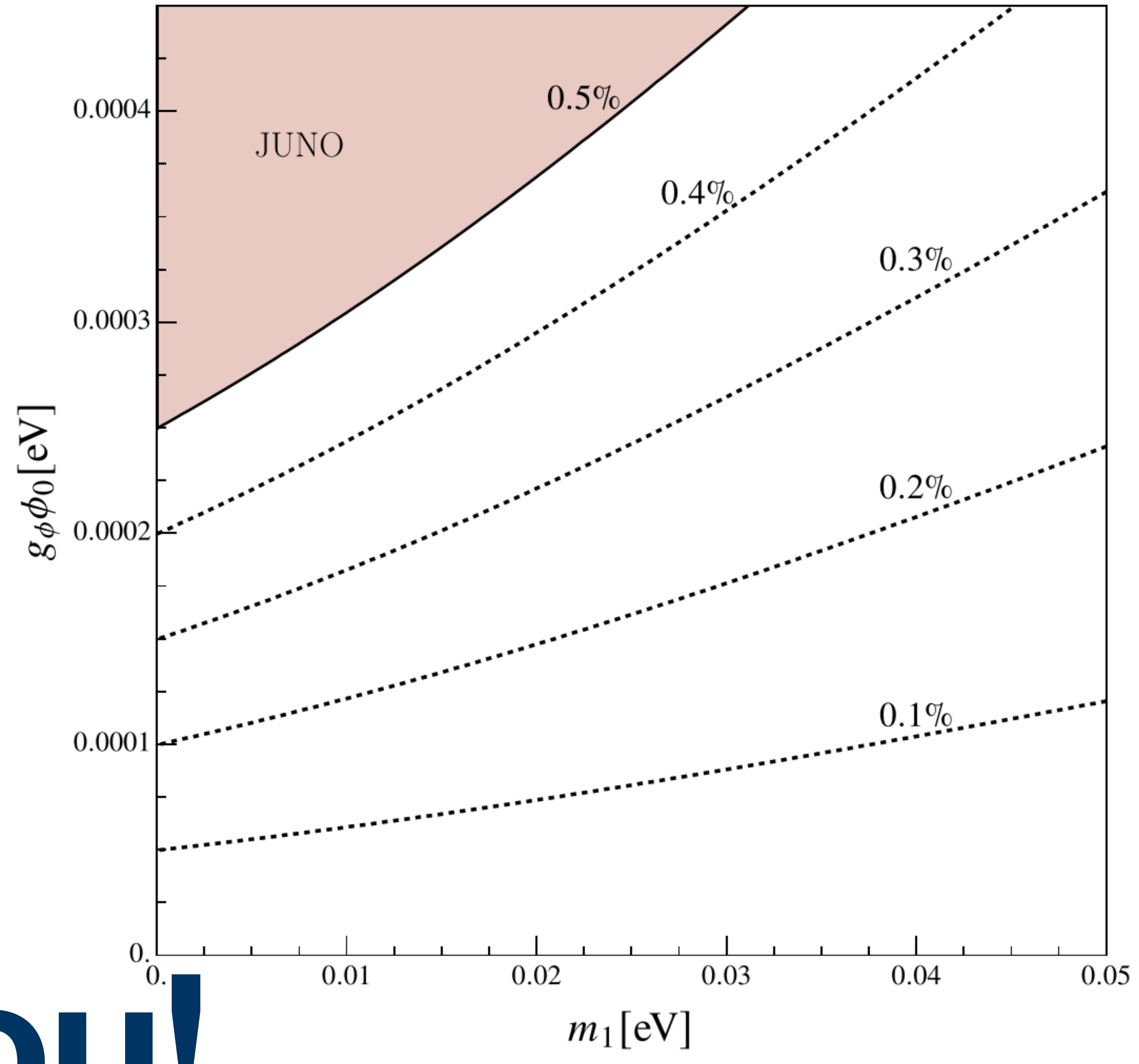

Thank you!

Backup

Ultralight scalars

Ultralight scalars have interesting properties

$$\frac{N}{V} = \frac{\rho_\phi}{m_\phi}$$

For ultralight scalar dark matter

$$[a, a^\dagger] = 1 \rightarrow aa^\dagger = \underbrace{a^\dagger a}_{N} + 1 \approx N$$

It's convenient to describe the scalar state as a coherent state

Quick review of coherent states

A coherent state is a superposition of states of the harmonic oscillator

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

R. J. Glauber, Phys. Rev. 131, 2766 (1963).
C. G. Sudarshan, Phys. Rev. Lett. 10, 277 (1963).

Quick review of coherent states

It is an eigenstate of the annihilation operator

$$a\,|\,\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n\,!}}\sqrt{n}\,|\,n-1\rangle$$

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$$= e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{(n-1)!}}\sqrt{n}|n-1\rangle = \alpha|\alpha\rangle$$

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Quick review of coherent states

Summary

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

$$\alpha = |\alpha|e^{i\theta}$$

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Quick review of coherent states

Summary

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

$$\alpha = |\alpha|e^{i\theta}$$

Effectively

$$a \rightarrow \alpha \qquad a^{\dagger} \rightarrow \alpha^{*}$$

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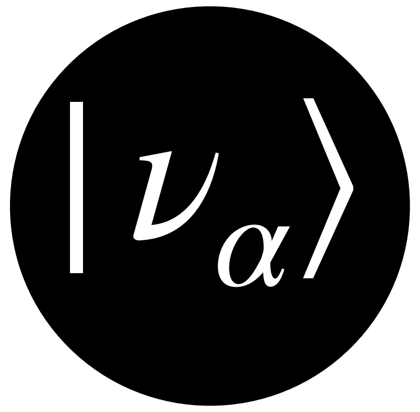
Neutrino oscillations in a nutshell

Production

$$|\nu_\alpha(0)\rangle = |\nu_e(0)\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

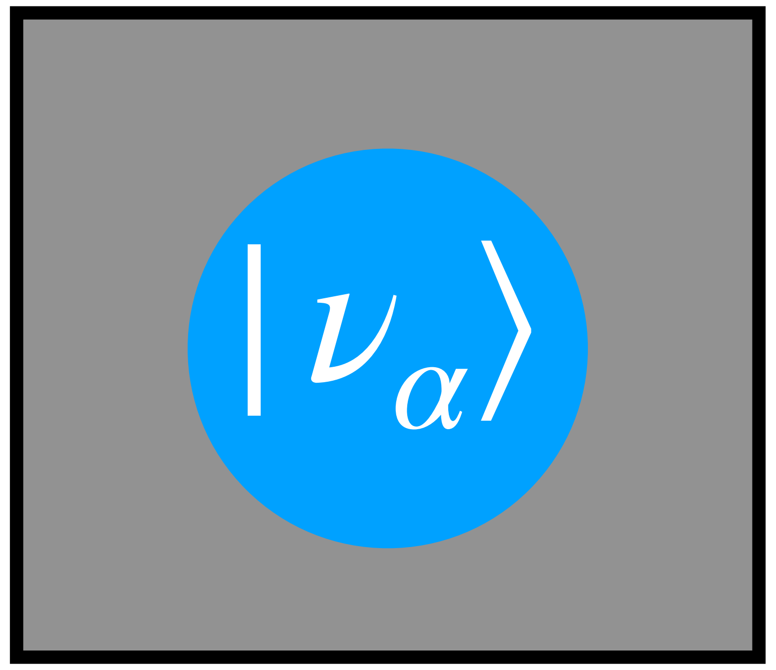


Propagation

$$|\nu_\alpha(t)\rangle = \cos\theta e^{-iE_1 t}|\nu_1\rangle + \sin\theta e^{-iE_2 t}|\nu_2\rangle$$

Detection

$$P(\nu_\alpha \to \nu_\alpha) = |\langle \nu_\alpha(t) | \nu_\alpha(0) \rangle|^2 = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$



Neutrinos interact very little

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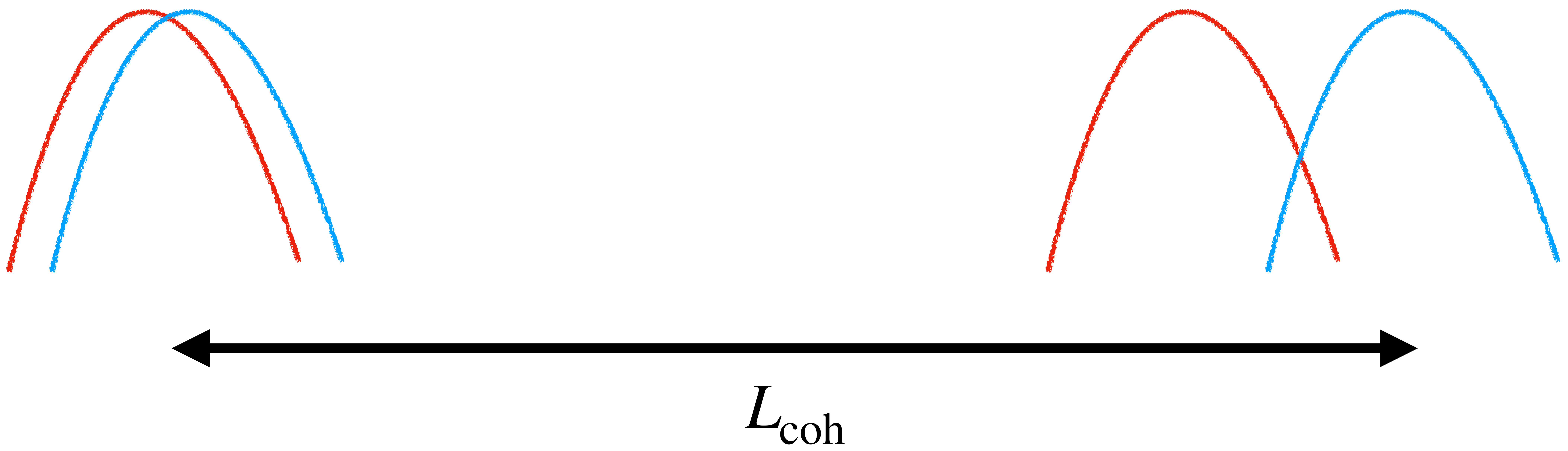
- The oscillation pattern is an interference effect between mass eigenstates

- Any new physics that modifies neutrino propagation
  will leave an imprint on the oscillation pattern

One type of decoherence

Wave packet decoherence



How does the probability change?

Scaling

Ultralight background

$$\exp\left[-\frac{(\eta\Delta m^2)^2L^2}{E^2}\right]$$

Wave packet

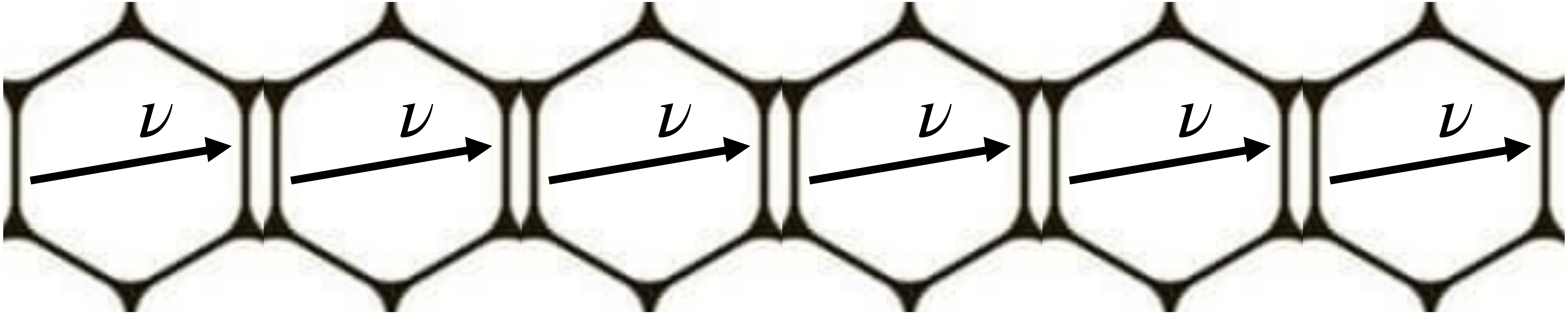
$$\exp\left[-\frac{|\Delta m^2|L^2}{32\sigma_x^2E^4}\right]$$

General approach

$$\exp\left[-\Gamma_0\left(\frac{E}{E_0}\right)^n L\right]$$

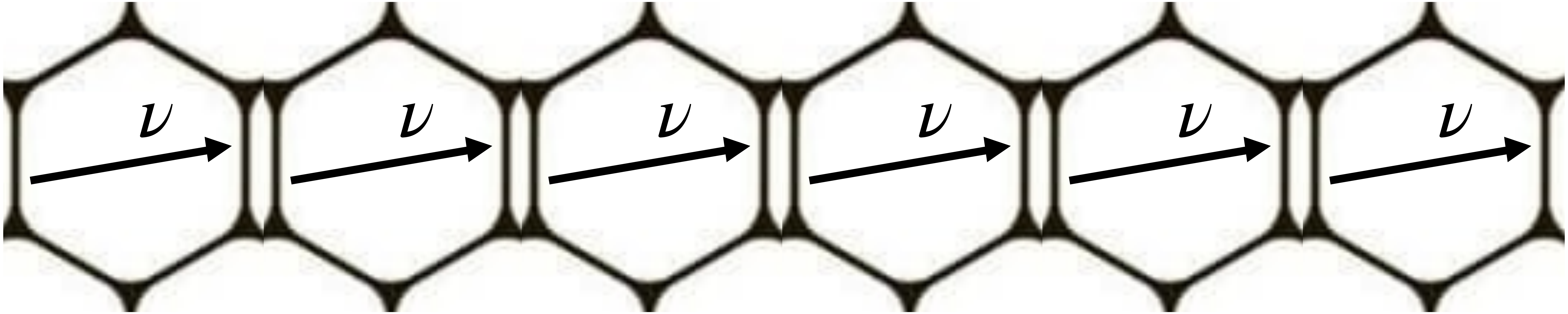
Quantum Decoherence

Statistical decoherence, not quantum



Statistical decoherence, not quantum

$\rho(t, \xi_1)$



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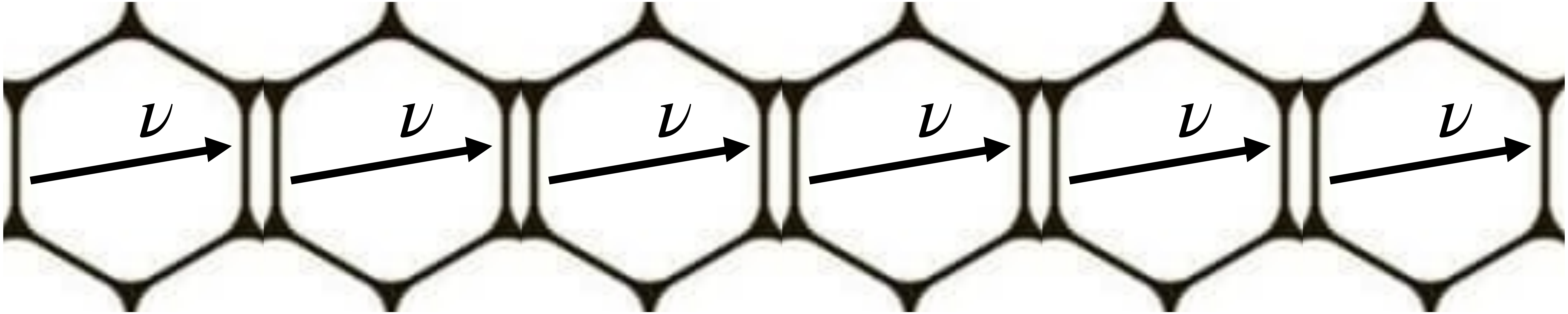

$$\mathrm{Tr}(\rho^2) = 1$$

Statistical decoherence, not quantum

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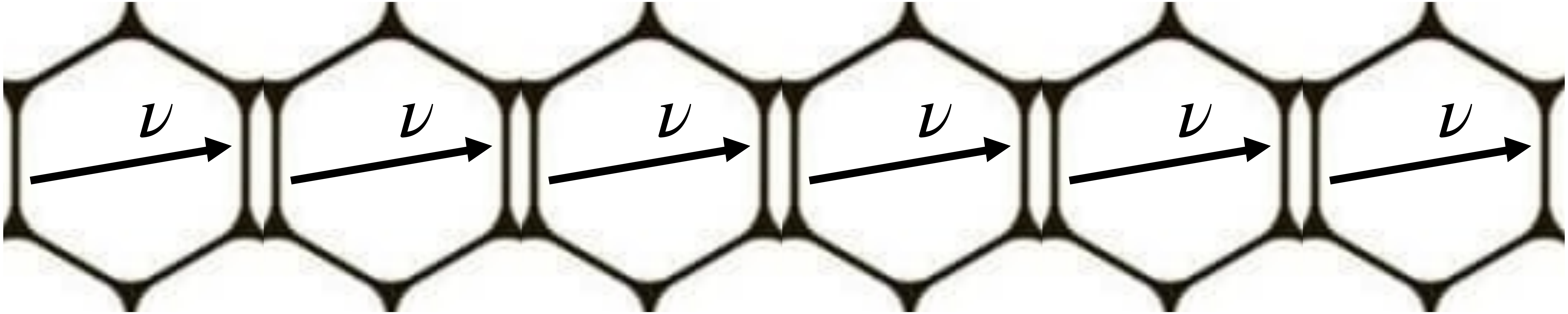
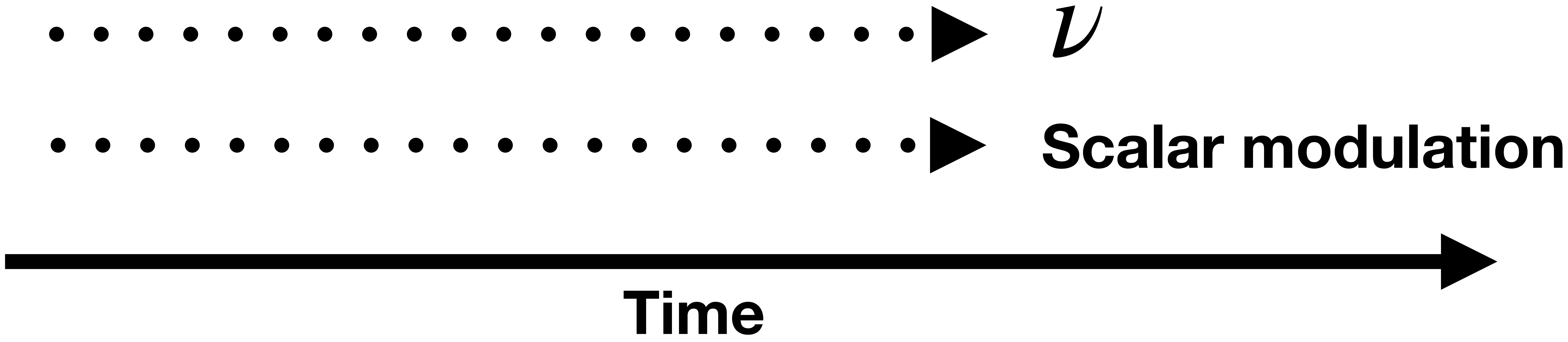


$$\overline{\rho}(t) = \frac{1}{2\pi}\int_0^{2\pi} d\xi U(t,\xi)\rho(0)U^\dagger(t,\xi)$$

$$\mathrm{Tr}(\overline{\rho}^2) < 1$$

Can we have an observable quantum decoherence effect?

Dynamical field regime



How the master equation should look

$$\partial_t \rho(t) = -i[H_0 + H_\phi(t), \rho(t)]$$

$$-\int d\tau \int d\tau' \frac{\mathrm{Tr}(\Delta\phi(\tau)\Delta\phi(\tau')\rho_\phi)}{2E^2}(\hat{m}_\nu^2 \rho(t) + \rho(t)\hat{m}_\nu^2 - 2\hat{m}_\nu \rho(t) \hat{m}_\nu)$$

$$\Delta\phi = \phi - \mathrm{Tr}(\phi\rho_\phi)$$

$$\rho_\phi = \int d\alpha P(\alpha)\,|\,\alpha\rangle\langle\alpha\,|$$

Requirements

1. We must fix the scalar state ρ_ϕ.

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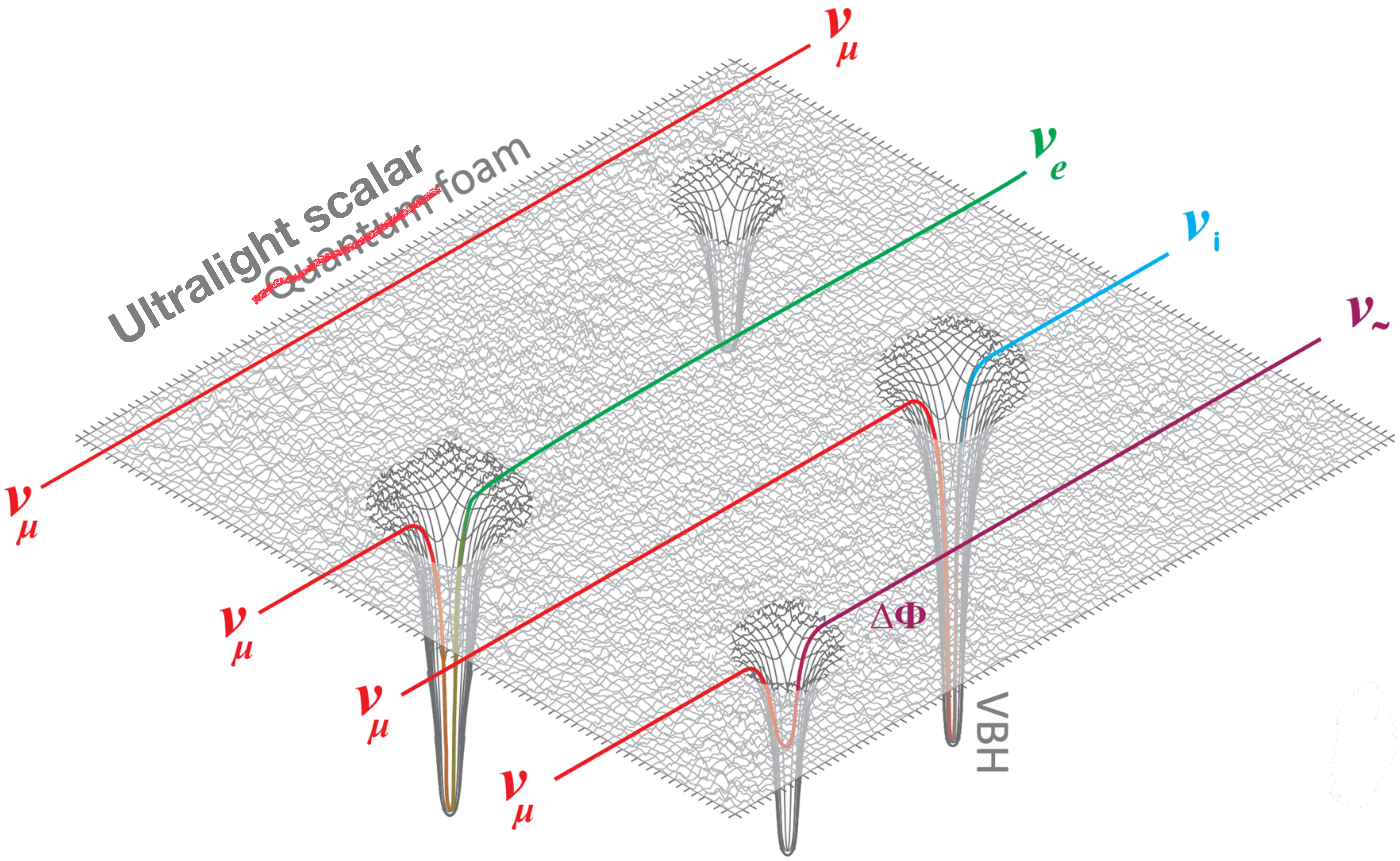
2. The field must have a nontrivial variance $\langle \Delta\phi \Delta\phi \rangle$
3. The field must not oscillate too fast

Routes to pursue



Can we probe true quantum decoherence instead of statistical decoherence?

Vectorization

Quick detour

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

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The “trick” (vectorization) in two generations

$$\rho = \rho_0 \mathbf{1} + \rho_i \sigma_i$$

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The “trick” (vectorization) in two generations

$$\rho = \rho_0 \mathbf{1} + \rho_i \sigma_i$$

Such that

$$\partial_t \begin{pmatrix} \rho_0 \\ \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} \\ M_{10} & M_{11} & M_{12} & M_{13} \\ M_{20} & M_{21} & M_{22} & M_{23} \\ M_{30} & M_{31} & M_{32} & M_{33} \end{pmatrix} \begin{pmatrix} \rho_0 \\ \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix}$$

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In three generations: $\rho = \rho_0 \mathbf{1} + \rho_i \lambda_i$

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$$\begin{pmatrix} \rho_0 \\ \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \\ \rho_5 \\ \rho_6 \\ \rho_7 \\ \rho_8 \end{pmatrix}$$

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$$\begin{pmatrix} \rho_0 \\ \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \\ \rho_5 \\ \rho_6 \\ \rho_7 \\ \rho_8 \end{pmatrix} \qquad \mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

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Diagonal of the density matrix

$$\text{diag}(\rho) = \left(\rho_0 + \rho_3 + \frac{1}{\sqrt{3}}\rho_8,\ \rho_0 - \rho_3 + \frac{1}{\sqrt{3}}\rho_8,\ \rho_0 - \frac{2}{\sqrt{3}}\rho_8 \right)$$

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$$\text{Tr}(\rho) = 3\rho_0$$

“Vectorizing” the time evolution

$$\rho(t,\xi) = \sum_{\mu=0}^{8} \rho^\mu \lambda_\mu = \begin{pmatrix} \frac{\rho_0}{\sqrt{3}} + \frac{\rho_3}{\sqrt{2}} + \frac{\rho_8}{\sqrt{6}} & \frac{\rho_1}{\sqrt{2}} - \frac{i\,\rho_2}{\sqrt{2}} & \frac{\rho_4}{\sqrt{2}} - \frac{i\,\rho_5}{\sqrt{2}} \\ \frac{\rho_1}{\sqrt{2}} + \frac{i\,\rho_2}{\sqrt{2}} & \frac{\rho_0}{\sqrt{3}} - \frac{\rho_3}{\sqrt{2}} + \frac{\rho_8}{\sqrt{6}} & \frac{\rho_6}{\sqrt{2}} - \frac{i\,\rho_7}{\sqrt{2}} \\ \frac{\rho_4}{\sqrt{2}} + \frac{i\,\rho_5}{\sqrt{2}} & \frac{\rho_6}{\sqrt{2}} + \frac{i\,\rho_7}{\sqrt{2}} & \frac{\rho_0}{\sqrt{3}} - \sqrt{\frac{2}{3}}\,\rho_8 \end{pmatrix}$$

$$H(\xi) = \sum_{\mu=0}^{8} h^\mu \lambda_\mu, \quad \lambda_\mu = \{\frac{\mathbf{1}_{3\times 3}}{\sqrt{3}}, \lambda_i\}, \quad \mathrm{Tr}(\lambda^\mu \lambda^\nu) = \delta_{\mu\nu}$$

“Vectorizing” the time evolution

Defining

$$|\rho(t,\xi)\rangle = \begin{pmatrix} \rho_0 \\ \rho_1 \\ \vdots \\ \rho_8 \end{pmatrix}$$

Expanding both sides

$$\partial_t \rho_\mu \lambda^\mu = -\, i h_\nu \rho_\theta [\lambda^\nu, \lambda^\theta]$$

Collect coefficients

$$\partial_t |\rho(t,\xi)\rangle = \mathscr{H}(\xi)\,|\rho(t)\rangle$$

This can be treated analytically

$$\mathscr{U}(t,\xi) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos\Delta_{21}^{\text{eff}} & \sin\Delta_{21}^{\text{eff}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\sin\Delta_{21}^{\text{eff}} & \cos\Delta_{21}^{\text{eff}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\Delta_{31}^{\text{eff}} & \sin\Delta_{31}^{\text{eff}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sin\Delta_{31}^{\text{eff}} & \cos\Delta_{31}^{\text{eff}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cos\Delta_{32}^{\text{eff}} & \sin\Delta_{32}^{\text{eff}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\sin\Delta_{32}^{\text{eff}} & \cos\Delta_{32}^{\text{eff}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Delta_{ij}^{\text{eff}} = \frac{\Delta m_{ij}^2}{2E_\nu}L + g_\phi \phi_0 \frac{(m_i - m_j)}{E_\nu}L \equiv \Delta_{ij} + \Delta_{ij}^{\phi}$$

Oscillation probability

Standard Oscillation probability

$$P(\overline{\nu}_e \to \overline{\nu}_e) \approx 1 - \frac{1}{2}\left(\sin^2(2\theta_{13}) + \sin^2(2\theta_{12})\right)$$

$$-P_\odot + \frac{\sin^2(2\theta_{13})}{2}\sqrt{1 - \sin^2(2\theta_{12})\sin^2\Delta_{21}}\cos(2\,|\,\Delta_{ee}\,|\,\pm\Phi_\odot)$$

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$$P_\odot \approx \sin^2 2\theta_{12}\sin^2\Delta_{21}$$

$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{4E}L$$

$$\Delta_{ee} = \cos^2\theta_{12}\Delta_{31} + \sin^2\theta_{12}\Delta_{32}$$

$$\Phi_\odot = \arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}$$

$$\cos^4\theta_{13} \approx 1$$

Details time evolution

How to get the master equation?

Define

$$\Delta U = U(t,\xi) - \overline{U}(t) \qquad \overline{U}(t) = \frac{1}{2\pi}\int_0^{2\pi} d\xi U(t,\xi)$$

We can rewrite

$$\overline{\rho}(t) = \overline{U}(t)\rho(0)\overline{U}^{\dagger}(t) + \frac{1}{2\pi}\int_0^{2\pi} d\xi \Delta U(t,\xi)\rho(0)\Delta U^{\dagger}(t,\xi)$$

How to get the master equation?

Define an effective hamiltonian

$$\partial_t \overline{U}(t) = -\, iV(t)\overline{U}(t)$$

Burgess, et al., Annals of Physics
Volume 256, Issue 1, 1

How to get the master equation?

Define an effective hamiltonian

$$\partial_t \overline{U}(t) = -iV(t)\overline{U}(t)$$

Burgess, et al., Annals of Physics
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We find the time evolution equation

$$\partial_t \overline{\rho}(t) = -i[V(t), \overline{\rho}(t)] + \partial_t \left(\frac{1}{2\pi} \int_0^{2\pi} d\xi \Delta U(t,\xi) \rho(0) \Delta U^\dagger(t,\xi) \right)$$