

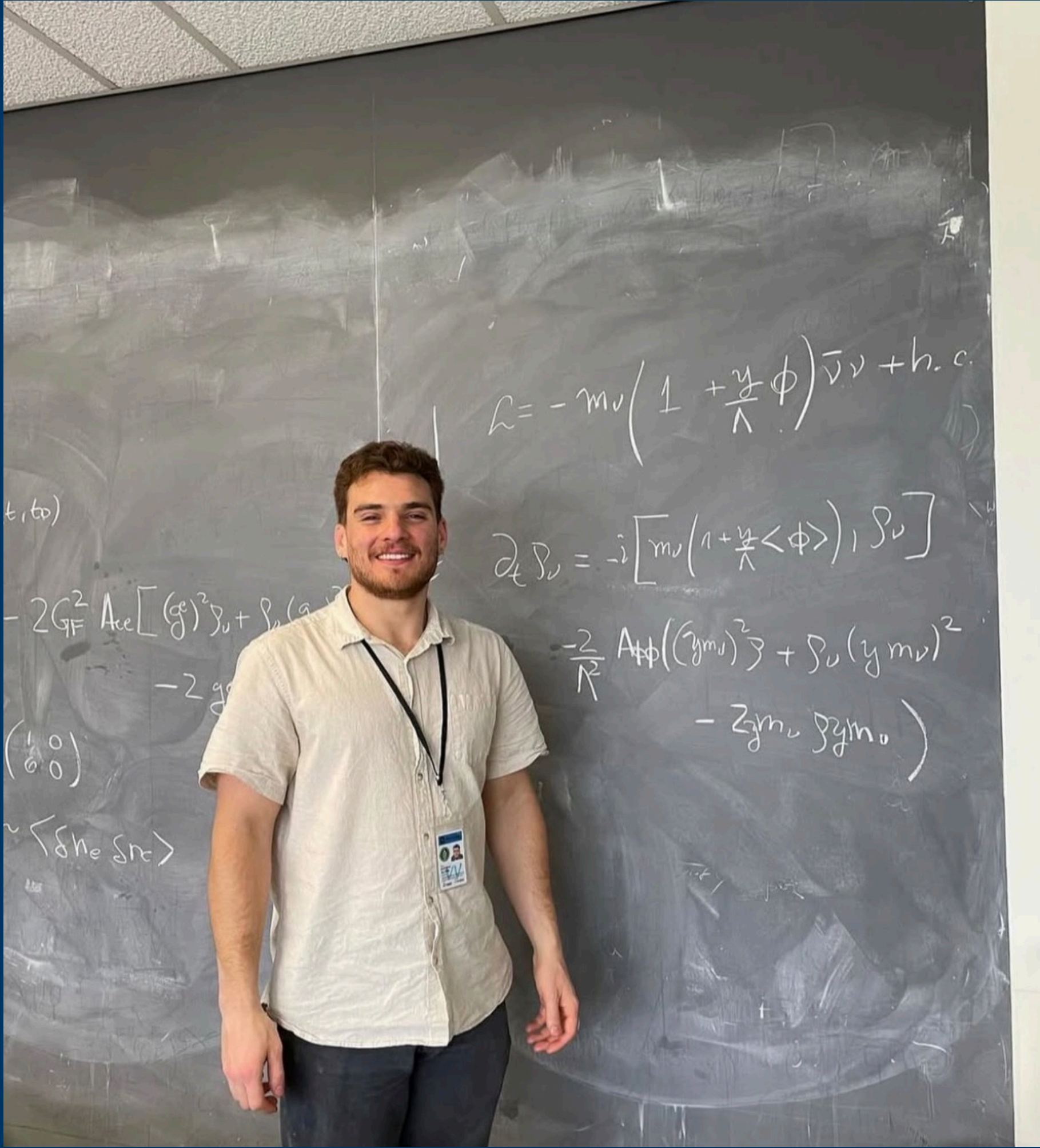
**If neutrino masses are clocks,
can we hear them tick?**



Gustavo F. S. Alves



Back to 2023



Ultralight scalars are compelling dark matter candidates

Hui, et al., Phys. Rev. D 95, 043541

Hu, et al., Phys. Rev. Lett. 85, 1158

Ferreira, Astron.Astrophys.Rev. 29 (2021) 1, 7.

Chadha-Day et al., Sci. Adv. 8, sciadv.abj3618

Arvanitaki, Phys. Rev. D 81, 123530

...

Neutrinos interacting with a new scalar

$$-\mathcal{L} \supset g_{\phi,1}\phi\bar{\nu}_1\nu_1 + g_{\phi,2}\phi\bar{\nu}_2\nu_2 + g_{\phi,3}\phi\bar{\nu}_3\nu_3$$

Neutrinos interacting with a new scalar

$$-\mathcal{L} \supset g_{\phi,1}\phi\bar{\nu}_1\nu_1 + g_{\phi,2}\phi\bar{\nu}_2\nu_2 + g_{\phi,3}\phi\bar{\nu}_3\nu_3$$

Neutrino masses get modified

$$m_{\nu_i} = m_{\nu_i}(g_{\phi} = 0) + g_{\phi,i}\phi$$

We will show that

$$\phi \sim \phi_0 \cos(m_\phi t + \theta) \quad \phi_0 = \frac{\sqrt{2\rho_\phi}}{m_\phi}$$

Neutrino masses become time dependent

$$m_{\nu_i}(t) = m_{\nu_i}^0 + g_{\phi,i}\phi_0 \cos(m_\phi t + \theta)$$

Neutrino oscillation frequency is modified

$$\Delta m_{ij}^2(t) \approx \Delta m_{ij}^2(\phi_0 = 0) + 2((g_\phi m)_i - (g_\phi m)_j)\phi_0 \cos(m_\phi t + \theta)$$

Theory Background

Ultralight scalars have interesting properties

$$\frac{N}{V} = \frac{\rho_\phi}{m_\phi}$$

For ultralight scalar dark matter

$$[a, a^\dagger] = 1 \rightarrow aa^\dagger = \underbrace{a^\dagger a}_N + 1 \approx N$$

Ultralight scalars have interesting properties

$$\frac{N}{V} = \frac{\rho_\phi}{m_\phi}$$

For ultralight scalar dark matter

$$[a, a^\dagger] = 1 \rightarrow aa^\dagger = \underbrace{a^\dagger a}_N + 1 \approx N$$

It's convenient to describe it as a coherent state

Quick review of coherent states

**A coherent state is a
superposition of states of the
harmonic oscillator**

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

Quick review of coherent states

**It is an eigenstate of the
annihilation operator**

$$a |\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \sqrt{n} |n-1\rangle$$

Quick review of coherent states

It is an eigenstate of the annihilation operator

$$\begin{aligned} a|\alpha\rangle &= e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \sqrt{n} |n-1\rangle \\ &= e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{(n-1)!}} \sqrt{n} |n-1\rangle = \alpha|\alpha\rangle \end{aligned}$$

Quick review of coherent states

Summary

$$a |\alpha\rangle = \alpha |\alpha\rangle$$

$$\alpha = |\alpha| e^{i\theta}$$

Quick review of coherent states

Summary

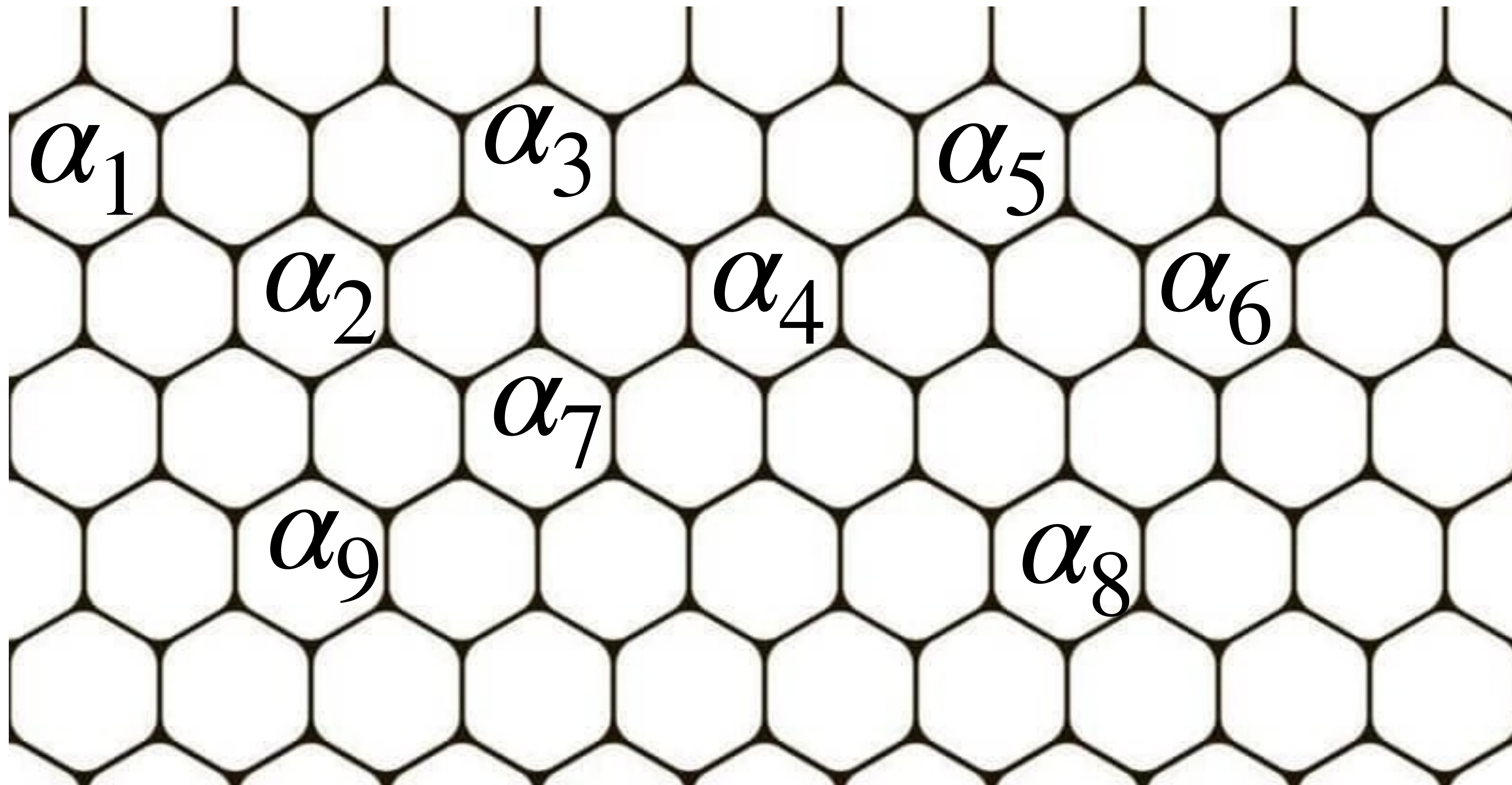
$$a |\alpha\rangle = \alpha |\alpha\rangle$$

$$\alpha = |\alpha| e^{i\theta}$$

Effectively

$$a \rightarrow \alpha \quad a^\dagger \rightarrow \alpha^*$$

The scalar field can be seen as



Time hierarchy

▶ \mathcal{V}



Time

Time hierarchy

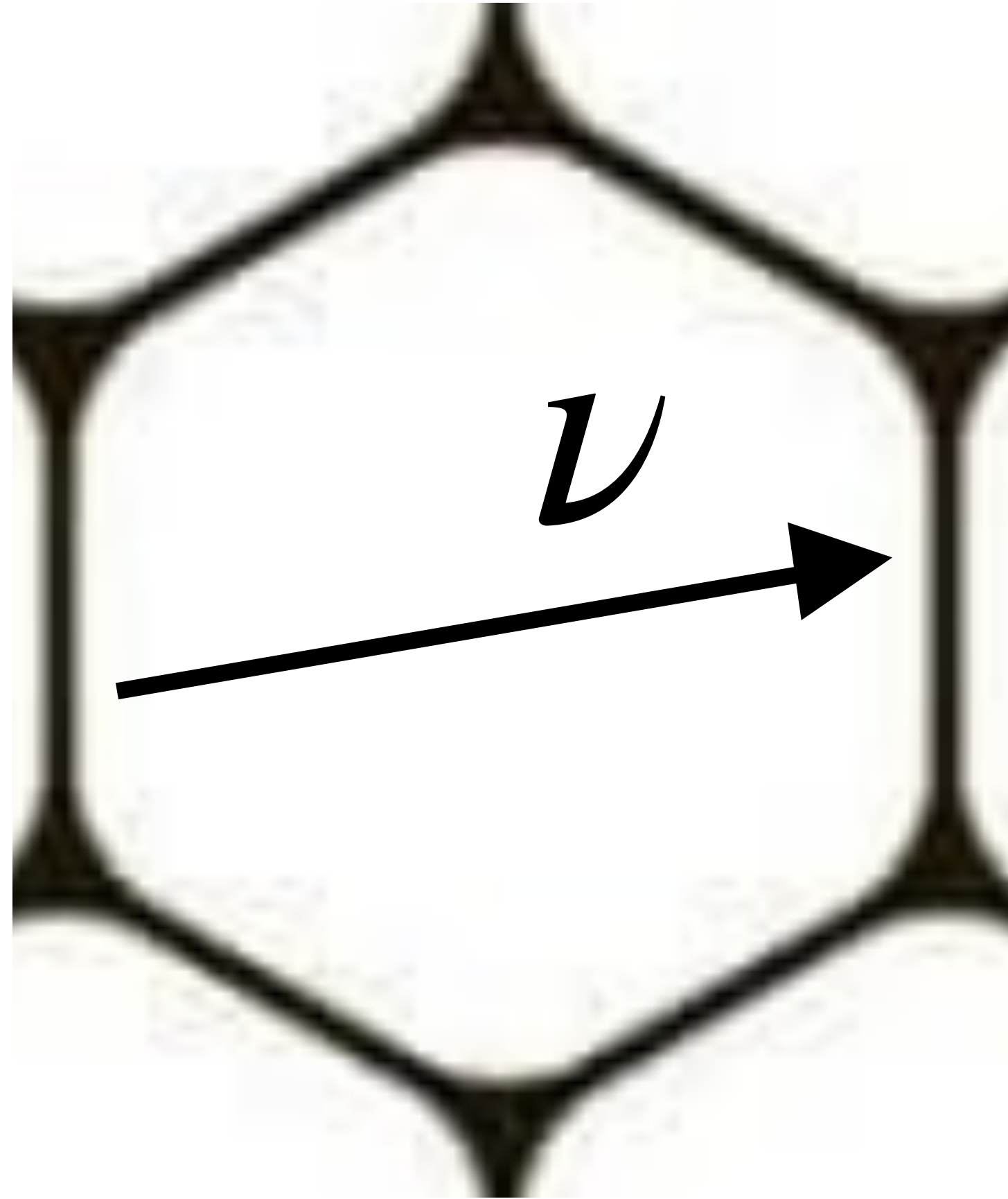
▶ \mathcal{V}

.....▶ **Scalar modulation**



Time

The scales determine the scalar state



Field state

$|\alpha\rangle$

The scales determine the scalar state

$$\phi(x, t) = \sum_k \frac{1}{\sqrt{2VE_k}} (a_k e^{-ikx} + a_k^\dagger e^{ikx})$$

The scales determine the scalar state

$$\phi(x, t) = \sum_k \frac{1}{\sqrt{2VE_k}} (a_k e^{-ikx} + a_k^\dagger e^{ikx})$$

Coherent states

$$\phi(x, t) = \sum_k \frac{1}{\sqrt{2VE_k}} (\alpha_k e^{-ikx} + \alpha_k^\dagger e^{ikx})$$

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Coherent states

$$\phi(x, t) = \sum_k \frac{1}{\sqrt{2VE_k}} (\alpha_k e^{-ikx} + \alpha_k^\dagger e^{ikx})$$

Non relativistic

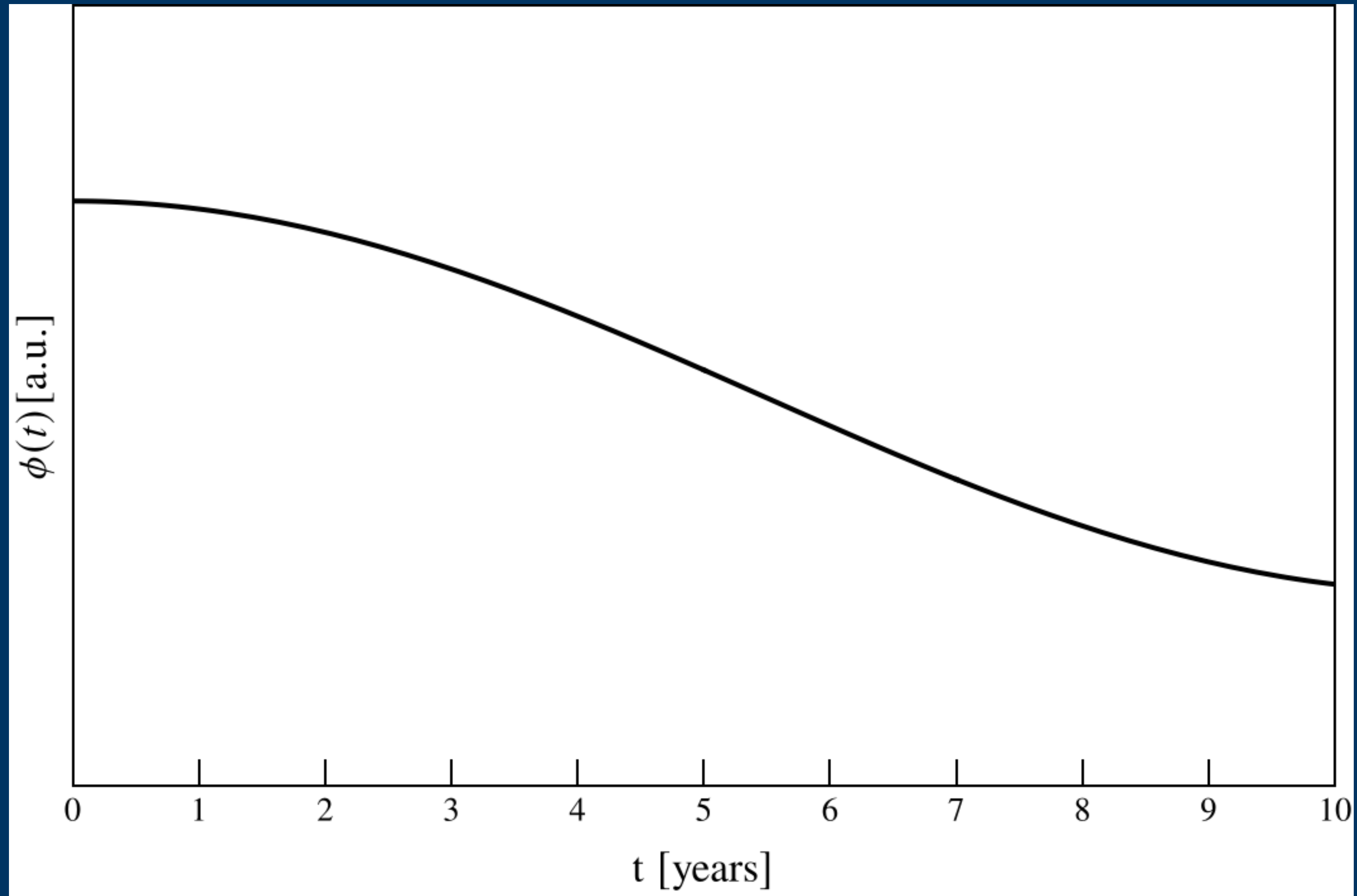
$$\phi(x, t) \approx \frac{\sqrt{2\rho_\phi}}{m_\phi} \cos(m_\phi t + \theta)$$

Neutrino masses become time dependent

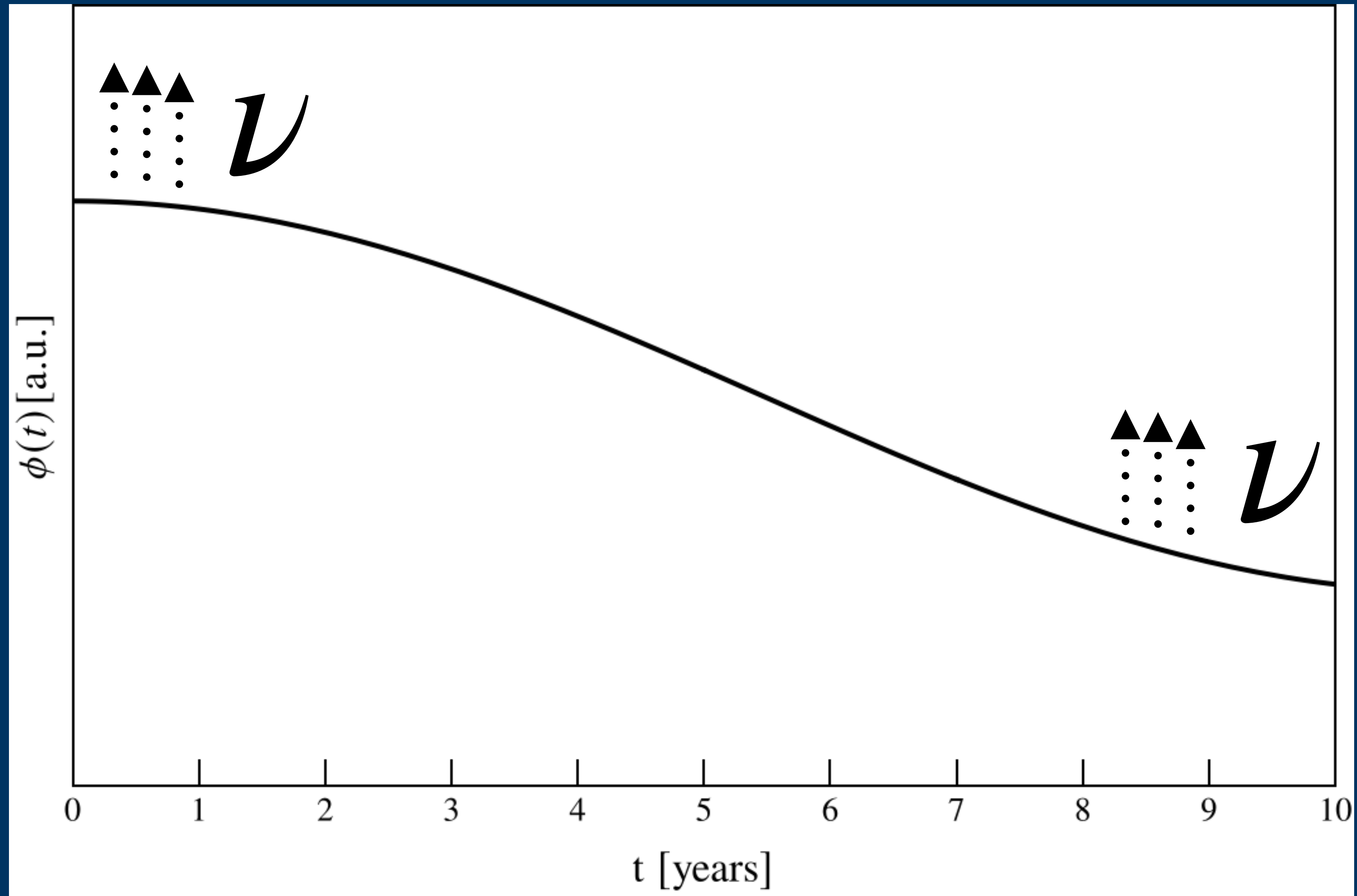
$$m_{\nu_i}(t) = m_{\nu_i}^0 + g_{\phi,i}\phi_0 \cos(m_\phi t + \theta)$$

First scenario

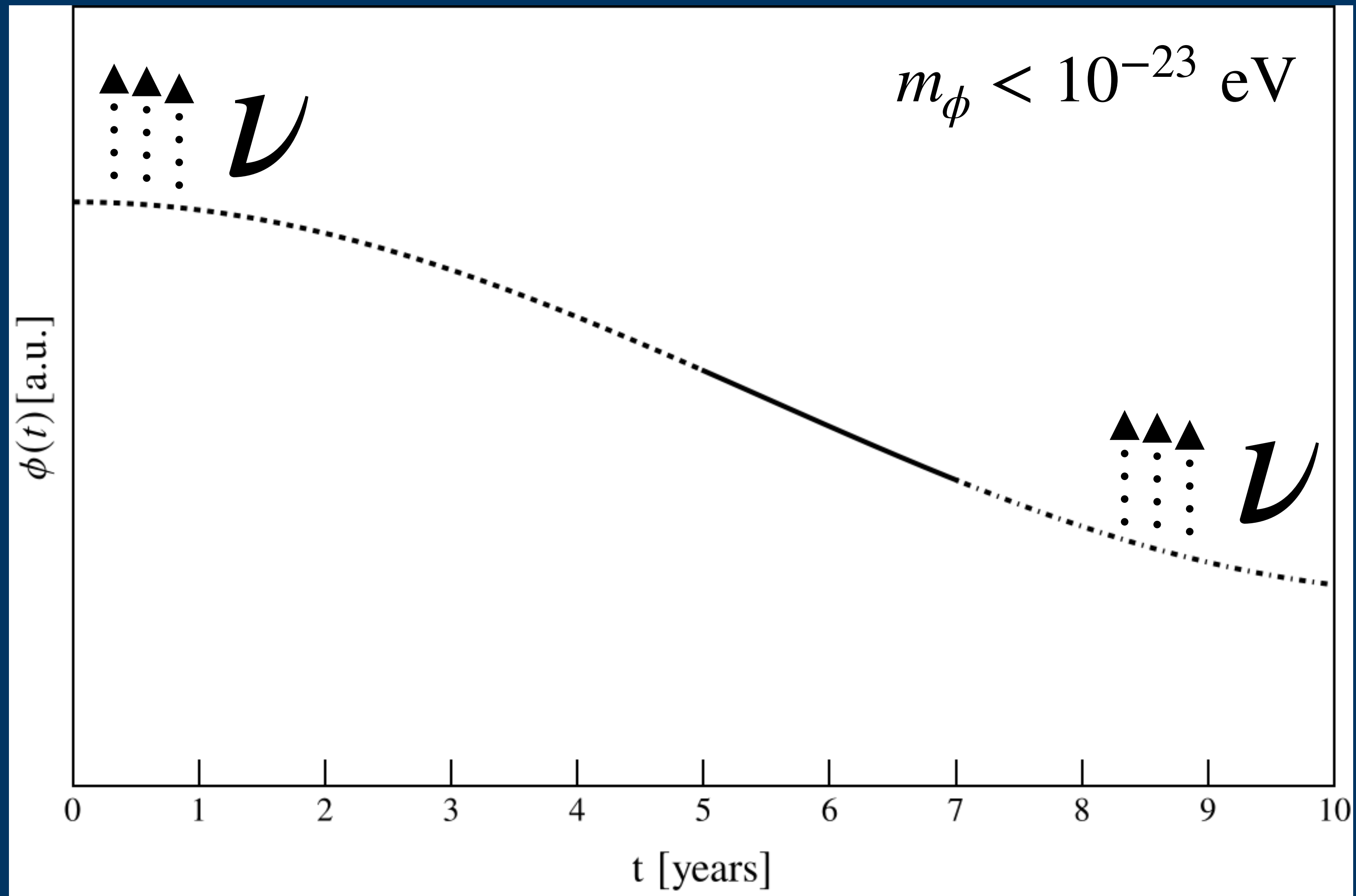
First: Very slow scalar modulations



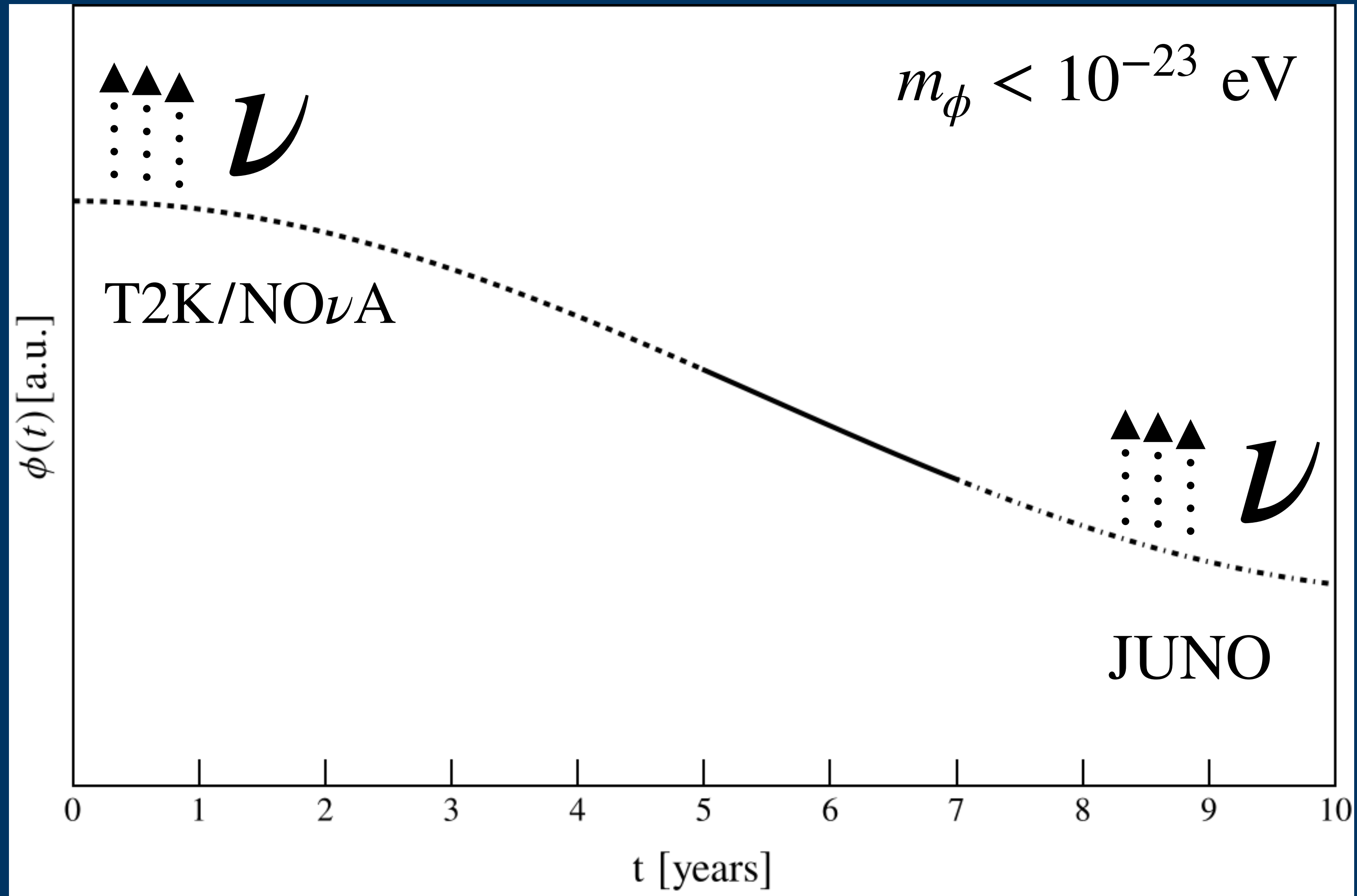
Very slow scalar modulations



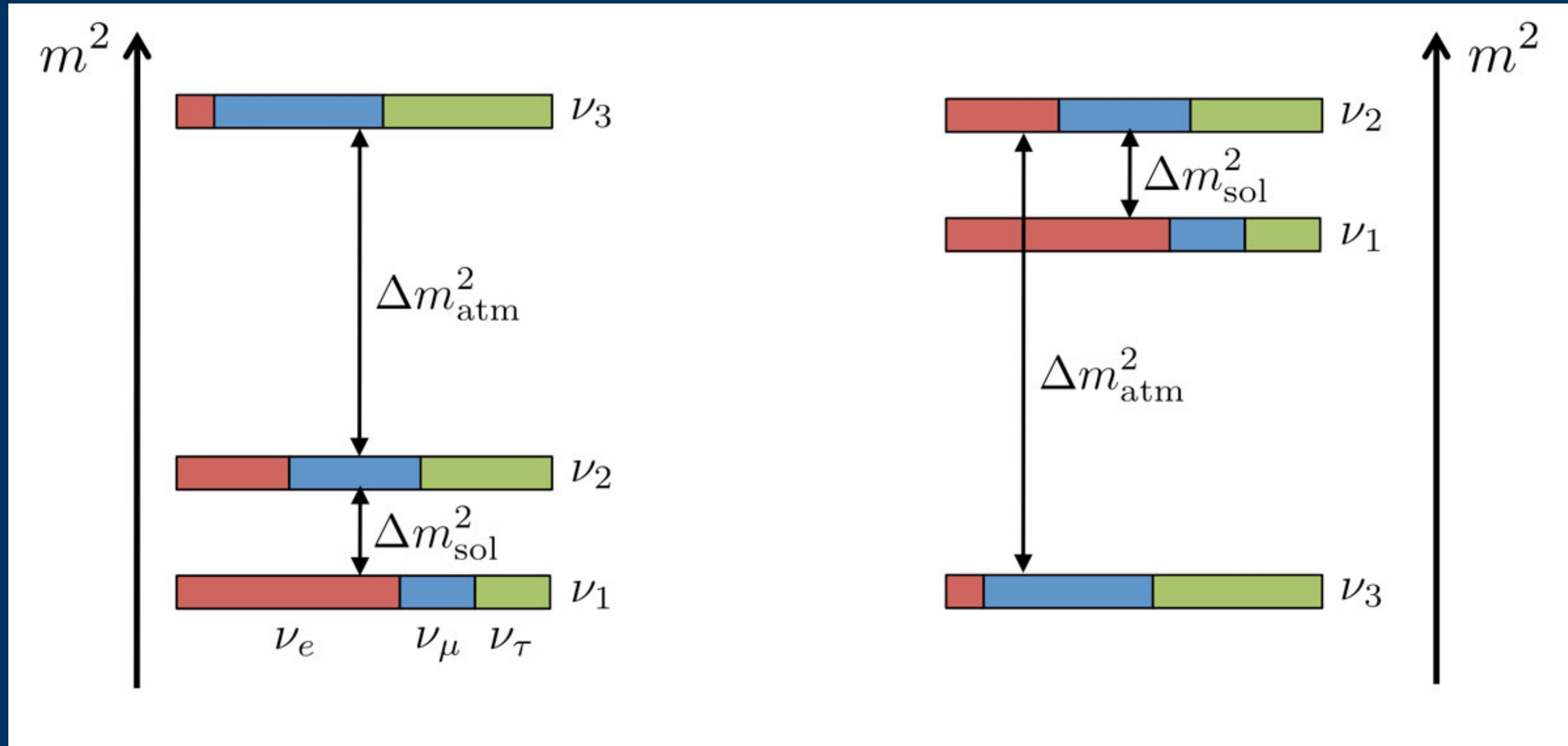
Very slow scalar modulations



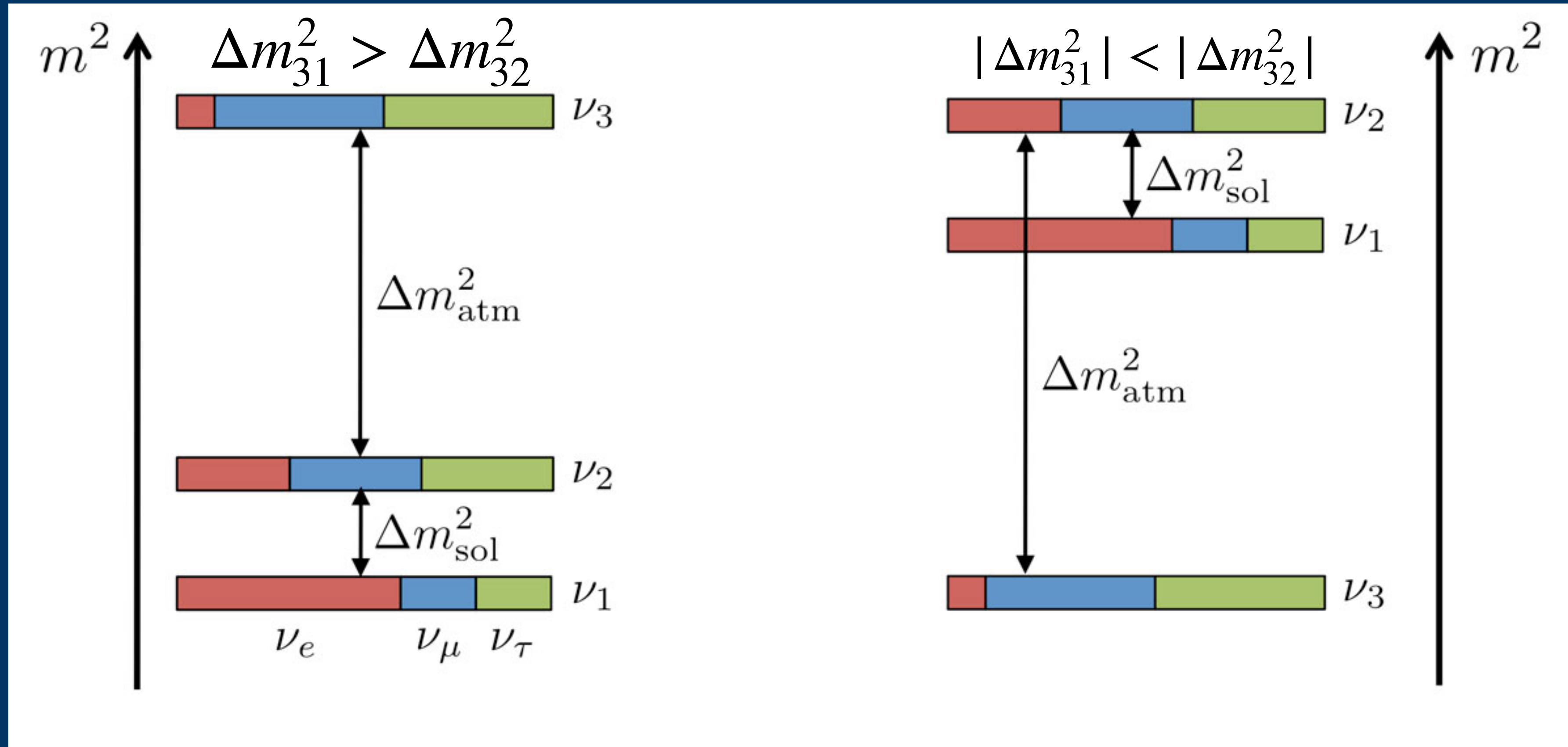
Very slow scalar modulations



What might happen?



What might happen?



One way to determine the ordering

T2K/NO ν A
(LBL)

One way to determine the ordering

$$\begin{array}{l} \text{T2K/NO}\nu\text{A} \\ \text{(LBL)} \end{array} \quad \begin{array}{l} P(\nu_{\mu} \rightarrow \nu_{\mu}) \\ P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\mu}) \end{array}$$

One way to determine the ordering

$$\begin{array}{l} \text{T2K/NO}\nu\text{A} \\ \text{(LBL)} \end{array} \quad \begin{array}{l} P(\nu_{\mu} \rightarrow \nu_{\mu}) \\ P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\mu}) \end{array}$$

JUNO

One way to determine the ordering

T2K/NO ν A
(LBL)

$$P(\nu_{\mu} \rightarrow \nu_{\mu})$$
$$P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\mu})$$

JUNO

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$$

One way to determine the ordering

T2K/NO ν A
(LBL)

$$P(\nu_{\mu} \rightarrow \nu_{\mu})$$

$$P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\mu})$$

$$\Delta m_{31}^2 ?$$

JUNO

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$$

$$\Delta m_{32}^2 ?$$

What is actually measured

T2K/NO ν A
(LBL)

$$\Delta m_{\mu\mu}^2$$

JUNO

$$\Delta m_{ee}^2$$

What is actually measured

T2K/NO ν A
(LBL)

$$\Delta m_{\mu\mu}^2 \approx \Delta m_{31}^2 \sin^2 \theta_{12} + \Delta m_{32}^2 \cos^2 \theta_{12} + \Delta m_{21}^2 \sin \theta_{13} \cos \delta$$

JUNO

$$\Delta m_{ee}^2 = \Delta m_{31}^2 \cos^2 \theta_{12} + \Delta m_{32}^2 \sin^2 \theta_{21}$$

Rephrasing the mass ordering question

NO

IO

$$|\Delta m_{ee}^2| > |\Delta m_{\mu\mu}^2|$$

$$|\Delta m_{ee}^2| < |\Delta m_{\mu\mu}^2|$$

Rephrasing the mass ordering question

NO

IO

$$|\Delta m_{ee}^2| > |\Delta m_{\mu\mu}^2| \quad |\Delta m_{ee}^2| < |\Delta m_{\mu\mu}^2|$$

This naturally requires precise measurements

Fortunately

another. JUNO claims that after 100 days of data taking they will be able to determine $|\Delta m_{\text{atm}}^2|$ at 0.8% precision and will continue to improve ultimately reaching 0.2%

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how many units of $\Delta\chi^2$. In this manner it is conceivable, if JUNO measures $|\Delta m_{\text{atm}}^2|$ close to the one given by combining Daya Bay and RENO data, that NO could be soon (in a year or so) determined, by the combined (T2K, NOvA and JUNO) disappearance measurements alone, to better than 3σ i.e. a confidence level of 99.73%.

The actual question

If nature's
choice is

$$|\Delta m_{ee}^2| > |\Delta m_{\mu\mu}^2|$$

NO

The actual question

If nature's
choice is

$$|\Delta m_{ee}^2| > |\Delta m_{\mu\mu}^2|$$

NO

**But neutrinos
couple to ULDM**

The actual question

If nature's
choice is

$$|\Delta m_{ee}^2| > |\Delta m_{\mu\mu}^2|$$

NO

But neutrinos
couple to ULDM

Could we get
confused?

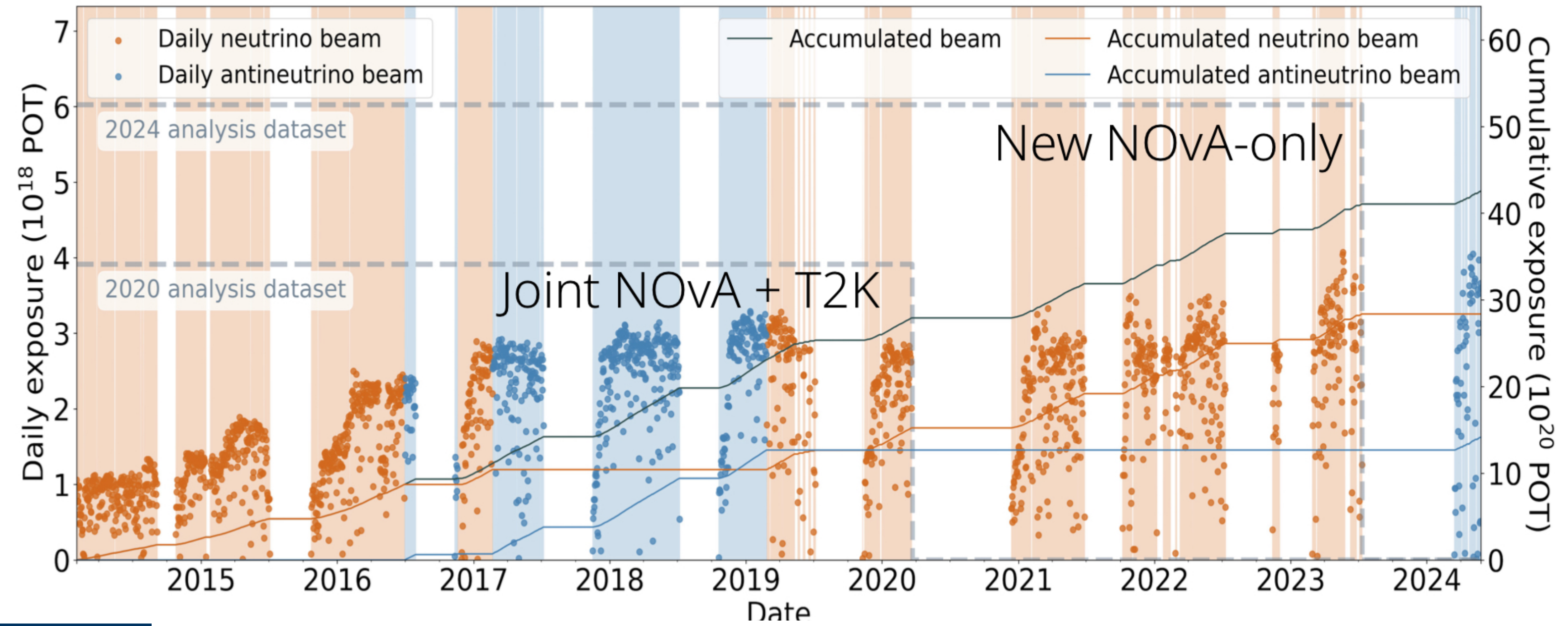
$$|\Delta m_{ee}^2| < |\Delta m_{\mu\mu}^2|$$

The actual question

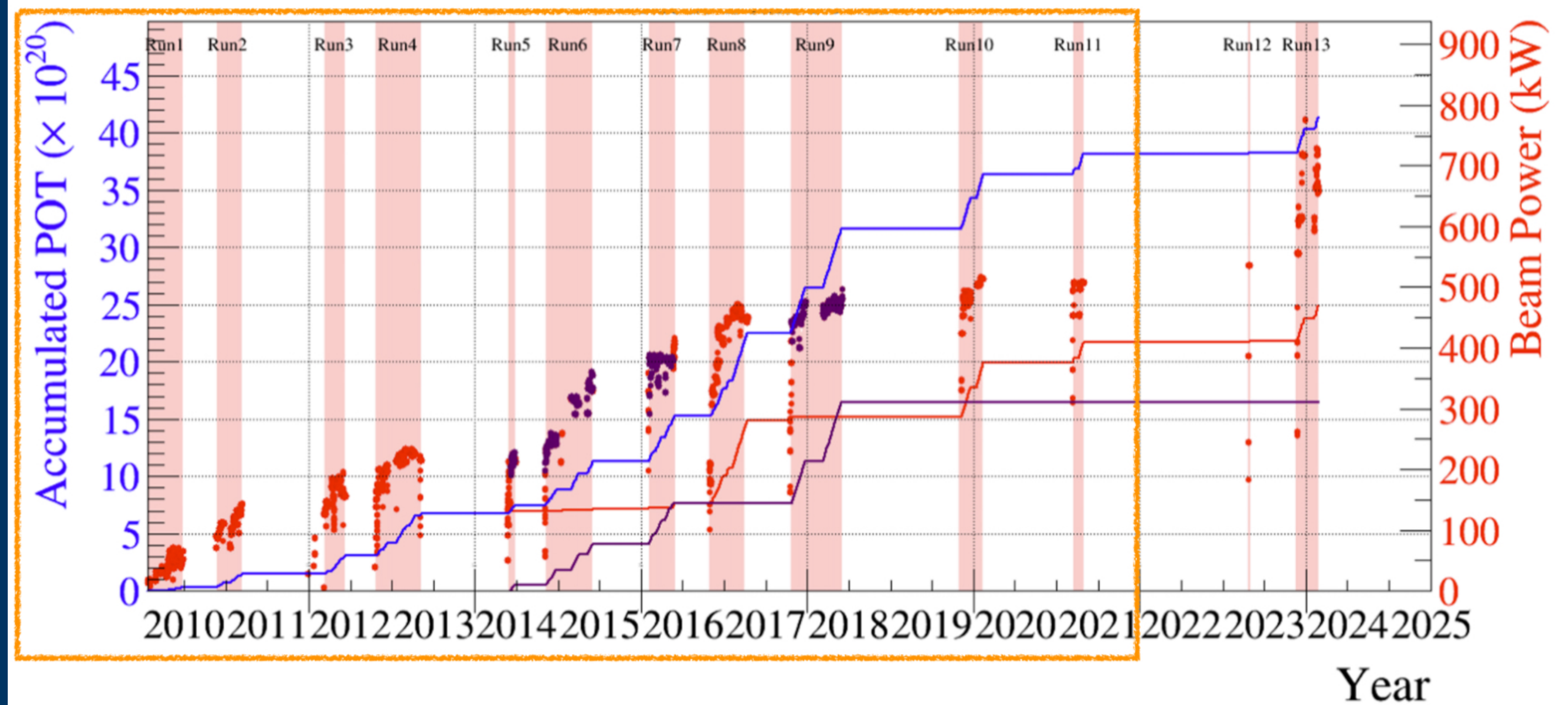
Can New Physics Spoil the JUNO–Long-Baseline Synergy in the Neutrino Mass Ordering Determination?

Gustavo F. S. Alves^{1,2,3,*} Hiroshi Nunokawa^{4,†} and Renata Zukanovich Funchal^{3,‡}

Wolcott, New neutrino oscillation results from NOvA with 10 years of data (talk-neutrino 2024)



LBL data taking periods



**JUNO has
just started!**

JUNO Releases First Physics Results
**The World's Most Precise Measurement
of Two Neutrino Oscillation Parameters**

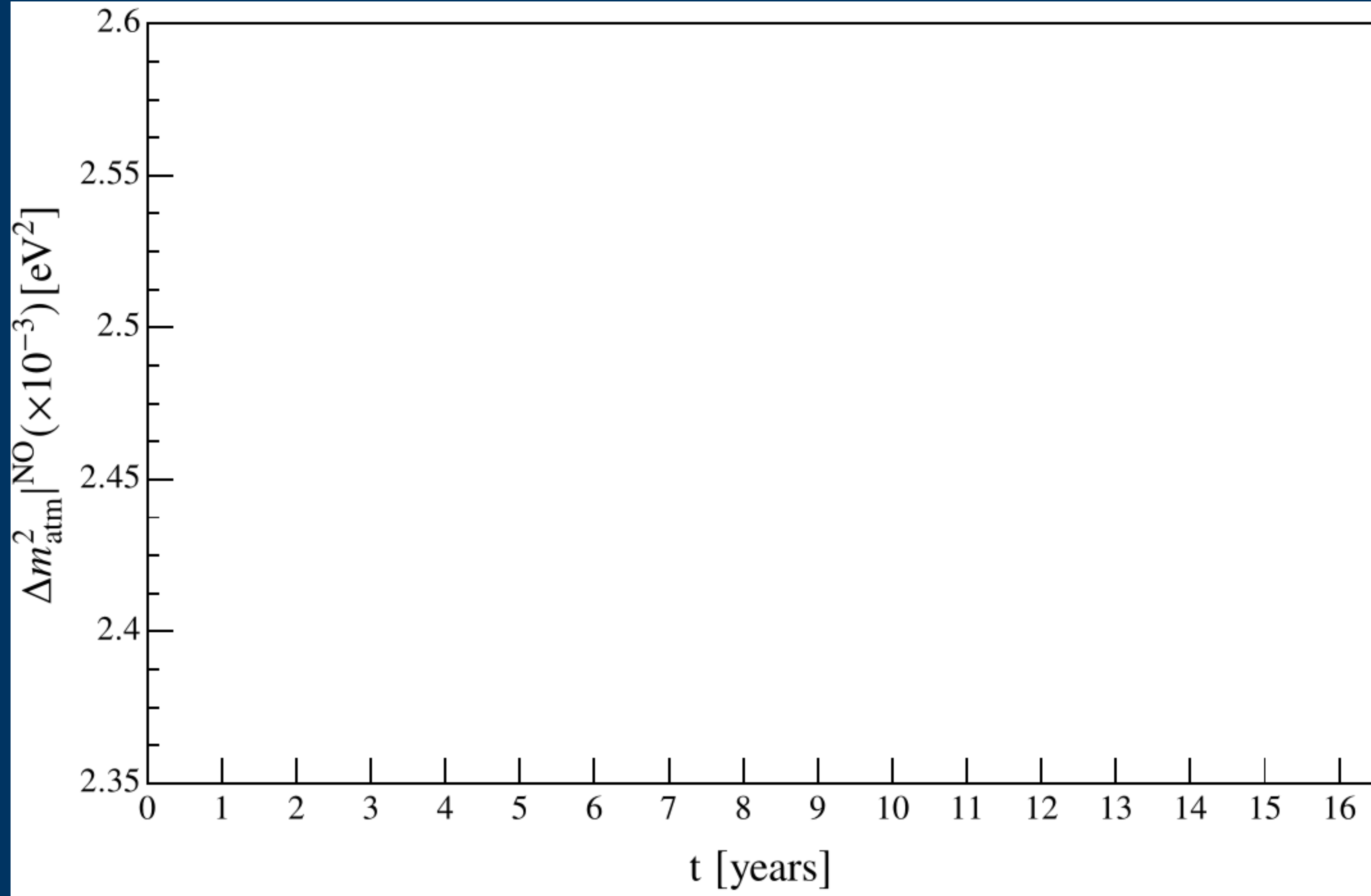


θ_{12} & Δm_{21}^2 : measured about twice precise
than previous experiments' average

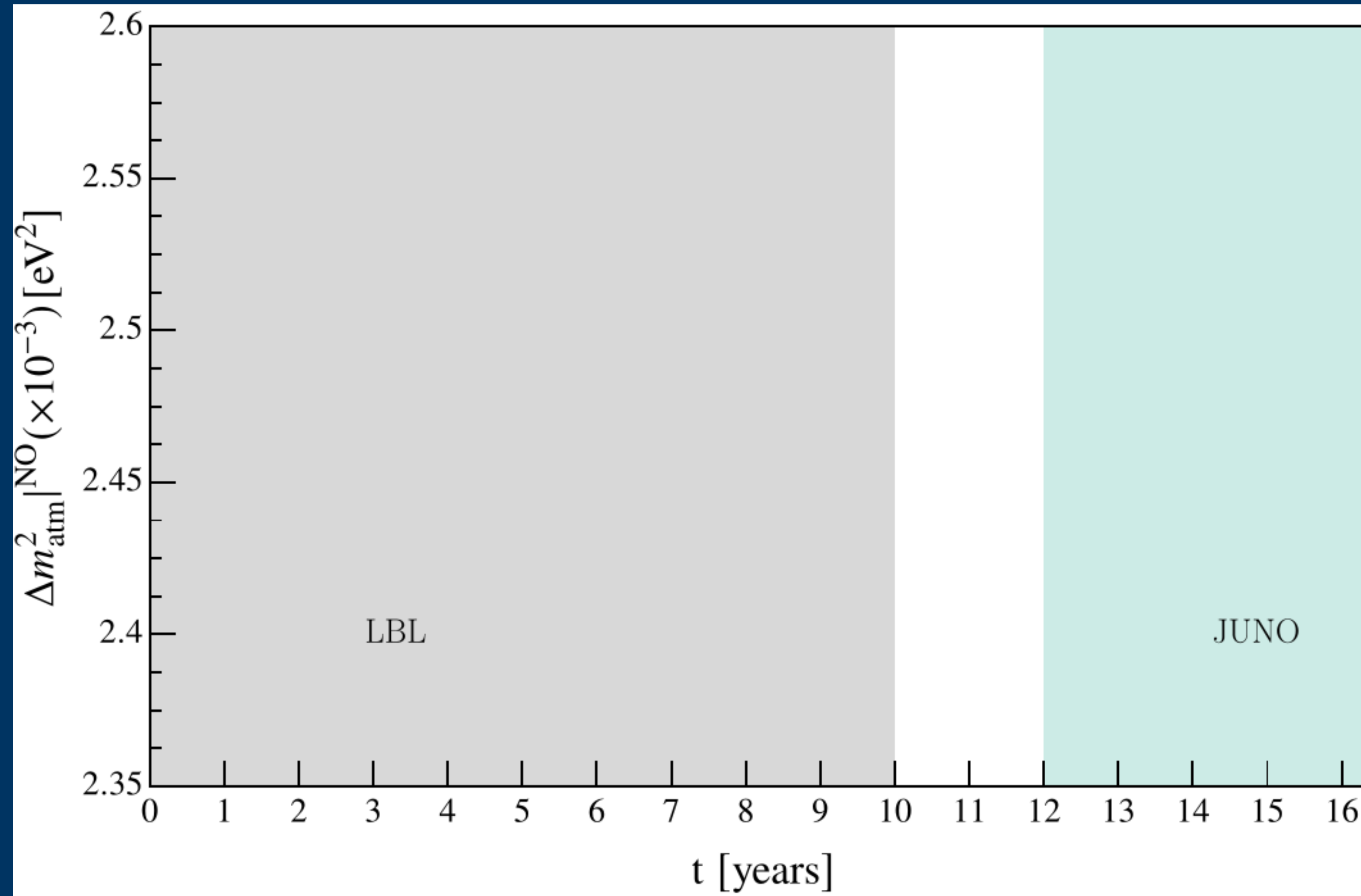
Setup

**We will assume the scalar only couples to the
third mass eigenstate**

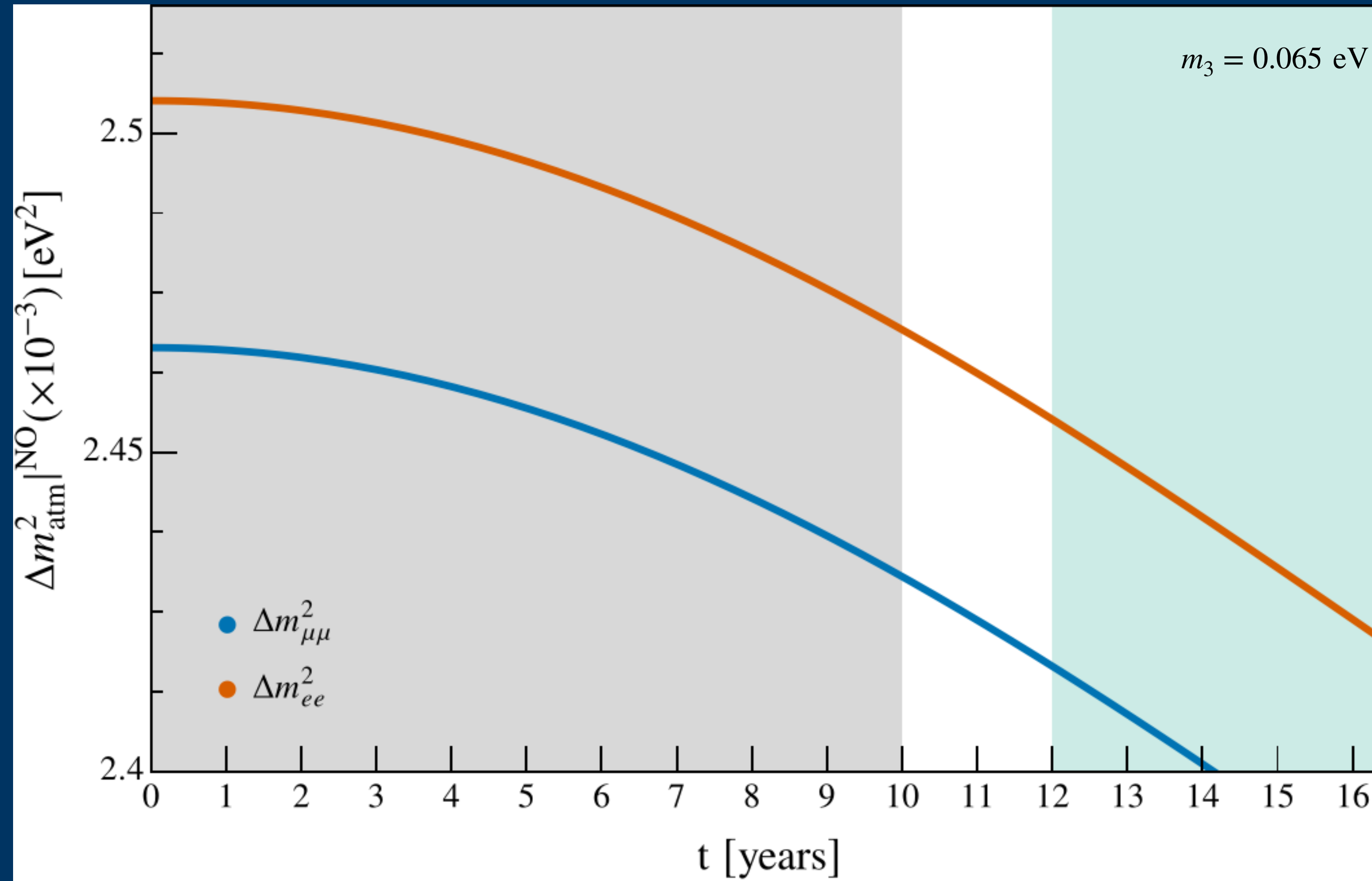
$$\Delta m_{31}^2(t) \approx \Delta m_{31}^2(g_\phi = 0) + 2m_3 g_{\phi,3} \phi_0 \cos(m_\phi t)$$



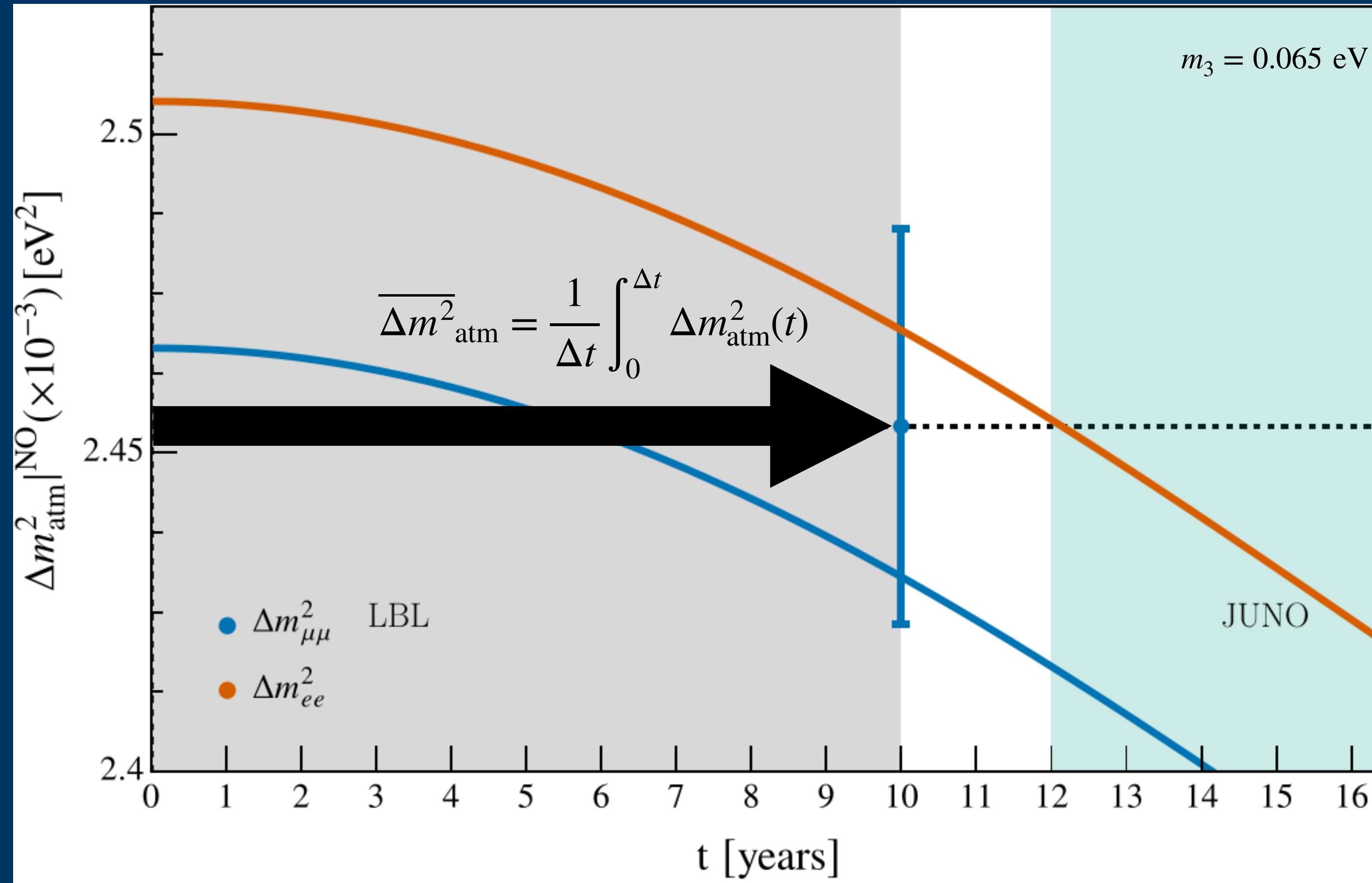
LBL and JUNO are asynchronous



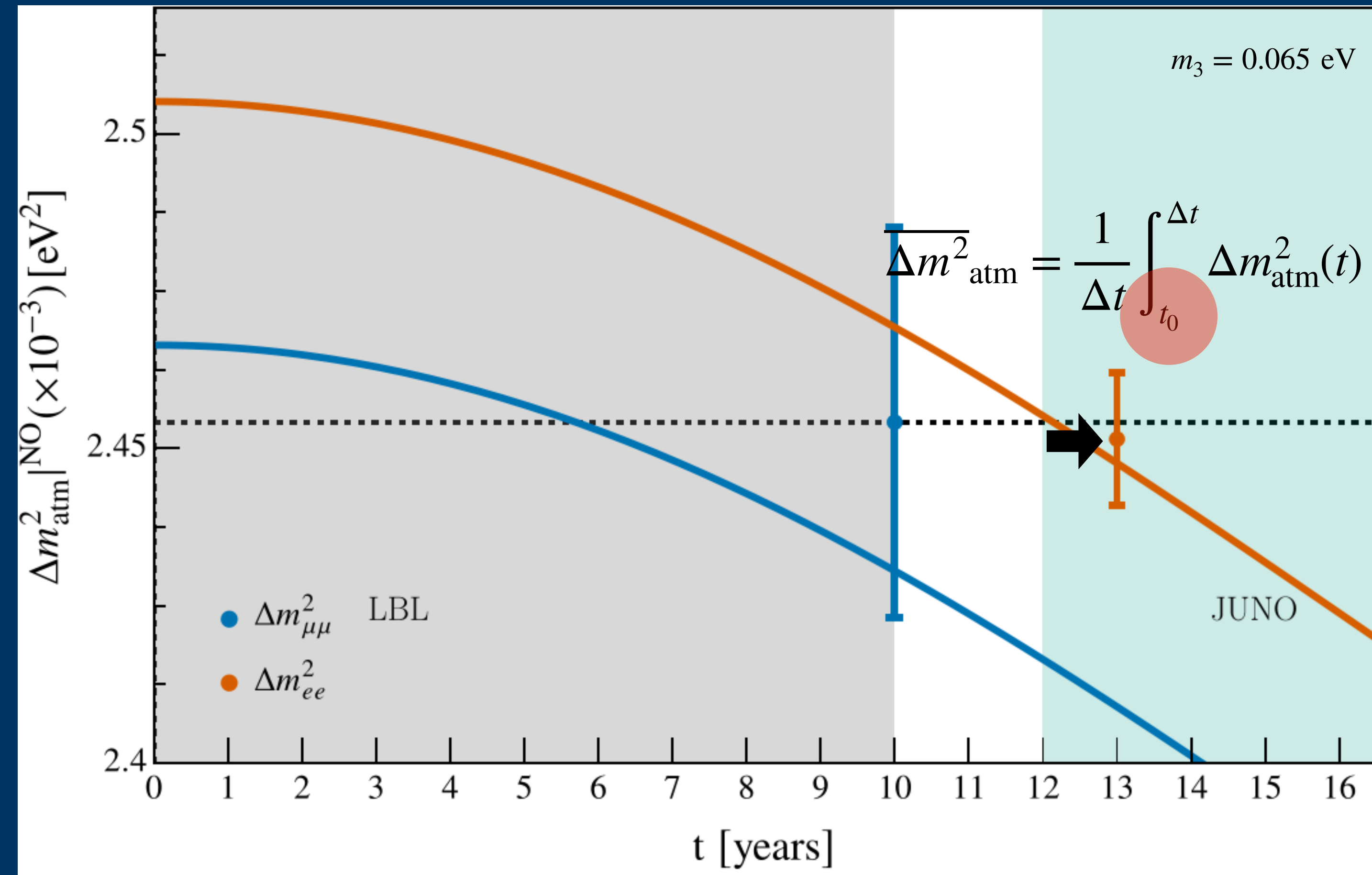
Instantaneous mass splittings



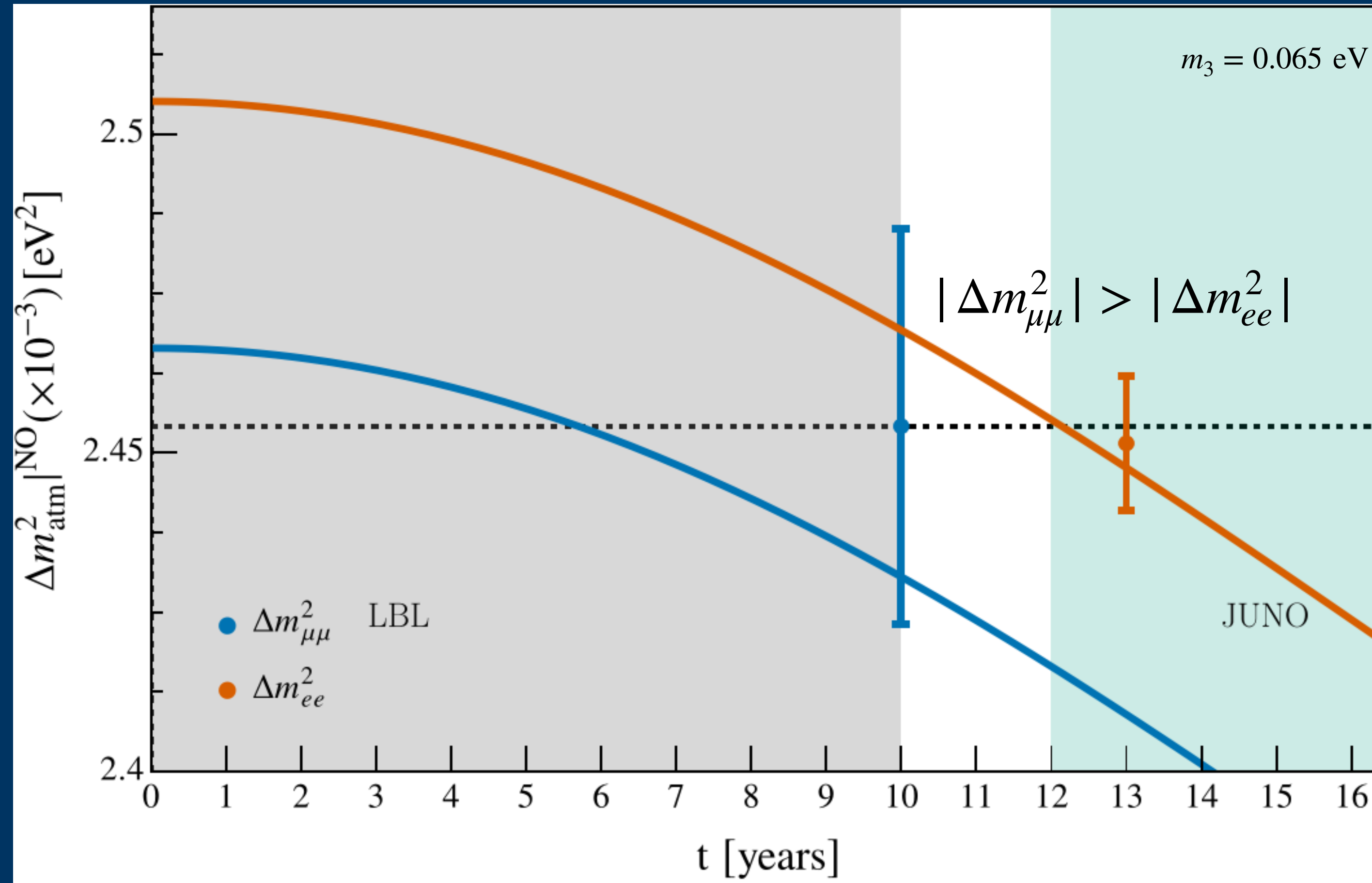
Experimental result LBL



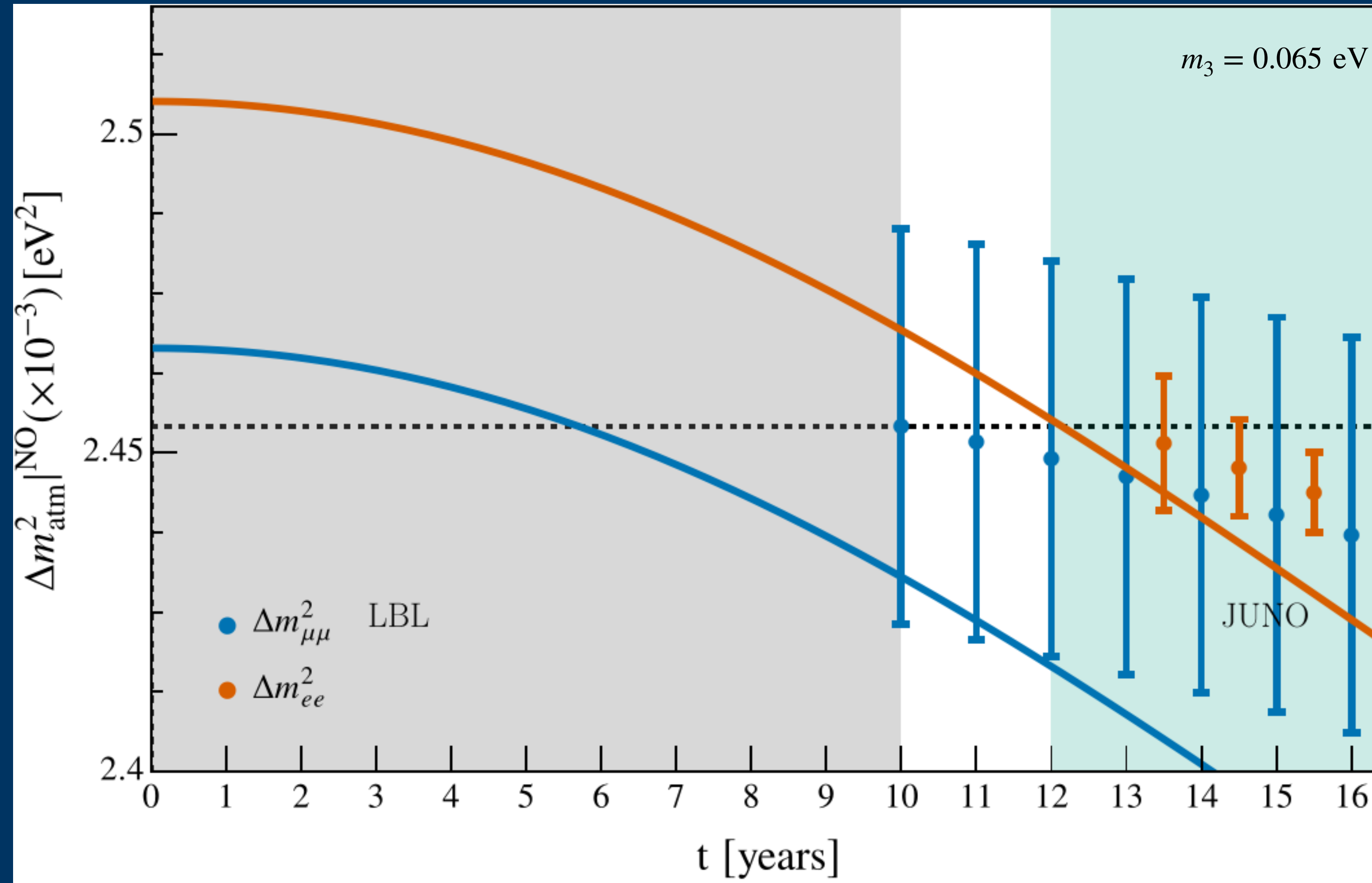
Experimental result JUNO



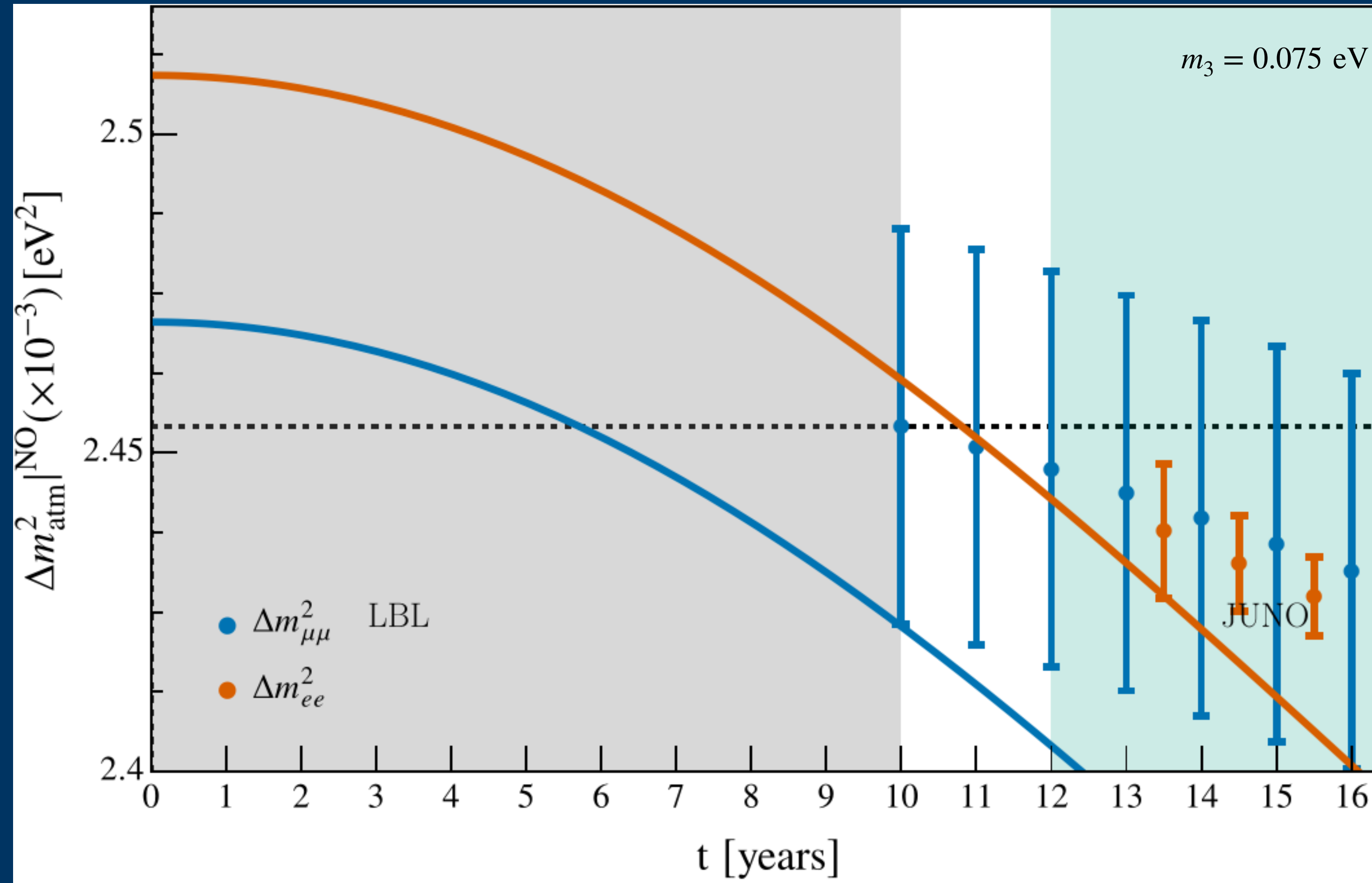
Apparent mass ordering confusion



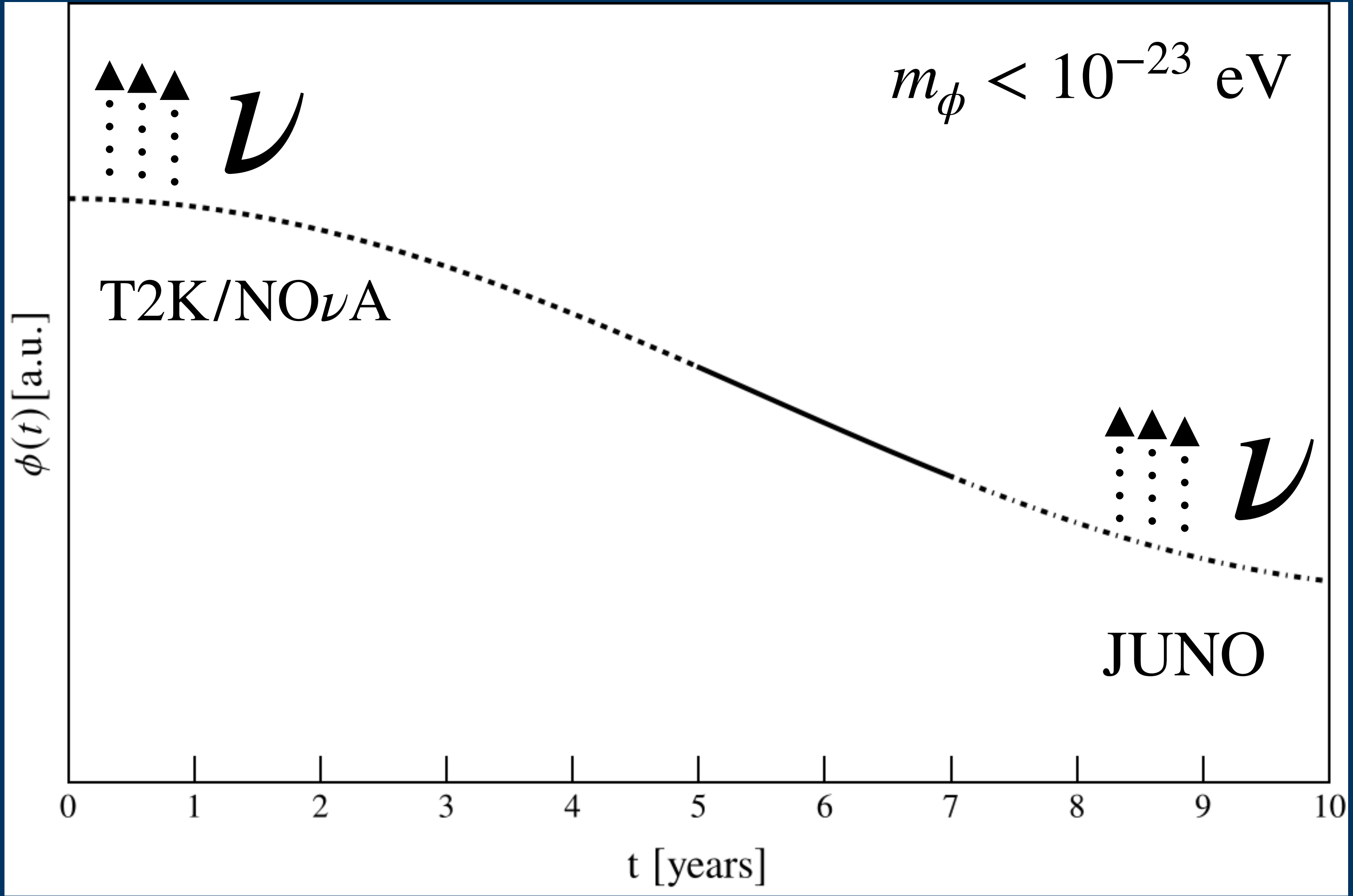
Conclusions are time dependent



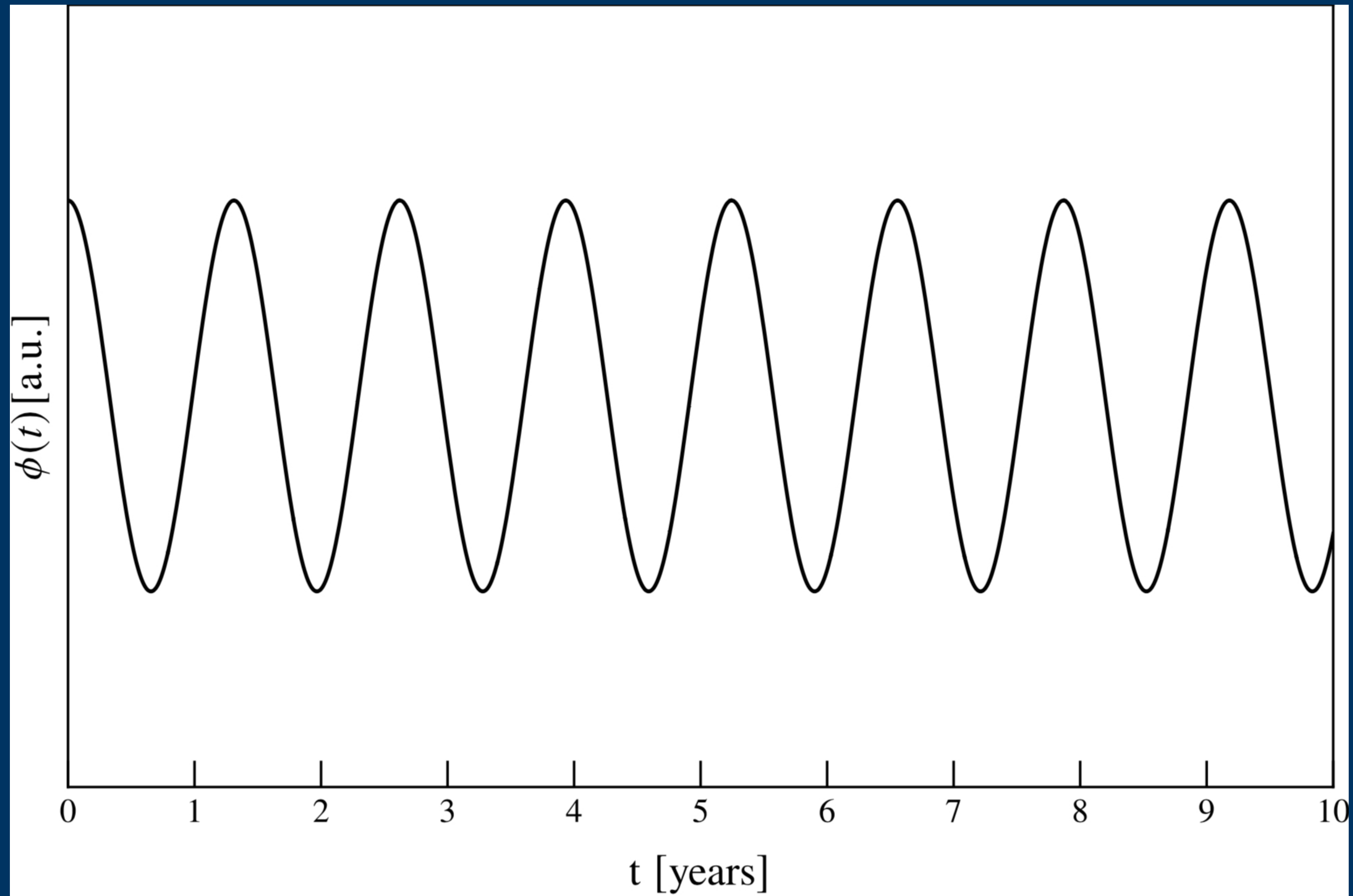
Conclusions are time dependent



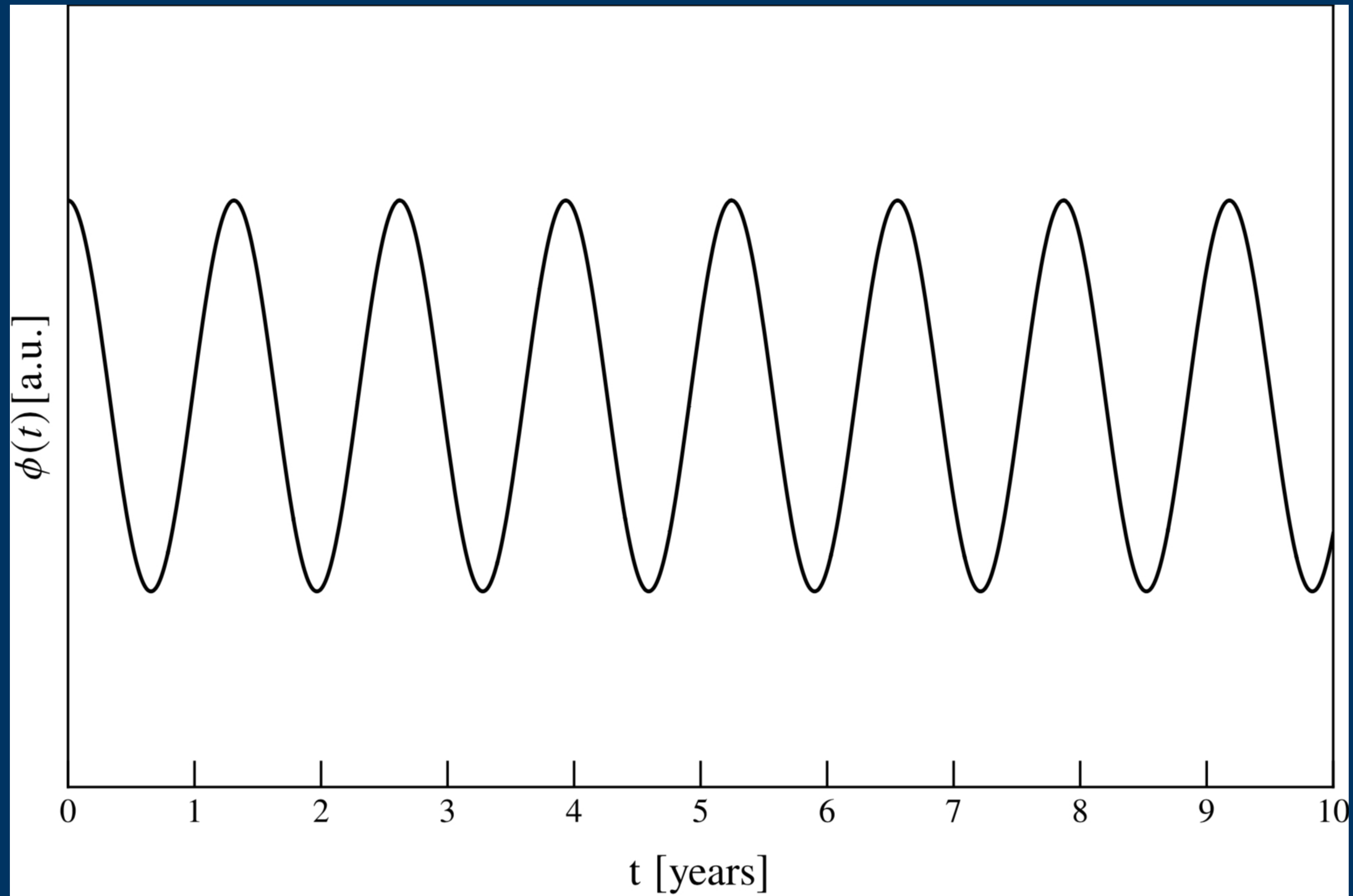
Done with the first scenario



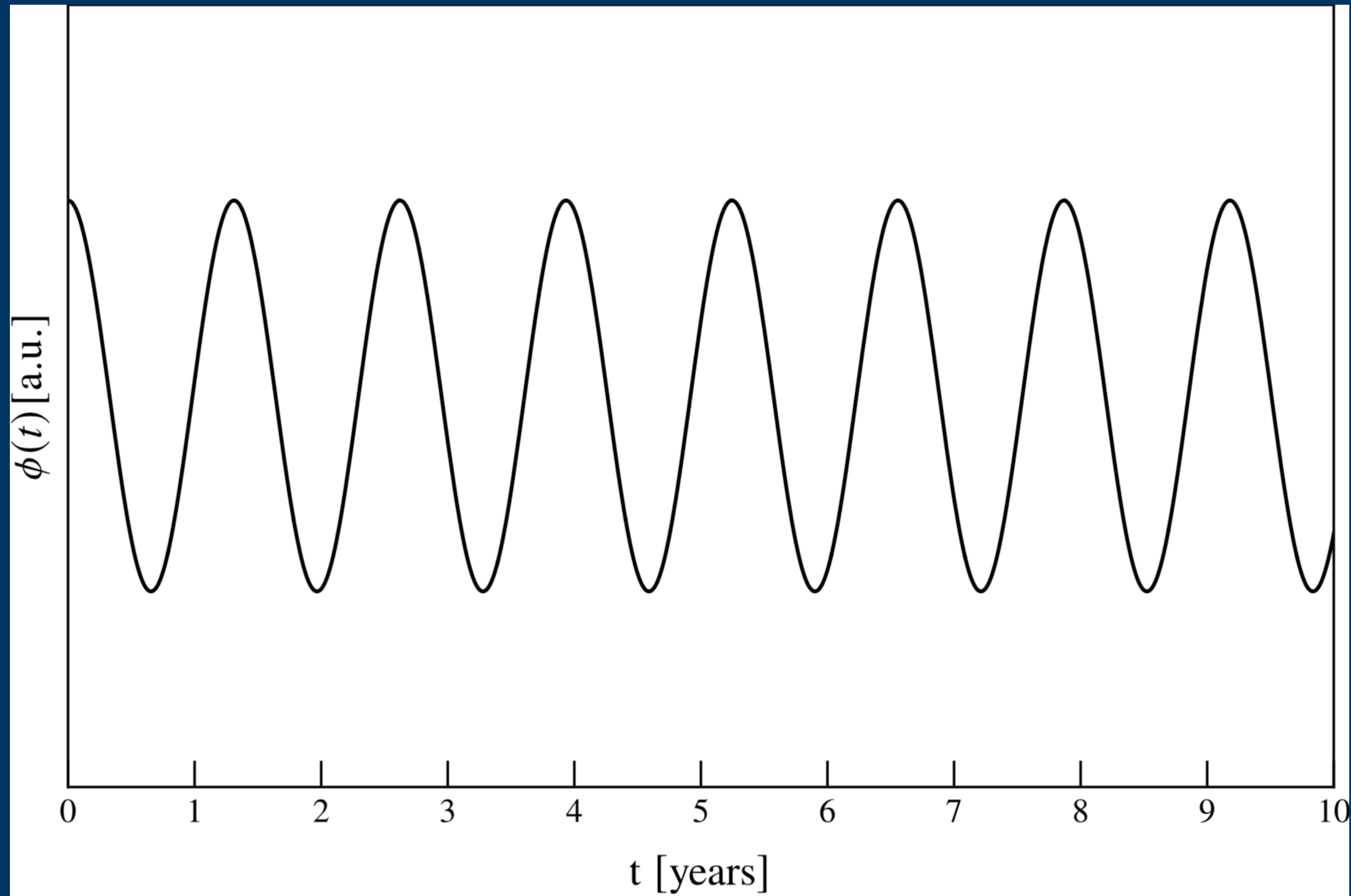
Second scenario



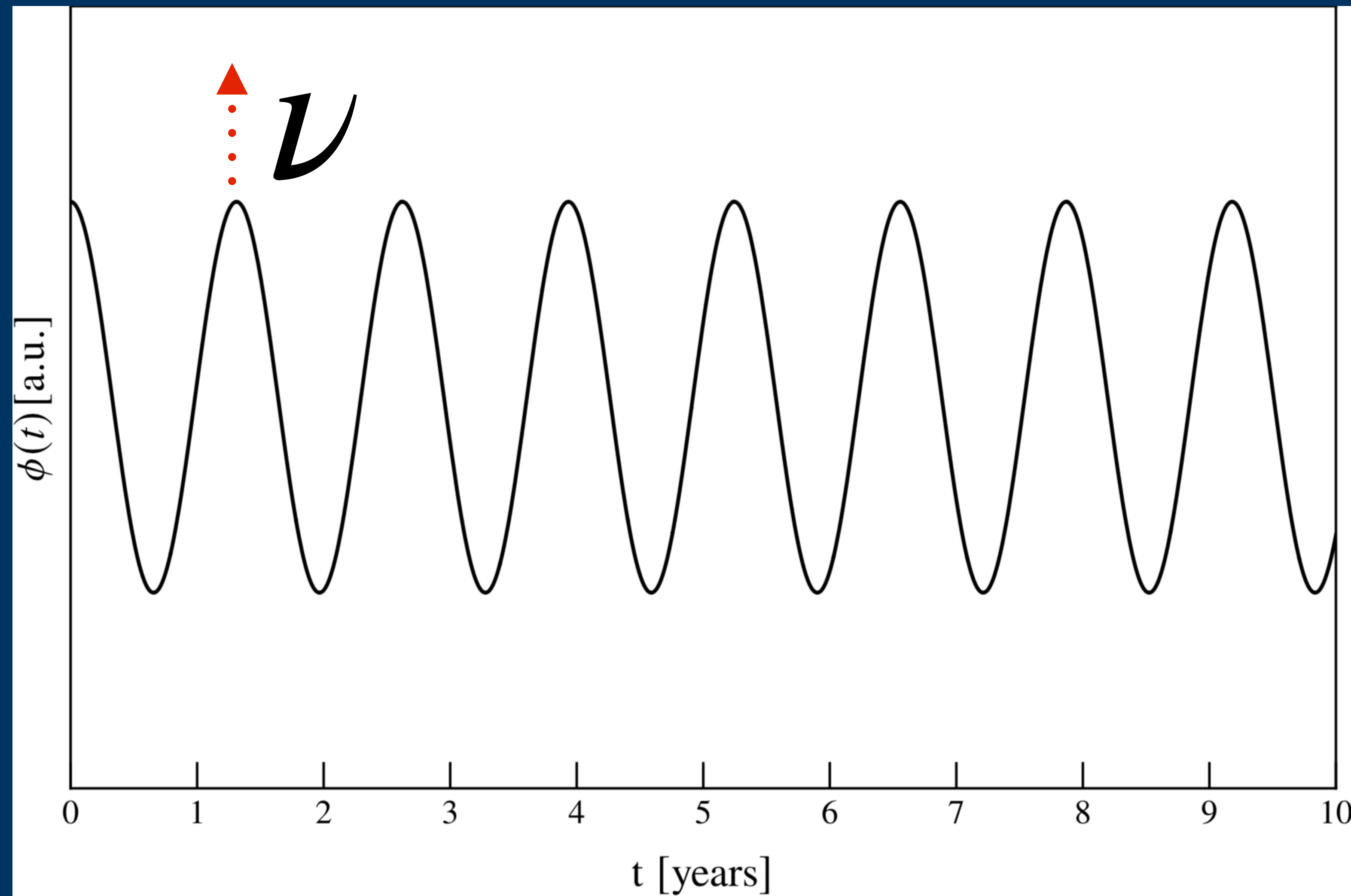
Several cycles over the experimental livetime



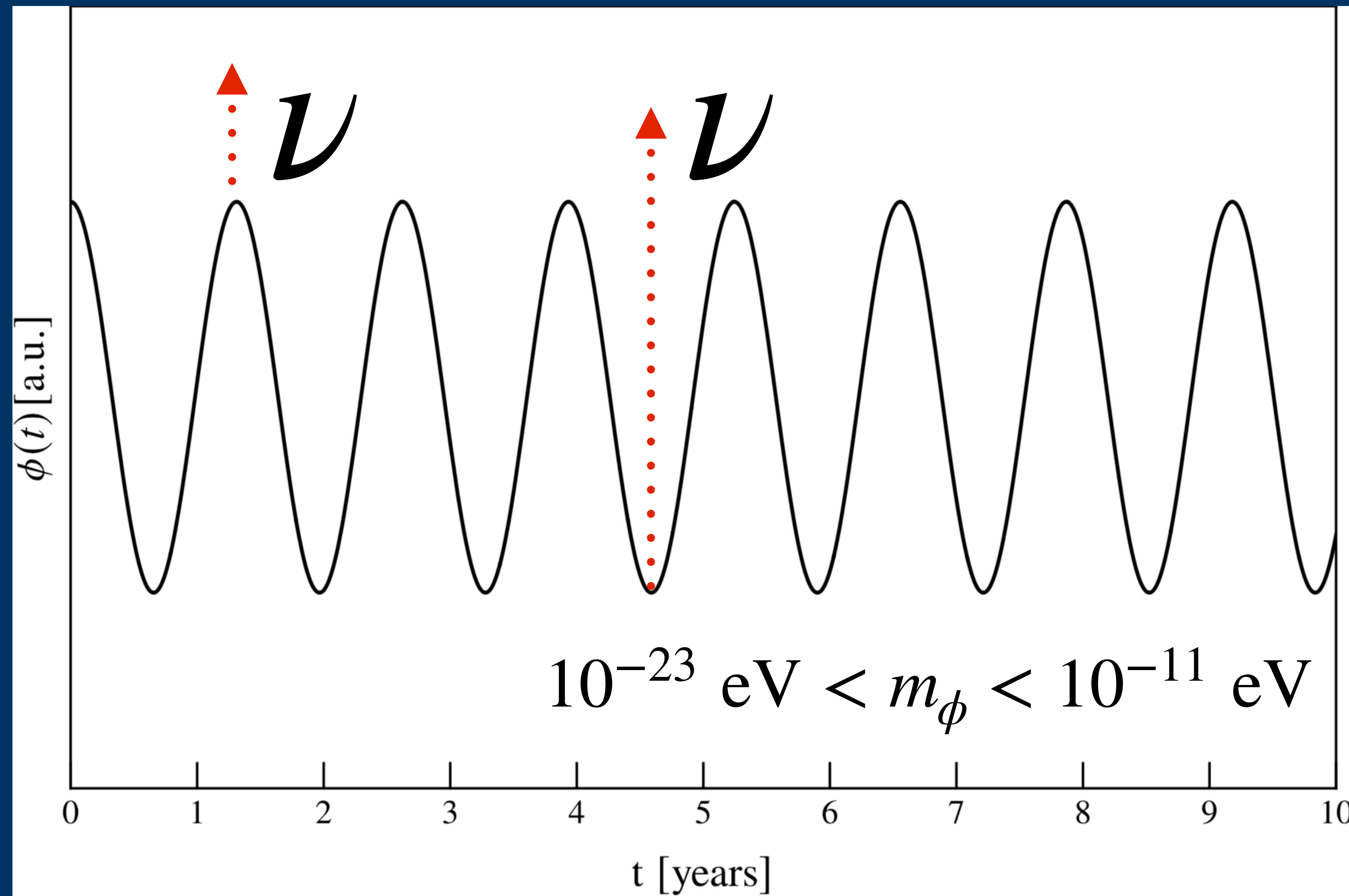
Neutrinos start with different initial conditions



Neutrinos start with different initial conditions



Neutrinos start with different initial conditions



Most explored scenario

$$10^{-23} \text{ eV} \leq m_\phi \leq 10^{-11} \text{ eV}$$

Neutrino Oscillations as a Probe of Light Scalar Dark Matter

[Asher Berlin](#)

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Phys. Rev. Lett. **117**, 231801 – Published 30 November, 2016

DOI: <https://doi.org/10.1103/PhysRevLett.117.231801>

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Distorted neutrino oscillations from time varying cosmic fields

[Gordan Krnjaic](#)^{1,*}, [Pedro A. N. Machado](#)^{1,†}, and [Lina Necib](#)^{2,‡}

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Phys. Rev. D **97**, 075017 – Published 16 April, 2018

DOI: <https://doi.org/10.1103/PhysRevD.97.075017>

Exp

[Submitted on 20 Dec 2025]

Ultralight dark matter search in a large liquid scintillator detector

[Luis A. Delgadillo](#), [O. G. Miranda](#), [Hiroshi Nunokawa](#)

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Signatures of ultralight dark matter in neutrino oscillation experiments

Regular Article – Theoretical Physics | [Open access](#) | Published: 18 January 2021

Volume 2021, article number 94, (2021) [Cite this article](#)

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[Abhish Dev](#) ✉, [Pedro A. N. Machado](#) & [Pablo Martínez-Miravé](#)

Most explored scenario

I will show that we can also approach the problem from an open systems perspective

Most explored scenario

I will show that we can also approach the problem from an open systems perspective

But why would one like to do that?

Neutrinos as open systems



Physics

Benatti, et. al., JHEP 10.1088

Gago, et. al., Phys. Rev. D., 10.1103

Guzzo, et. al., EPJ C 10.1140

Lisi, et. al., PRL. 85.1166

Guzzo, et. al., Phys. Rev. D., 89.053002

Schwetz, et. al, JHEP JHEP05 (2015) 007

.....

Neutrinos as open systems

Extra CP sources

Physics

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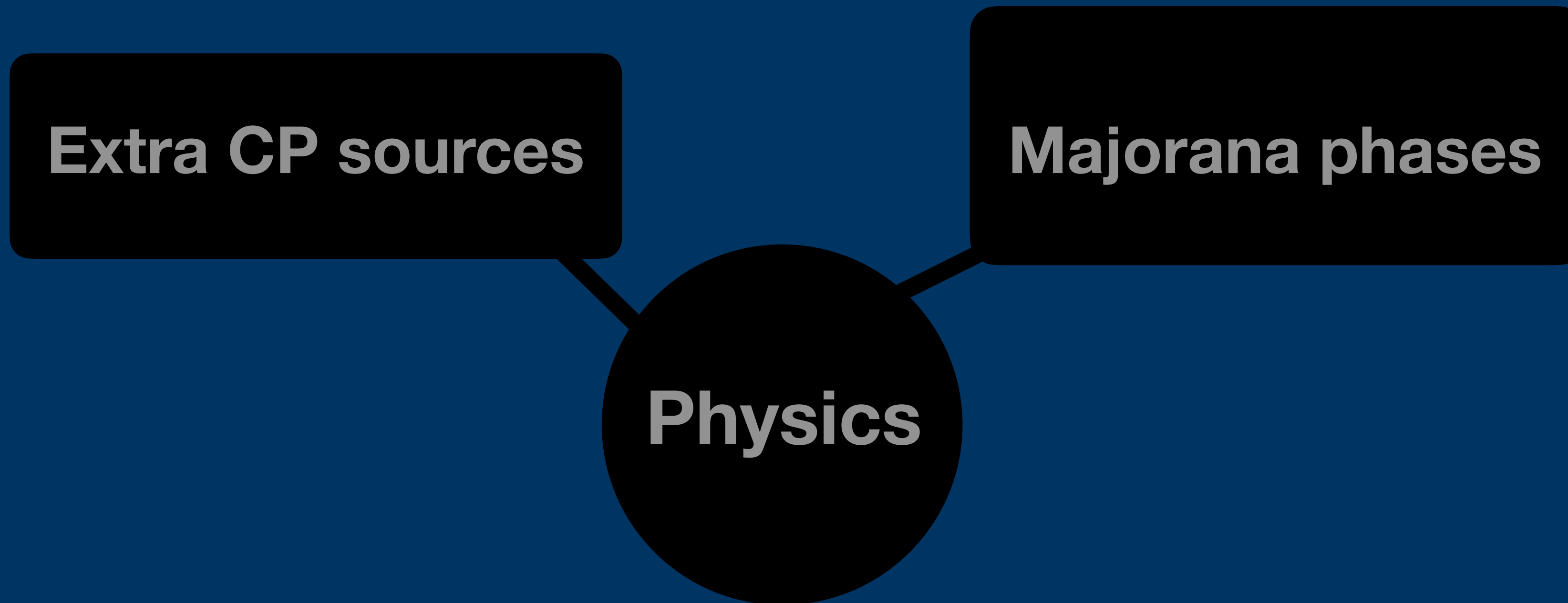
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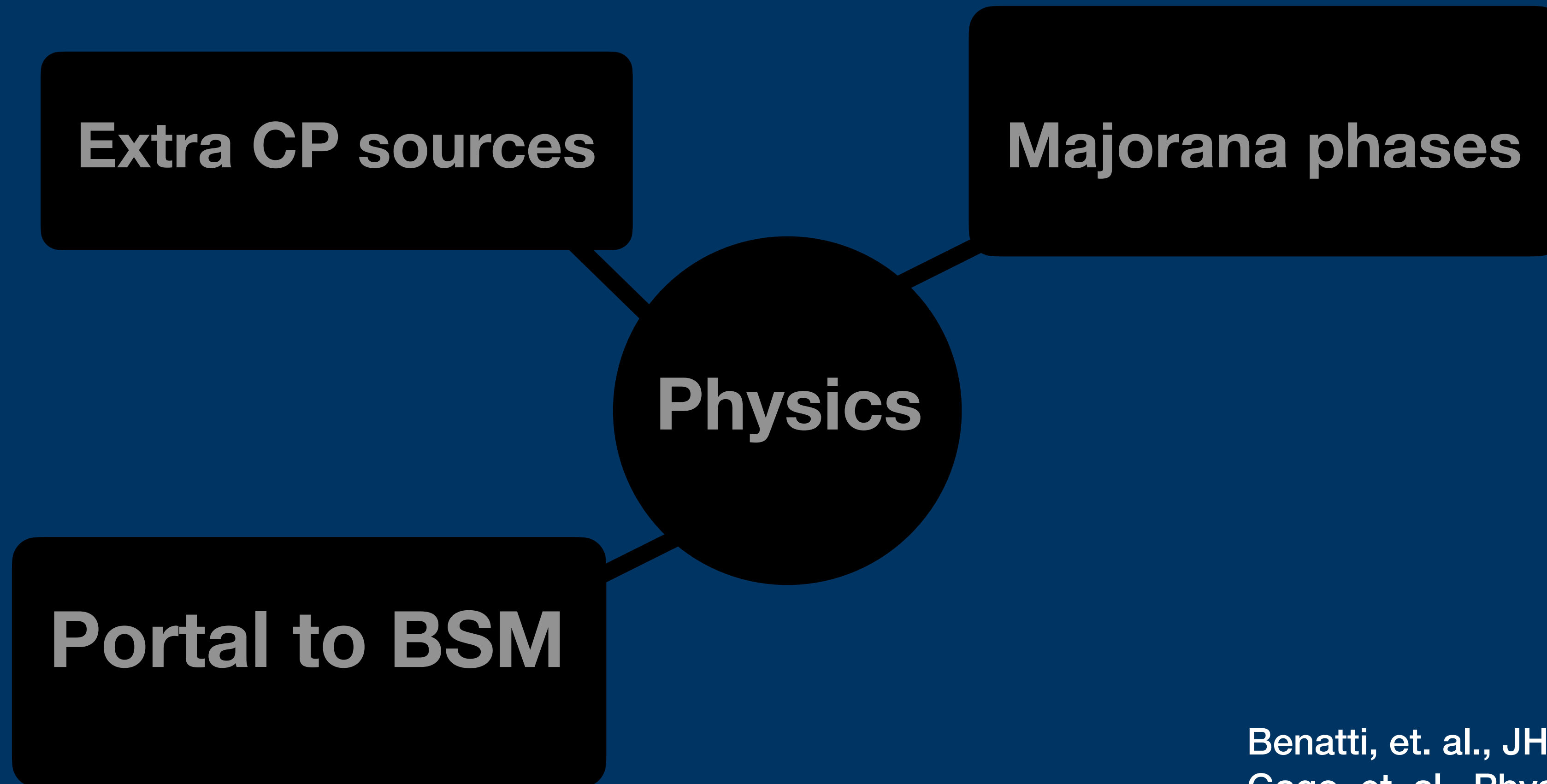
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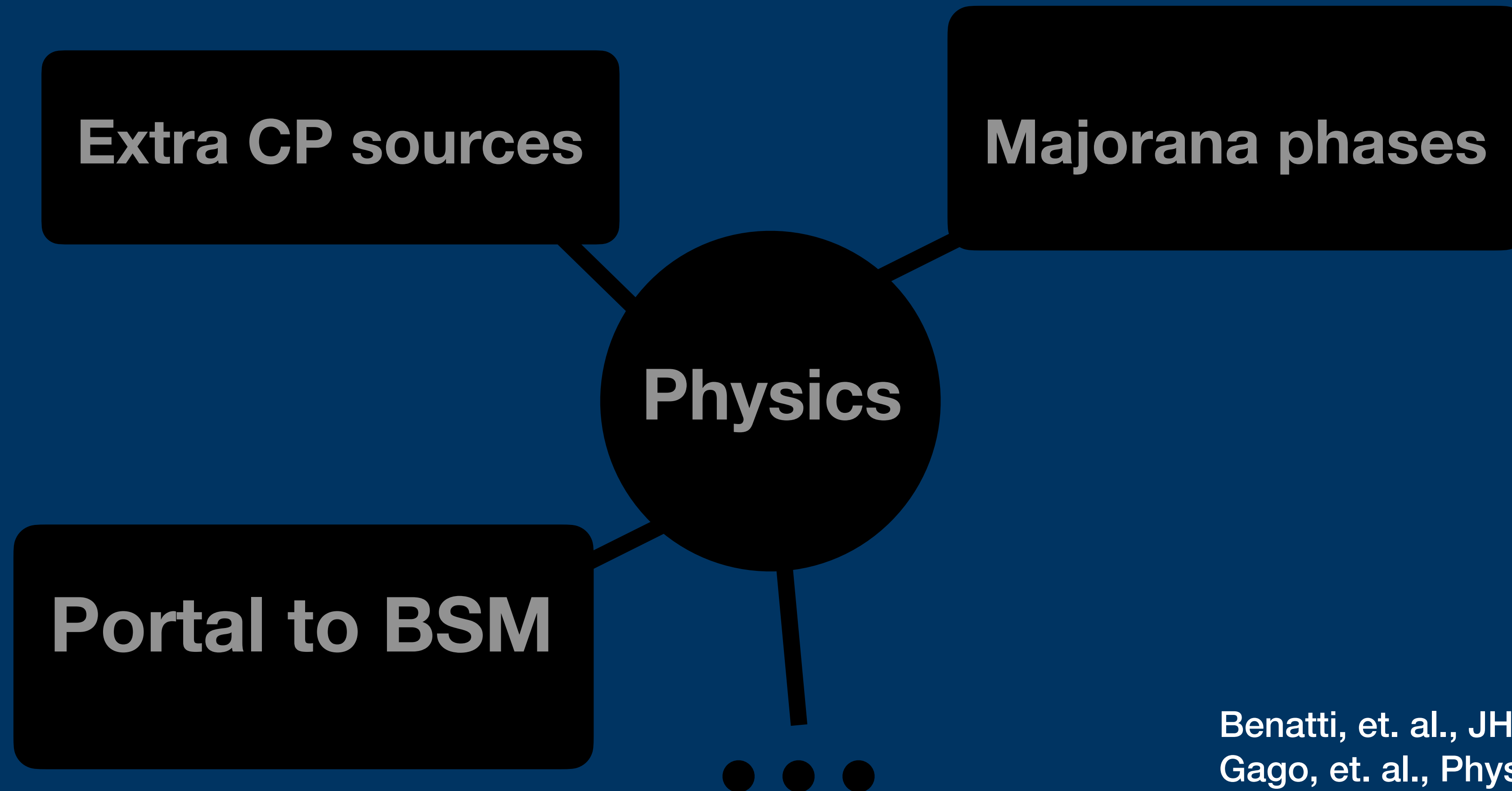
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Schwetz, et. al, JHEP JHEP05 (2015) 007

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How the framework has been applied to neutrino physics?

Open systems, open possibilities

Article | Published: 26 March 2024

Search for decoherence from quantum gravity with atmospheric neutrinos

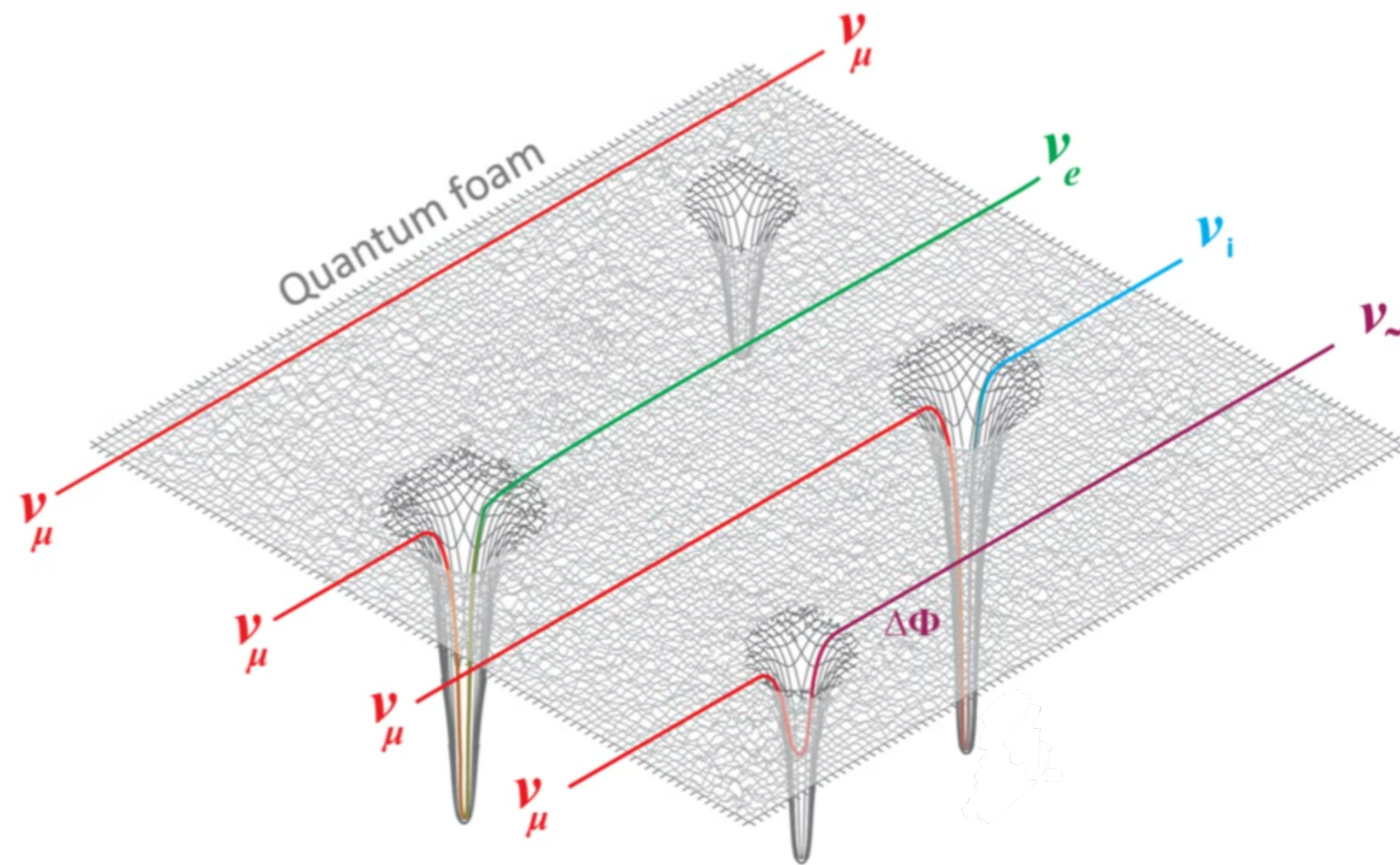
[The IceCube Collaboration](#)

[Nature Physics](#) **20**, 913–920 (2024) | [Cite this article](#)

3483 Accesses | **4** Citations | **315** Altmetric | [Metrics](#)

How the framework is applied to neutrino physics

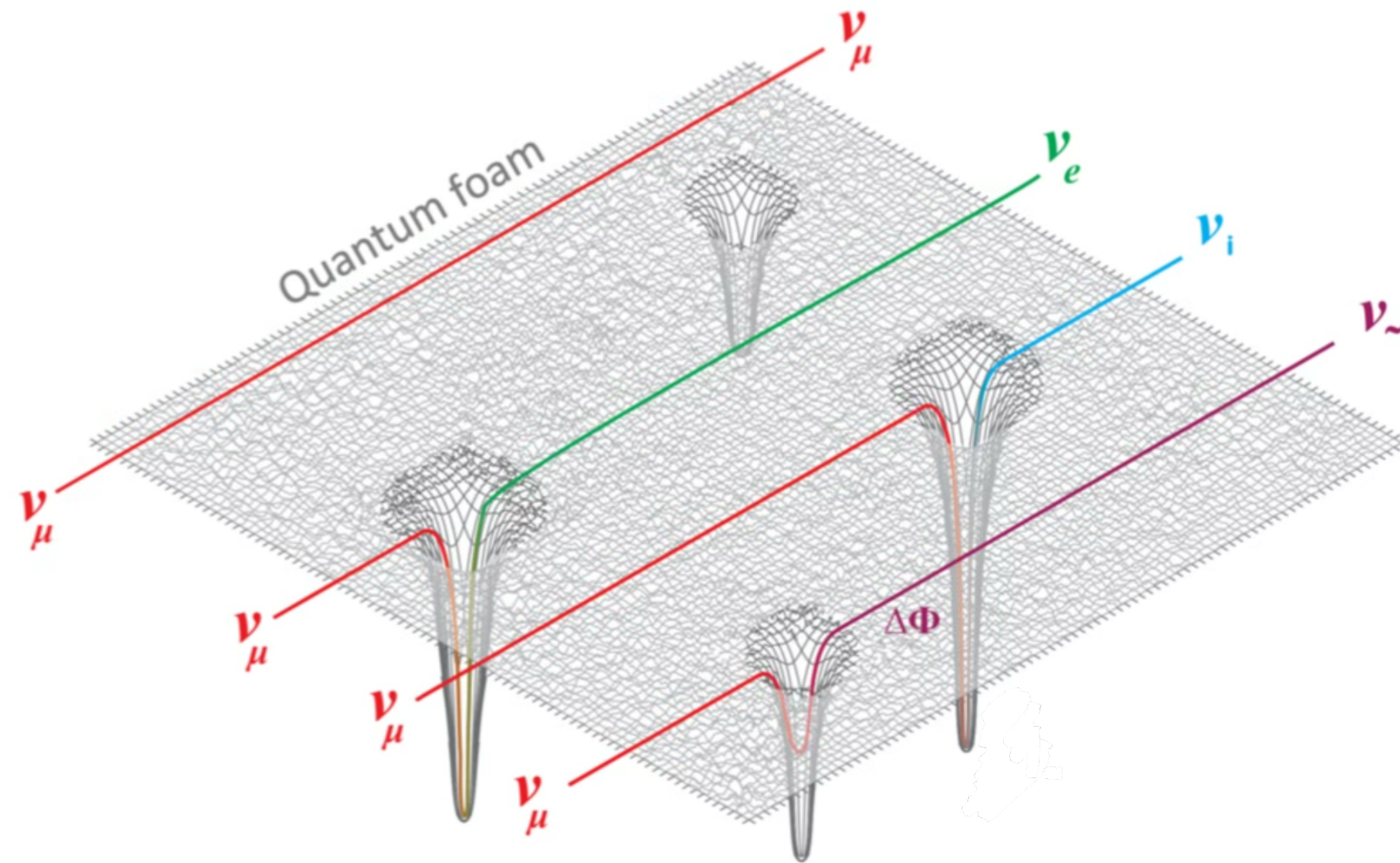
Motivation:



From: The IceCube collaboration, Nature Physics 20, 913-920

How the framework is applied to neutrino physics

Motivation:



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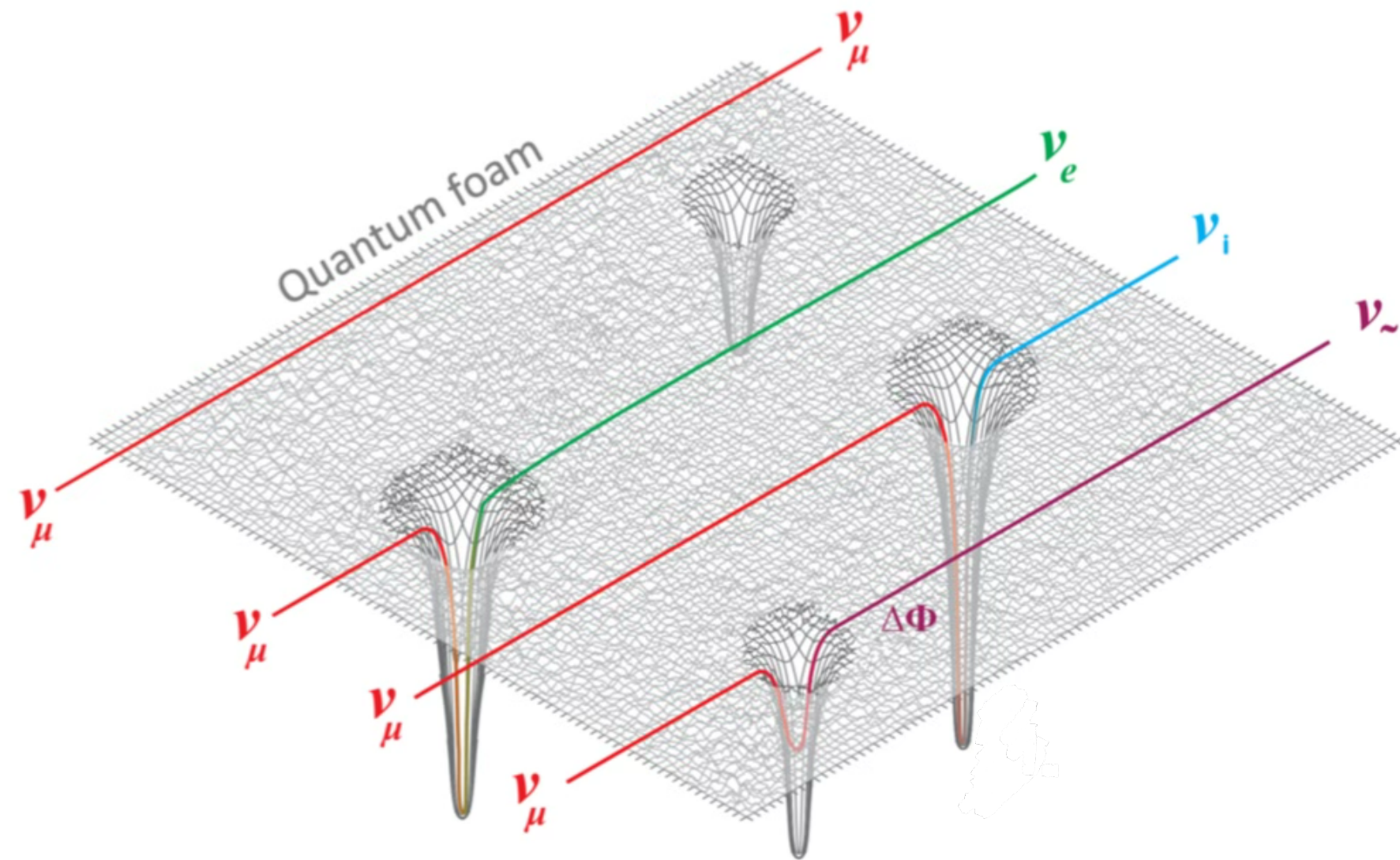
Method:

$$\dot{\rho} = -i[H, \rho] - \mathcal{D}[\rho]$$

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

How the framework is applied to neutrino physics

Motivation:



From: The IceCube collaboration, Nature Physics 20, 913-920

Method:

$$\dot{\rho} = -i[H, \rho] - \mathcal{D}[\rho]$$

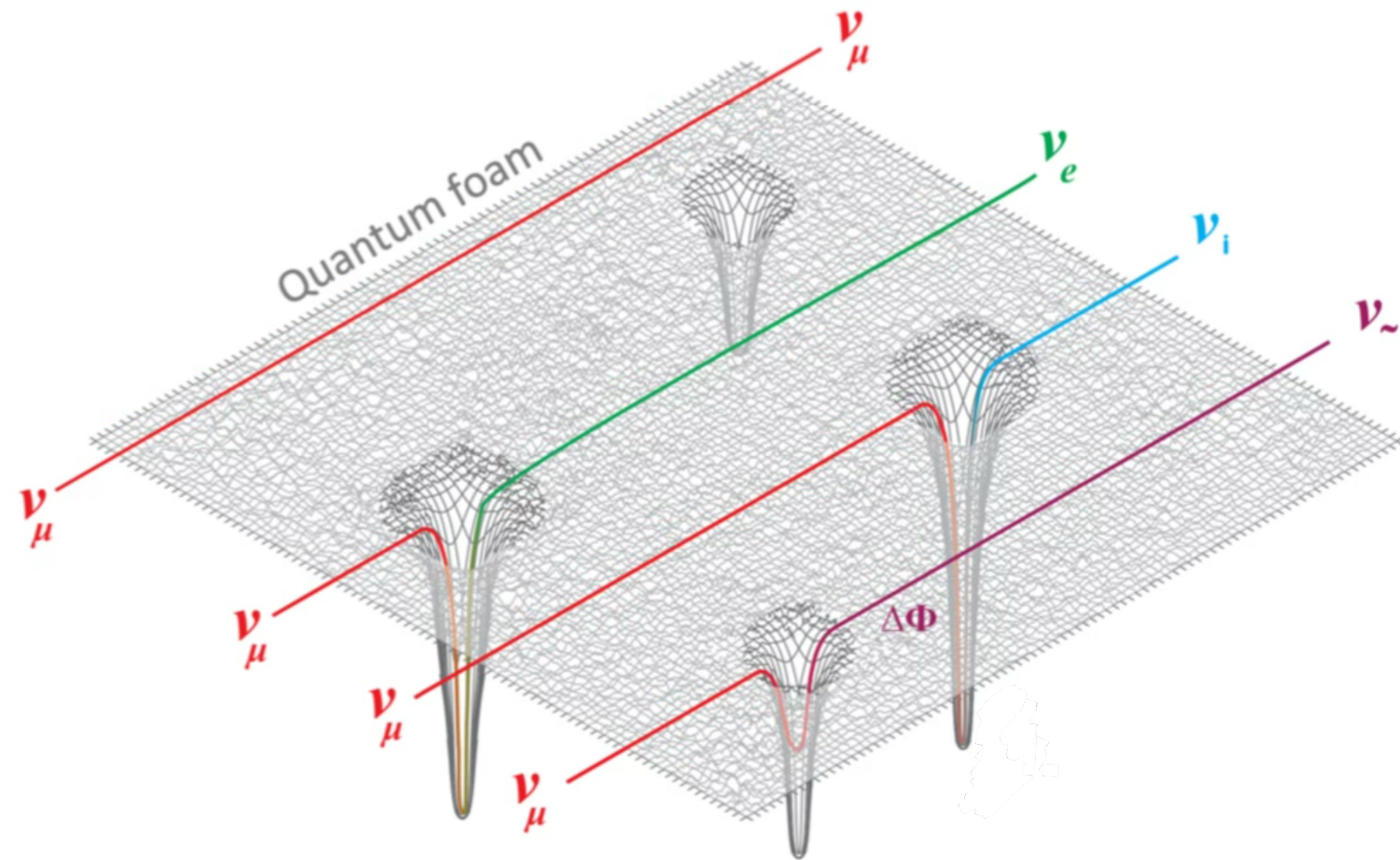
$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

Pheno:

$$\Gamma(E_\nu) = \Gamma_0 \left(\frac{E_\nu}{E_0} \right)^n$$

How the framework is applied to neutrino physics

Motivation:



From: The IceCube collaboration, Nature Physics 20, 913-920

Method:

$$\dot{\rho} = -i[H, \rho] - \mathcal{D}[\rho]$$

$$D_{\text{phase perturbation}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0)$$

Pheno:

$$\Gamma(E_\nu) = \Gamma_0 \left(\frac{E_\nu}{E_0} \right)^n$$

What is Γ ?

Where does it come from? How?

Complete framework?

I will give you one concrete example

An open system approach to neutrino propagating through a ultralight scalar background

Lua F. T. Airoldi,^{1,2,*} Gustavo F. S. Alves,^{1,2,3,†} Pedro A. N. Machado,^{2,‡} and Peter Vander Griend^{2,4,§}

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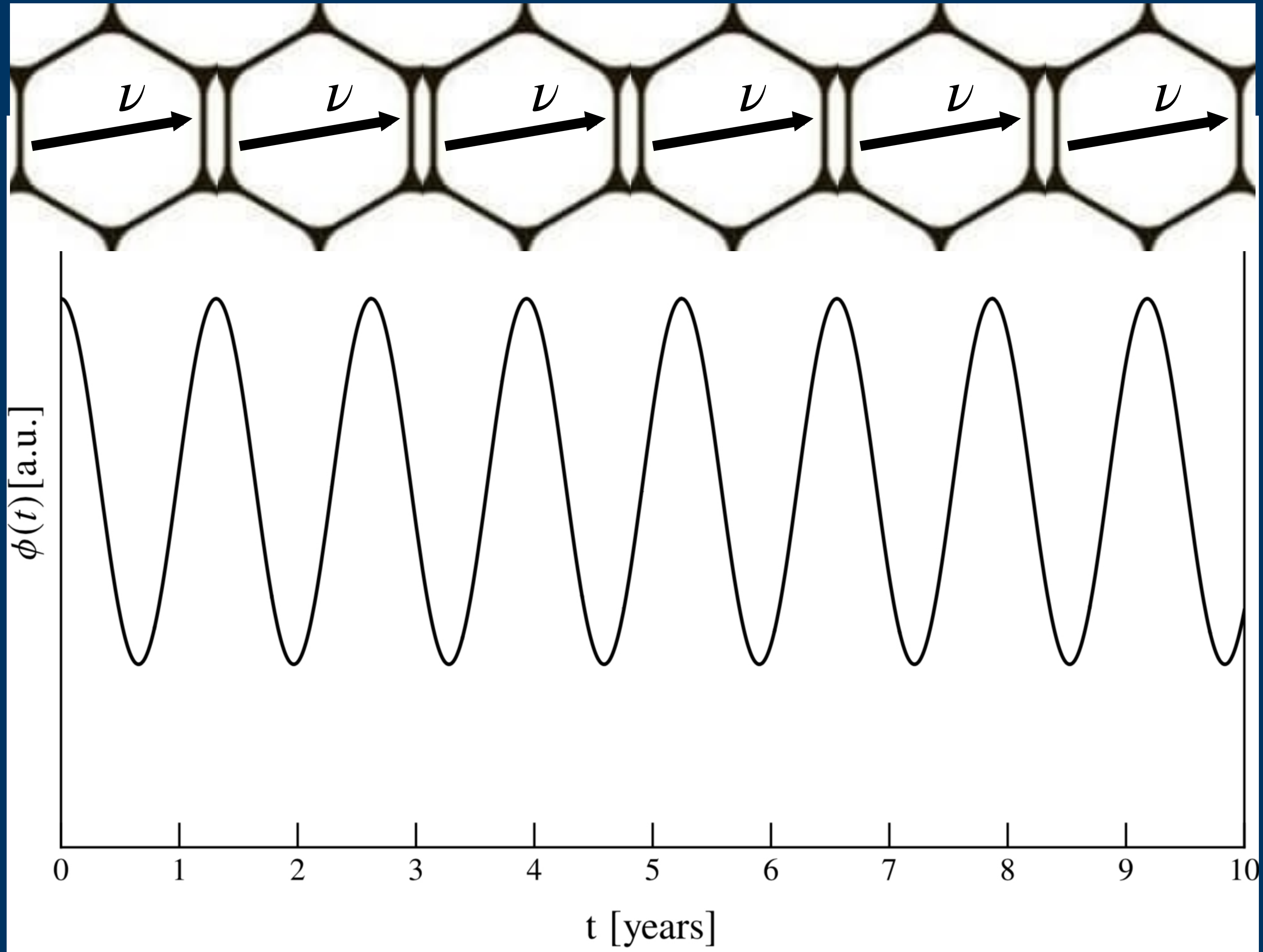
1. Mapping between model parameters and open systems framework

I will give you one concrete example

An open system approach to neutrino propagating through a ultralight scalar background

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1. Mapping between model parameters and open system framework
2. This simple model eludes previous analyses



The field can be effectively described as

$$\phi(x, t) = \phi_0 \cos(\xi)$$

$$\xi \in [0, 2\pi]$$

Setup

We will assume the scalar couples with all mass eigenstates with the same coupling constant

Impact on neutrino propagation

Neutrino Hamiltonian

$$H_\nu = H_0 + H_\phi$$

Impact on neutrino propagation

Neutrino Hamiltonian

$$H_\nu = H_0 + H_\phi$$

$$H_0 = \frac{1}{2E} \text{diag}(m_1^2, m_2^2, m_3^2)$$

Impact on neutrino propagation

Neutrino Hamiltonian

$$H_\nu = H_0 + H_\phi$$

$$H_0 = \frac{1}{2E} \text{diag}(m_1^2, m_2^2, m_3^2)$$

$$H_\phi = g_\phi \frac{\phi_0}{E} \text{diag}(m_1, m_2, m_3) \cos \xi$$

How to approach the problem?

Typically:

1. Compute the time evolution for one neutrino.

$$U(t, \xi) = \exp \left(-iH_\nu(\xi)t \right)$$

How to approach the problem?

Typically:

- 1. Compute the time evolution for one neutrino.**
- 2. Compute the oscillation probability. It will be a function of an unknown phase.**

How to approach the problem?

Typically:

1. Compute the time evolution for one neutrino.
2. Compute the oscillation probability. It will be a function of an unknown phase.
3. Average over the phase.

$$\bar{P} = \frac{1}{2\pi} \int_0^{2\pi} d\xi P(\xi)$$

**This particular example
is easy enough to treat
analytically**

Think of this problem as a door with a key



We will explore other methods





Density Operators

Quick review of density operators

Quick review of density operators

The density operator satisfies

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This is a pure state.

Any pure state satisfies $\text{Tr}(\rho^2) = 1$

Quick review of density operators

Simple example

$$|\nu_e\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle$$

$$\rho_e = |\nu_e\rangle\langle\nu_e| = \begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{pmatrix}$$

Quick review of density operators

Mixed states have the form

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

With $\text{Tr}(\rho^2) < 1$

Quick review of density operators

Simple example

$$\rho = \cos^2 \theta |\nu_1\rangle\langle\nu_1| + \sin^2 \theta |\nu_2\rangle\langle\nu_2|$$

$$\rho = \begin{pmatrix} \cos^2 \theta & 0 \\ 0 & \sin^2 \theta \end{pmatrix}$$

$$\text{Tr}(\rho^2) < 1$$

Quick review of density operators

Time evolution

$$\partial_t \rho(t) = -i[H, \rho(t)]$$

$$\rho(t) = U(t)\rho(0)U^\dagger(t)$$

Quick review of density operators

Time evolved neutrino state

$$|\nu_e\rangle = \cos\theta e^{-iE_1 t} |\nu_1\rangle + \sin\theta e^{-iE_2 t} |\nu_2\rangle$$

$$\rho_e = |\nu_e\rangle\langle\nu_e| = \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta e^{-i\Delta E_{12}t} \\ \cos\theta\sin\theta e^{i\Delta E_{12}t} & \sin^2\theta \end{pmatrix}$$

Quick review of density operators

Time evolution with open systems

$$\partial_t \rho(t) = -i[H, \rho(t)] - L^\dagger L \rho(t) - \rho(t) L^\dagger L + 2L \rho(t) L^\dagger$$

A taste of open systems effects

Time evolved neutrino state

$$\rho_e = |\nu_e\rangle\langle\nu_e| = \begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta e^{-i\Delta E_{12}t - \Gamma L} \\ \cos \theta \sin \theta e^{i\Delta E_{12}t - \Gamma L} & \sin^2 \theta \end{pmatrix}$$

A taste of open systems effects

Time evolved neutrino state

$$\rho_e = |\nu_e\rangle\langle\nu_e| = \begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta e^{-i\Delta E_{12}t - \Gamma L} \\ \cos \theta \sin \theta e^{i\Delta E_{12}t - \Gamma L} & \sin^2 \theta \end{pmatrix}$$

For large L

$$\rho_e = |\nu_e\rangle\langle\nu_e| = \begin{pmatrix} \cos^2 \theta & 0 \\ 0 & \sin^2 \theta \end{pmatrix}$$

How to find the master equation

How to find the master equation

This is not a AI generated output*

How to get the master equation?

Recall

$$\rho(t, \xi) = U(t, \xi) \rho(0) U^\dagger(t, \xi)$$

How to get the master equation?

Recall

$$\rho(t, \xi) = U(t, \xi) \rho(0) U^\dagger(t, \xi)$$

Average

$$\bar{\rho}(t) = \frac{1}{2\pi} \int_0^{2\pi} d\xi U(t, \xi) \rho(0) U^\dagger(t, \xi)$$

How to get the master equation?

Define

$$\Delta U = U(t, \xi) - \bar{U}(t) \quad \bar{U}(t) = \frac{1}{2\pi} \int_0^{2\pi} d\xi U(t, \xi)$$

How to get the master equation?

Define

$$\Delta U = U(t, \xi) - \bar{U}(t) \quad \bar{U}(t) = \frac{1}{2\pi} \int_0^{2\pi} d\xi U(t, \xi)$$

We can rewrite

$$\bar{\rho}(t) = \bar{U}(t)\rho(0)\bar{U}^\dagger(t) + \frac{1}{2\pi} \int_0^{2\pi} d\xi \Delta U(t, \xi)\rho(0)\Delta U^\dagger(t, \xi)$$

How to get the master equation?

Define an effective hamiltonian

$$\partial_t \bar{U}(t) = -iV(t)\bar{U}(t)$$

Burgess, et al., Annals of Physics
Volume 256, Issue 1, 1

How to get the master equation?

Define an effective hamiltonian

$$\partial_t \bar{U}(t) = -iV(t)\bar{U}(t)$$

Burgess, et al., Annals of Physics
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We find the time evolution equation

$$\partial_t \bar{\rho}(t) = -i[V(t), \bar{\rho}(t)] + \partial_t \left(\frac{1}{2\pi} \int_0^{2\pi} d\xi \Delta U(t, \xi) \rho(0) \Delta U^\dagger(t, \xi) \right)$$

How to get the master equation?

To second order in perturbation theory

$$U(t, \xi) \approx \mathbf{1} - i \int_0^t d\tau H(\tau, \xi) - \int_0^t d\tau \int_0^\tau d\tau' H(\tau, \xi) H(\tau', \xi)$$

We can determine $V(t)$ and $\Delta U(t, \xi)$

The master equation

$$\partial_t \bar{\rho}(t) = -i[H_0, \bar{\rho}(t)] - \frac{g_\phi^2 \phi_0^2 t}{2E^2} (\hat{m}_\nu^2 \bar{\rho}(t) + \bar{\rho}(t) \hat{m}_\nu^2 - 2\hat{m}_\nu \bar{\rho}(t) \hat{m}_\nu)$$

$$\hat{m}_\nu = \text{diag}(m_1, m_2, m_3)$$

The master equation

$$\partial_t \bar{\rho}(t) = -i[H$$

A photograph of a chalkboard with handwritten mathematical equations. The equations are:

$$\partial_t \rho_\nu = -i \left[m_\nu \left(1 + \frac{y}{\kappa} \langle \phi \rangle \right), \rho_\nu \right]$$

$$- \frac{2}{\kappa^2} A_{\phi\phi} \left((y m_\nu)^2 \rho + \rho_\nu (y m_\nu)^2 - 2 y m_\nu \rho y m_\nu \right)$$

$$2 \hat{m}_\nu \bar{\rho}(t) \hat{m}_\nu)$$

The master equation

$$\partial_t \bar{\rho}(t) = -i[H_0, \bar{\rho}(t)] - \frac{g_\phi^2 \phi_0^2 t}{2E^2} (\hat{m}_\nu^2 \bar{\rho}(t) + \bar{\rho}(t) \hat{m}_\nu^2 - 2\hat{m}_\nu \bar{\rho}(t) \hat{m}_\nu)$$

JUNO is the best candidate to search for these effects.

The master equation

$$\partial_t \bar{\rho}(t) = -i[H_0, \bar{\rho}(t)] - \frac{g_\phi^2 \phi_0^2 t}{2E^2} (\hat{m}_\nu^2 \bar{\rho}(t) + \bar{\rho}(t) \hat{m}_\nu^2 - 2\hat{m}_\nu \bar{\rho}(t) \hat{m}_\nu)$$

Let's have a look at the probability

How does the probability change?

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \frac{1}{2} (\sin^2(2\theta_{13}) + \sin^2(2\theta_{12}))$$
$$- \exp\left(-\frac{(\Delta\phi_{21})^2}{4}\right) P_{\odot}$$
$$+ \exp\left(-\frac{(\Delta\phi_{atm})^2}{4}\right) \frac{\sin^2(2\theta_{13})}{2} \sqrt{1 - \sin^2(2\theta_{12}) \sin^2 \Delta_{21}} \cos(2|\Delta_{ee}| \pm \Phi_{\odot})$$

How does the probability change?

$$\begin{aligned}
 P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx & 1 - \frac{1}{2} (\sin^2(2\theta_{13}) + \sin^2(2\theta_{12})) \\
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 \end{aligned}$$

$$\begin{aligned}
 \Delta_{ee} &= \cos^2 \theta_{12} \Delta_{31} + \sin^2 \theta_{12} \Delta_{32} \\
 \Phi_{\odot} &= \arctan(\cos 2\theta_{12} \tan \Delta_{21}) - \Delta_{21} \cos 2\theta_{12} \\
 \cos^4 \theta_{13} &\approx 1
 \end{aligned}$$

How does the probability change?

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \frac{1}{2} (\sin^2(2\theta_{13}) + \sin^2(2\theta_{12})) - \exp\left(-\frac{(\Delta\phi_{21})^2}{4}\right) P_{\odot} + \exp\left(-\frac{(\Delta\phi_{atm})^2}{4}\right) \frac{\sin^2(2\theta_{13})}{2} \sqrt{1 - \sin^2(2\theta_{12})\sin^2 \Delta_{21}} \cos(2|\Delta_{ee}| \pm \Phi_{\odot})$$
$$P_{\odot} \approx \sin^2 2\theta_{12} \sin^2 \Delta_{21}$$
$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{4E} L$$

How does the probability change?

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \frac{1}{2} (\sin^2(2\theta_{13}) + \sin^2(2\theta_{12}))$$

$$- \exp\left(-\frac{(\Delta\phi_{21}^\phi)^2}{4}\right) P_\odot$$

$$+ \exp\left(-\frac{(\Delta\phi_{atm}^\phi)^2}{4}\right) \frac{\sin^2(2\theta_{13})}{2} \sqrt{1 - \sin^2(2\theta_{12}) \sin^2 \Delta_{21}} \cos(2|\Delta_{ee}| \pm \Phi_\odot)$$

$$\Delta_{ij}^\phi = g_\phi \phi_0 \frac{(m_i - m_j)}{E} L$$

How does the probability change?

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \frac{1}{2} (\sin^2(2\theta_{13}) + \sin^2(2\theta_{12}))$$

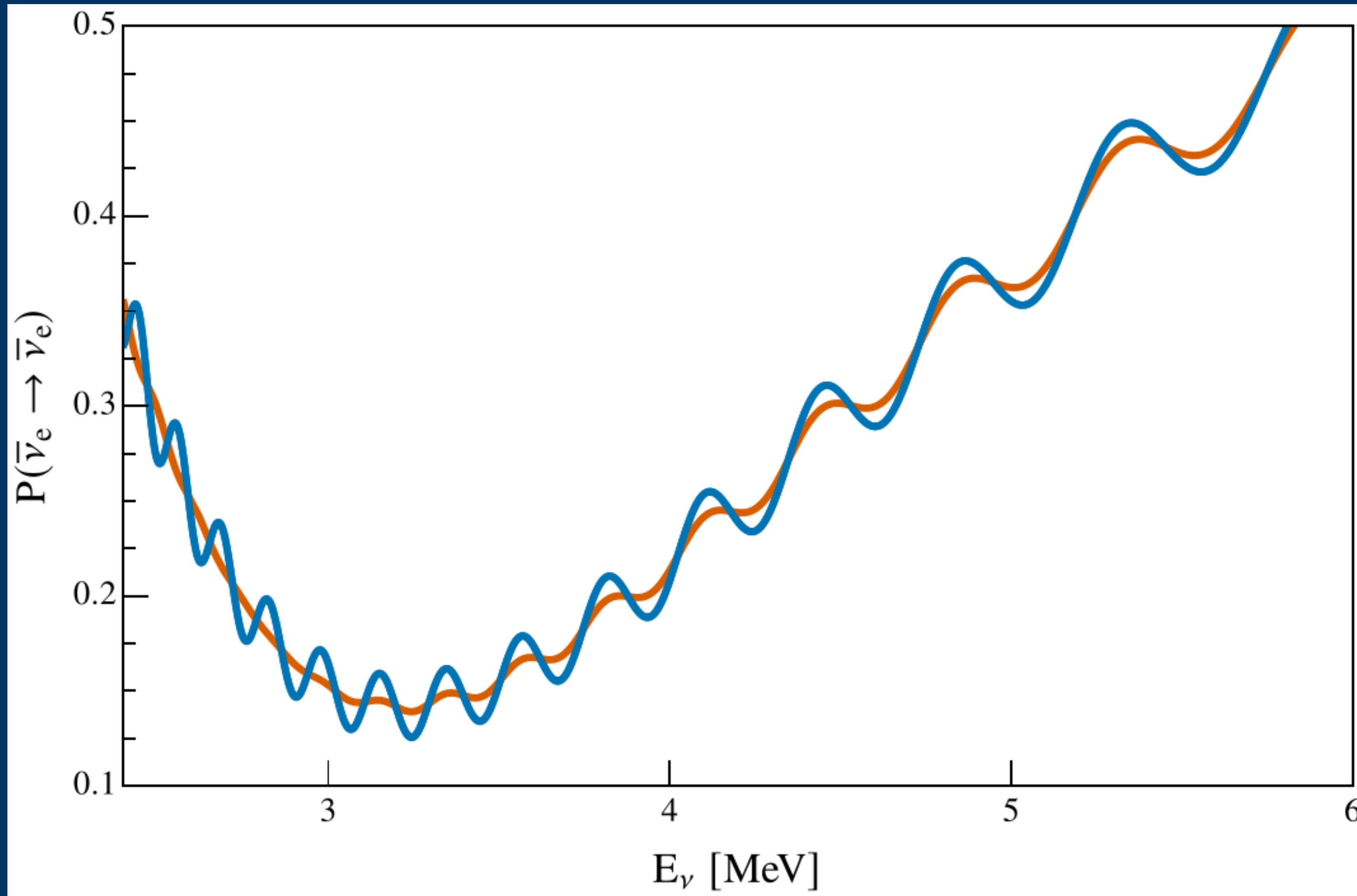
$$- \exp\left(-\frac{(\Delta\phi_{21})^2}{4}\right) P_{\odot}$$

$$+ \exp\left(-\frac{(\Delta\phi_{atm})^2}{4}\right) \frac{\sin^2(2\theta_{13})}{2} \sqrt{1 - \sin^2(2\theta_{12}) \sin^2 \Delta_{21}} \cos(2|\Delta_{ee}| \pm \Phi_{\odot})$$

$$\Delta_{ij}^{\phi} = g_{\phi} \phi_0 \frac{(m_i - m_j)}{E} L$$

Scaling as L^2/E^2

Consequences



Consequences

For large L

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \rightarrow 1 - \frac{1}{2} \left(\sin^2(2\theta_{13}) + \sin^2(2\theta_{12}) \right)$$

The exact solution

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \frac{1}{2} (\sin^2(2\theta_{13}) + \sin^2(2\theta_{12})) \\ - J_0(\Delta_{21}^\phi) P_\odot \\ + J_0(\Delta_{\text{atm}}^\phi) \frac{\sin^2(2\theta_{13})}{2} \sqrt{1 - \sin^2(2\theta_{12}) \sin^2 \Delta_{21}} \cos(2|\Delta_{ee}| \pm \Phi_\odot)$$

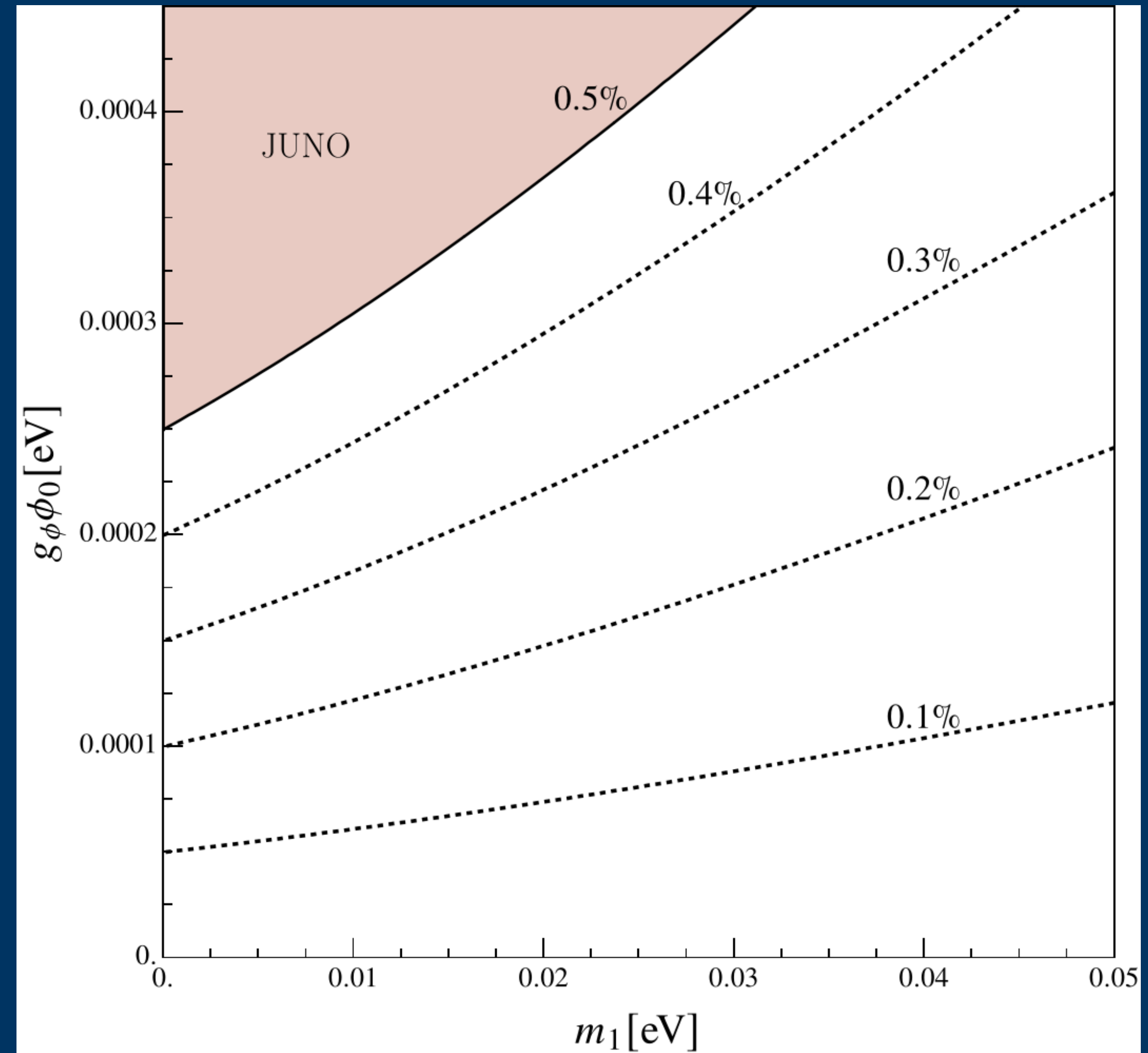
To leading order they coincide

$$J_0(x) = 1 - \frac{x^2}{4} + \mathcal{O}(x^4)$$

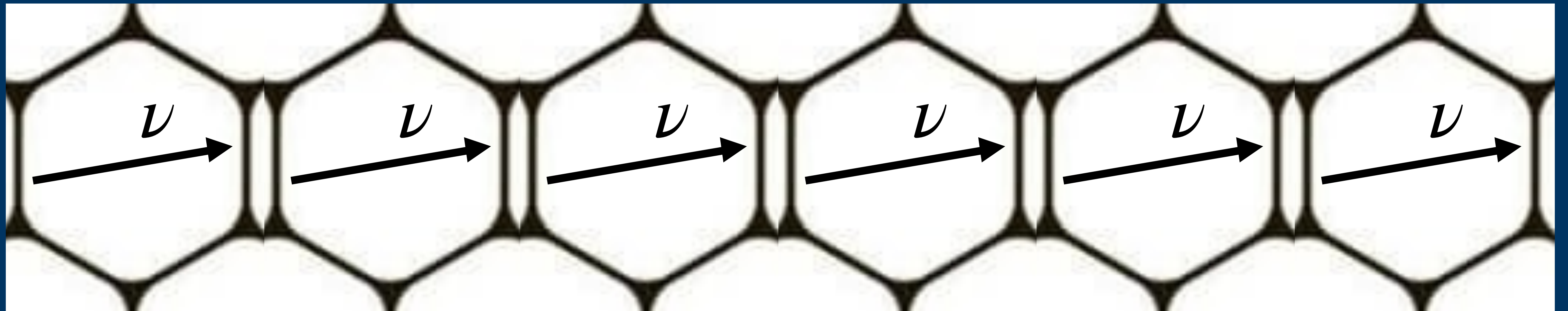
$$\exp\left(-\frac{x^2}{4}\right) = 1 - \frac{x^2}{4} + \mathcal{O}(x^4)$$

Open system can be mapped to model parameters

$$g_\phi \phi_0 (m_i - m_j) = \Delta m_{ij}^2 (\eta_\phi)_{ij}$$

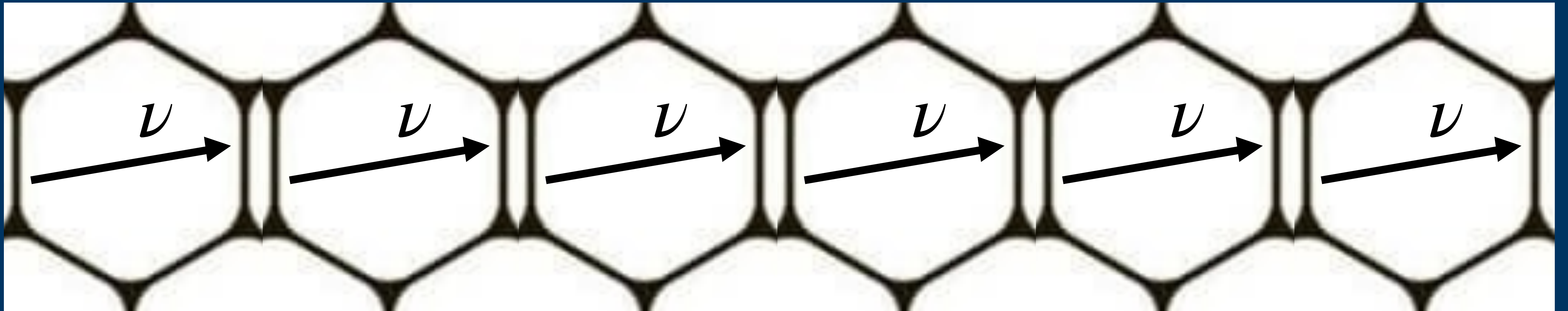


Statistical decoherence, not quantum



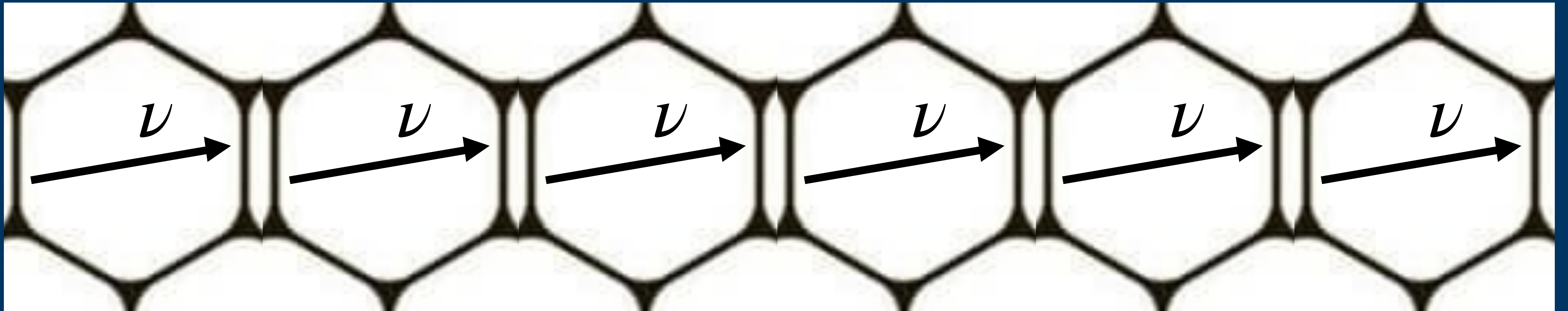
Statistical decoherence, not quantum

$$\rho(t, \xi_1)$$



Statistical decoherence, not quantum

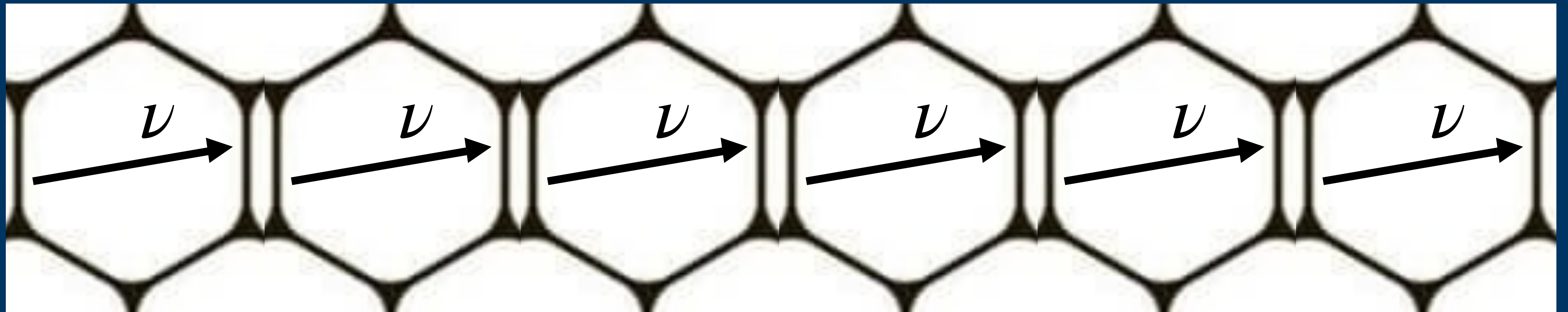
$$\rho(t, \xi_1) \quad \rho(t, \xi_2) \quad \rho(t, \xi_3) \quad \rho(t, \xi_4) \quad \rho(t, \xi_5) \quad \rho(t, \xi_6)$$



$$\text{Tr}(\rho^2) = 1$$

Statistical decoherence, not quantum

$$\rho(t, \xi_1) \quad \rho(t, \xi_2) \quad \rho(t, \xi_3) \quad \rho(t, \xi_4) \quad \rho(t, \xi_5) \quad \rho(t, \xi_6)$$



$$\bar{\rho}(t) = \frac{1}{2\pi} \int_0^{2\pi} d\xi U(t, \xi) \rho(0) U^\dagger(t, \xi) \quad \text{Tr}(\bar{\rho}^2) < 1$$

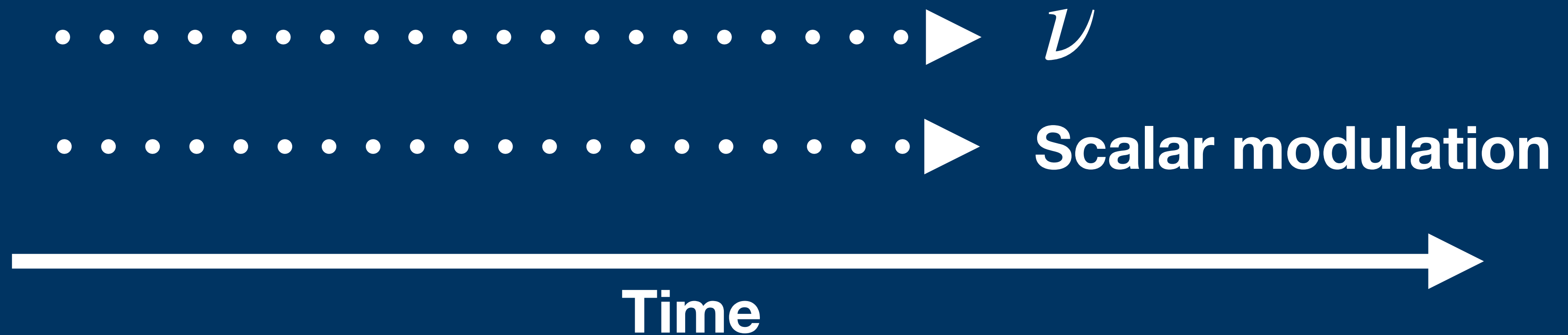
Routes to pursue

**“The end of a story is like the end of a good meal.
We’re sated but it doesn’t mean that we will never be
hungry again.”**

— Kate Tellers

Can we have an observable quantum decoherence effect?

Dynamical field regime



How the master equation should look

$$\partial_t \rho(t) = -i[H_0 + H_\phi(t), \rho(t)] - \int d\tau \int d\tau' \frac{\text{Tr}(\Delta\phi(\tau)\Delta\phi(\tau')\rho_\phi)}{2E^2} (\hat{m}_\nu^2 \rho(t) + \rho(t) \hat{m}_\nu^2 - 2\hat{m}_\nu \rho(t) \hat{m}_\nu)$$

$$\Delta\phi = \phi - \text{Tr}(\phi\rho_\phi) \quad \rho_\phi = \int d\alpha P(\alpha) |\alpha\rangle\langle\alpha|$$

Requirements

1. We must fix the scalar state ρ_ϕ .

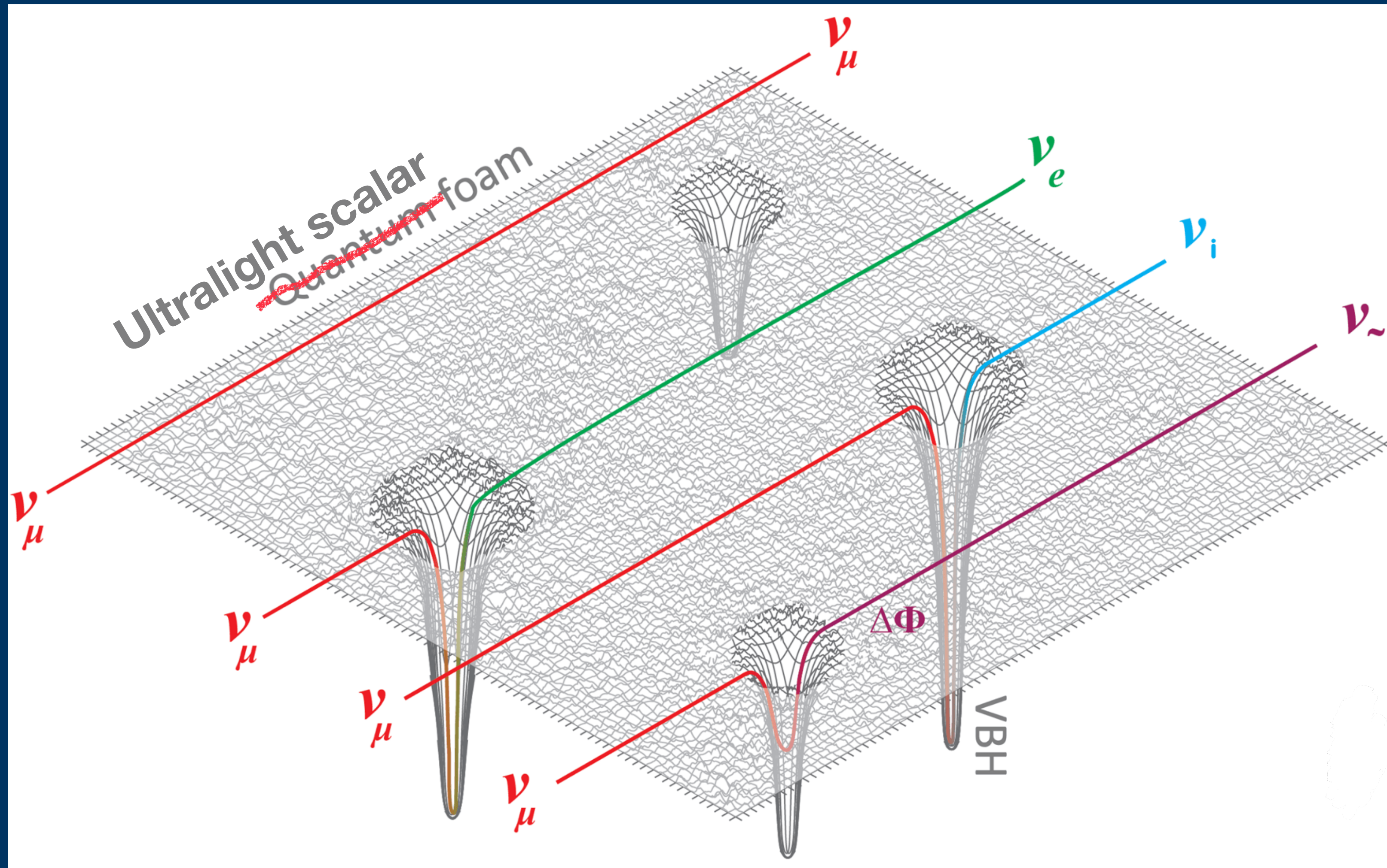
Requirements

1. We must fix the scalar state ρ_ϕ .
2. The field must have a nontrivial variance $\langle \Delta\phi\Delta\phi \rangle$

Requirements

1. We must fix the scalar state ρ_ϕ .
2. The field must have a nontrivial variance $\langle \Delta\phi\Delta\phi \rangle$
3. The field must not oscillate too fast

Routes to pursue



Can we probe true quantum decoherence instead of statistical decoherence?

Routes to pursue

Intrinsically Quantum Effects of Axion Dark Matter are Undetectable

Yunjia Bao,^{1,2,3,4} Dhong Yeon Cheong,^{1,2,3,4,5} Nicholas L. Rodd,^{6,7}
Joey Takach,^{6,7} Lian-Tao Wang,^{1,2,3,4} and Kevin Zhou^{6,7}

$$\rho_\phi = \int d\alpha P(\alpha) |\alpha\rangle\langle\alpha|$$

Routes to pursue

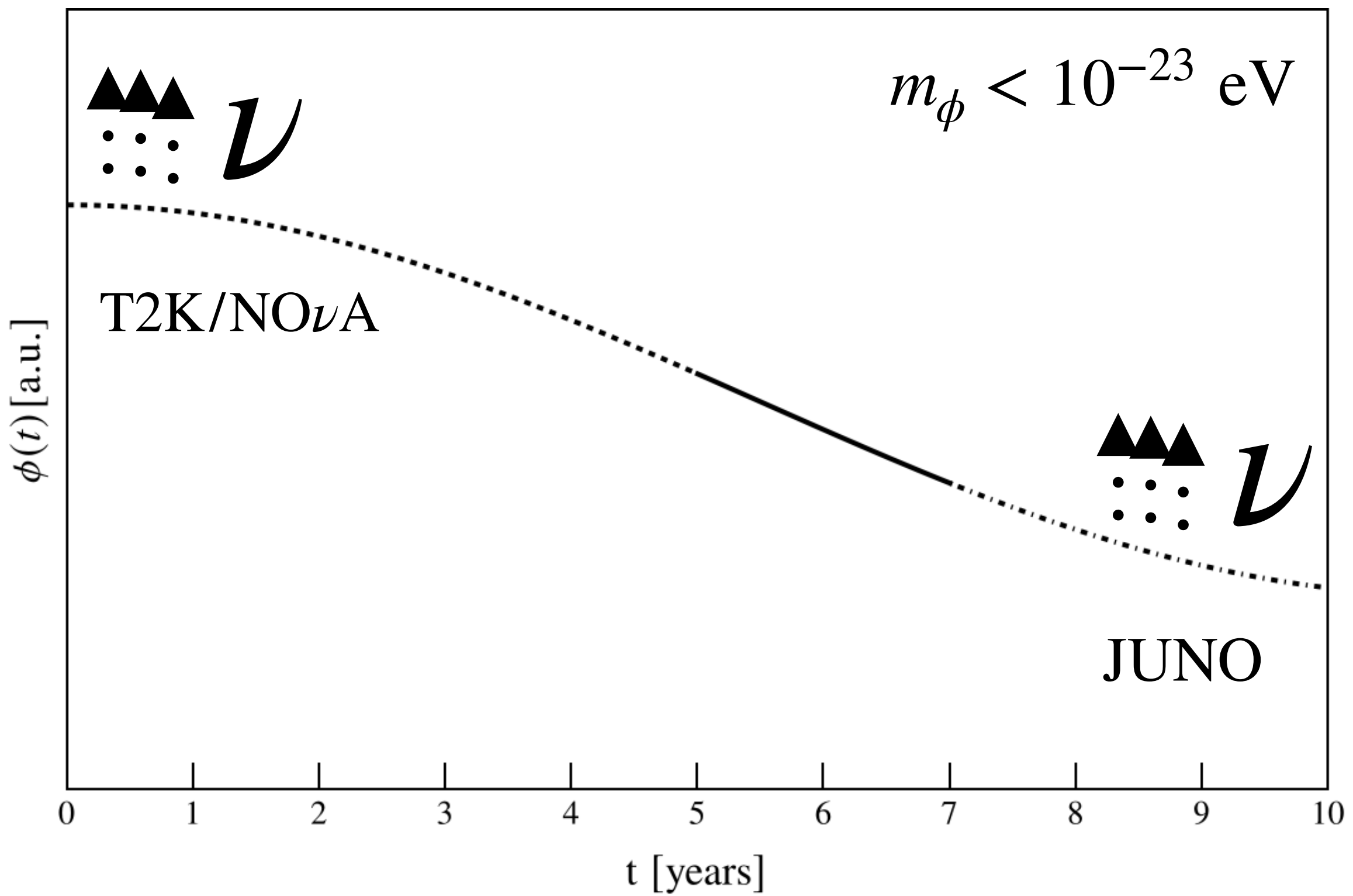
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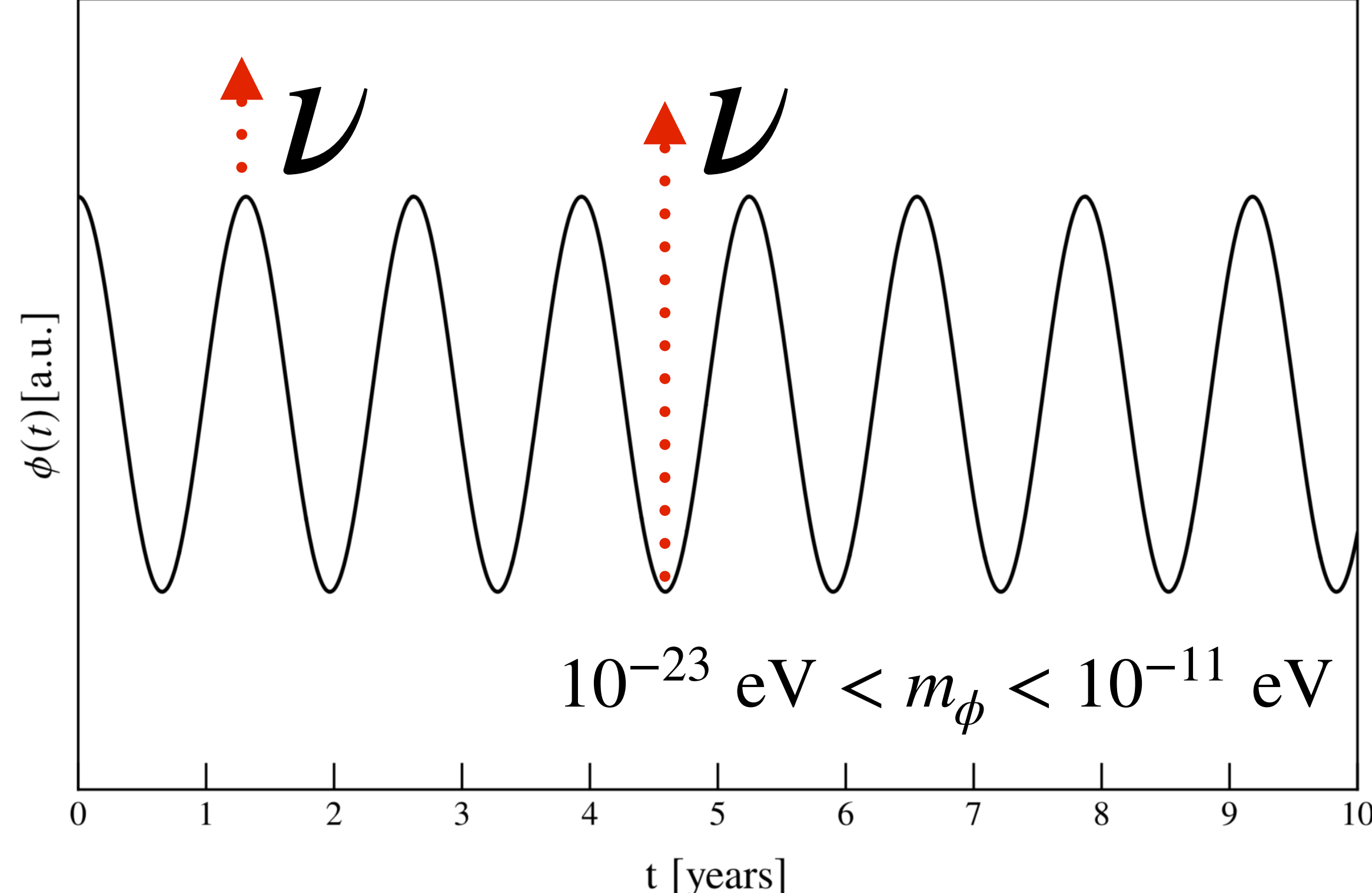
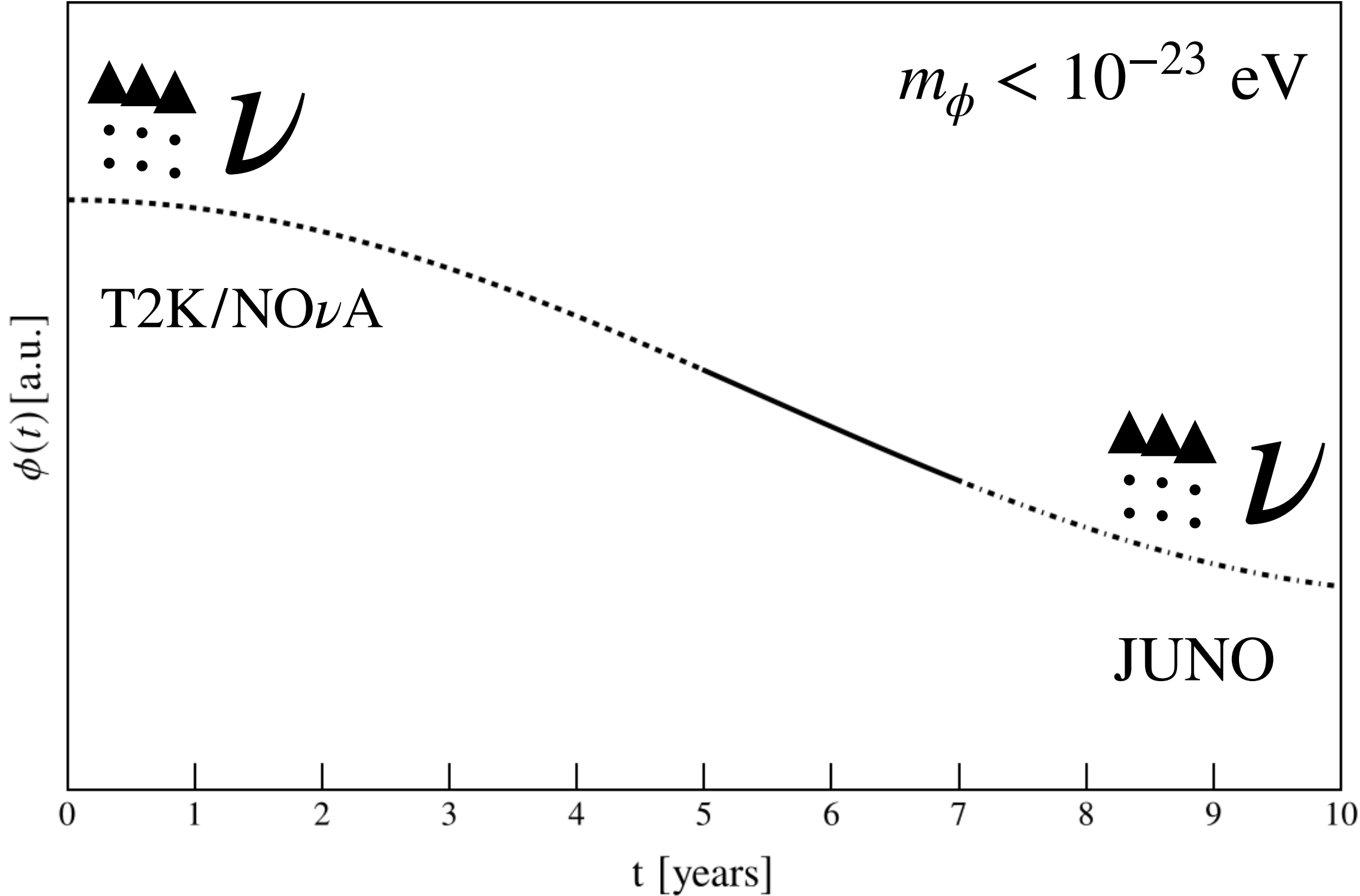
Any hope from the neutrino side?

Summary

Quick recap



Quick recap



**If neutrino masses are clocks,
can we hear them tick?**

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can we hear them tick?**

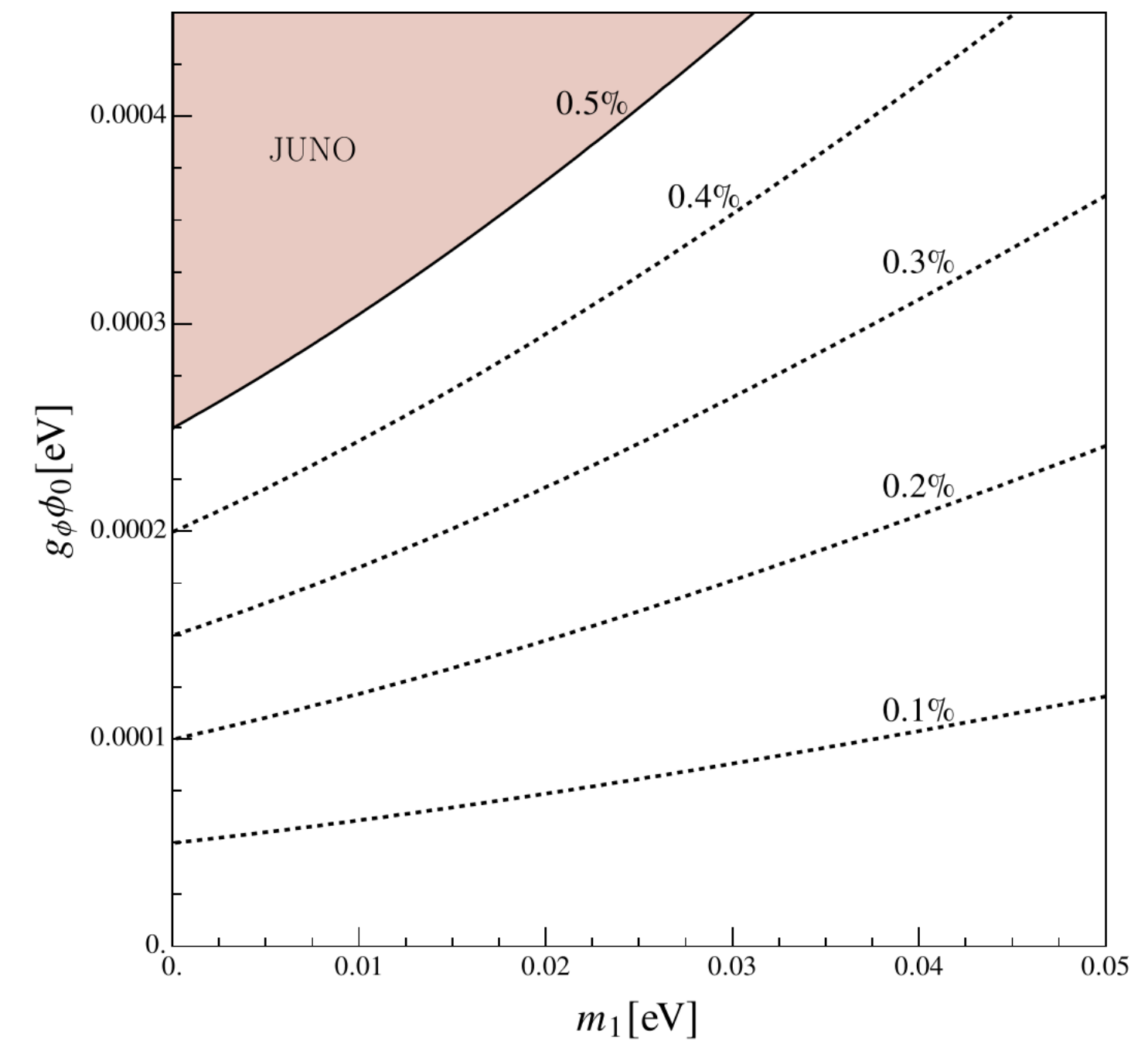
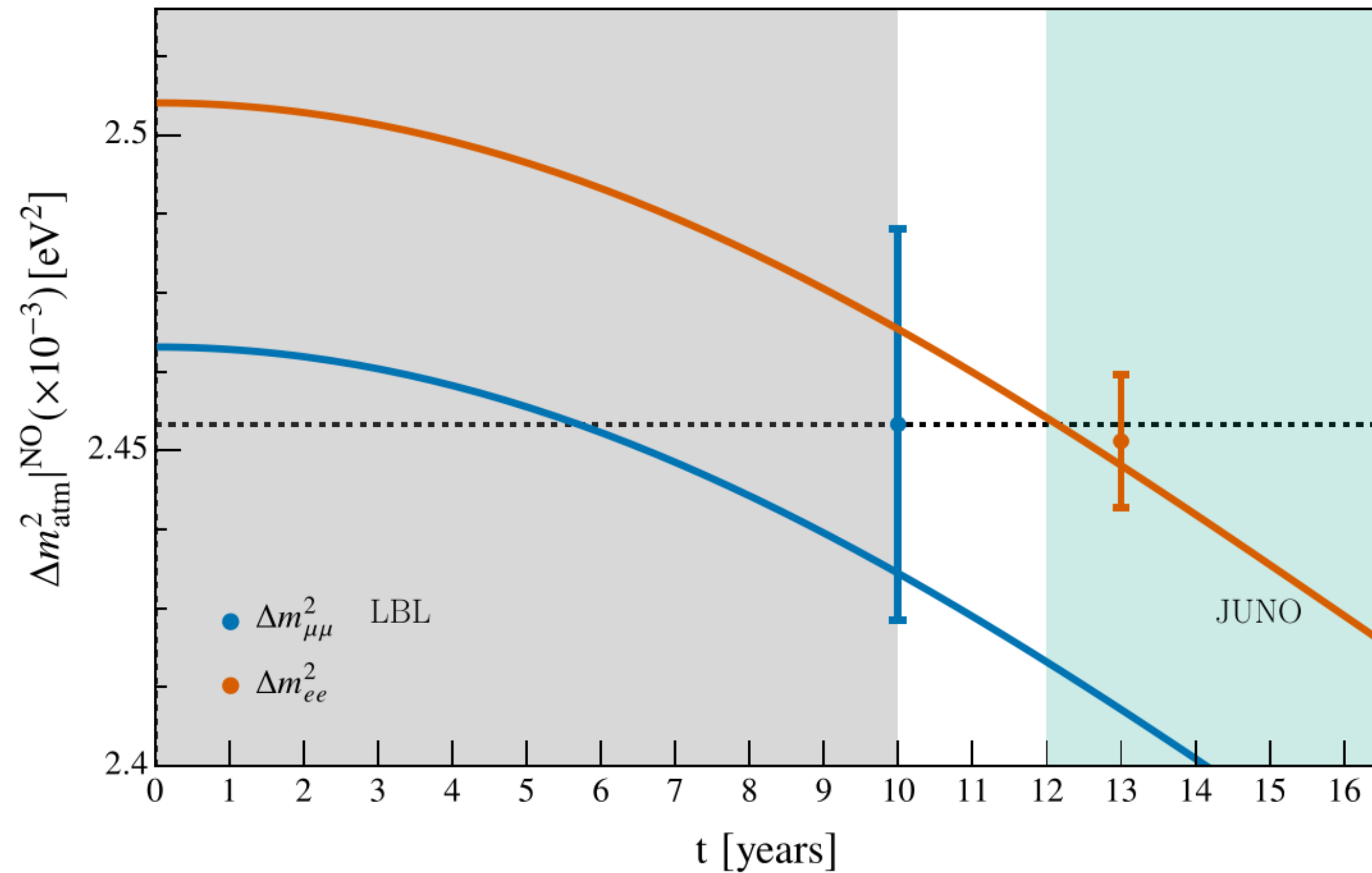


Yes.

**And even if we listen only to the
sound of silence...**

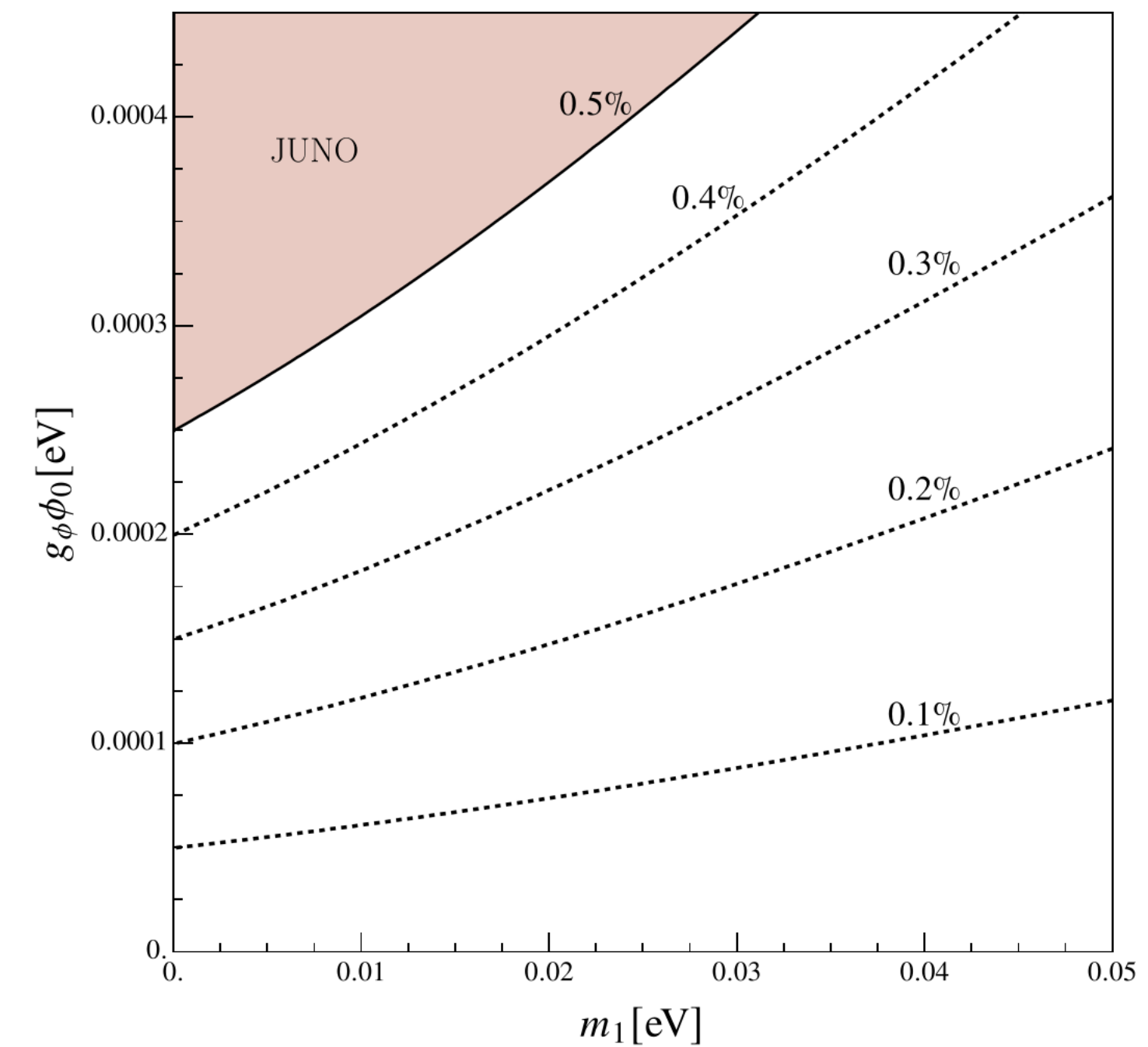
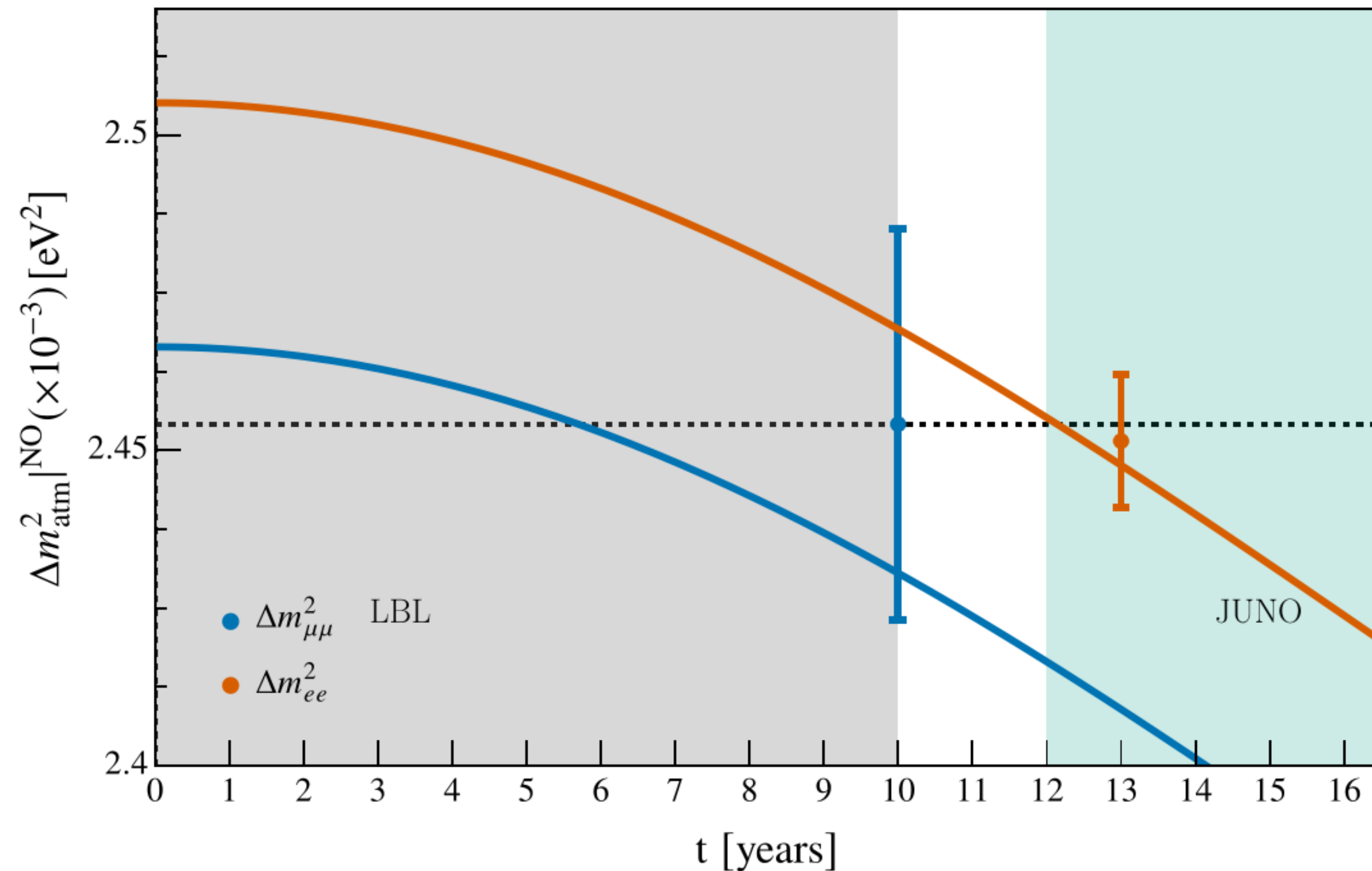
And even if we listen only to the sound of silence...

We can still learn something new!



And even if we listen only to the sound of silence...

We can still learn something new!



Thank you!

BACKUP

“Vectorizing” the time evolution

$$\rho(t, \xi) = \sum_{\mu=0}^8 \rho^\mu \lambda_\mu = \begin{pmatrix} \frac{\rho_0}{\sqrt{3}} + \frac{\rho_3}{\sqrt{2}} + \frac{\rho_8}{\sqrt{6}} & \frac{\rho_1}{\sqrt{2}} - \frac{i\rho_2}{\sqrt{2}} & \frac{\rho_4}{\sqrt{2}} - \frac{i\rho_5}{\sqrt{2}} \\ \frac{\rho_1}{\sqrt{2}} + \frac{i\rho_2}{\sqrt{2}} & \frac{\rho_0}{\sqrt{3}} - \frac{\rho_3}{\sqrt{2}} + \frac{\rho_8}{\sqrt{6}} & \frac{\rho_6}{\sqrt{2}} - \frac{i\rho_7}{\sqrt{2}} \\ \frac{\rho_4}{\sqrt{2}} + \frac{i\rho_5}{\sqrt{2}} & \frac{\rho_6}{\sqrt{2}} + \frac{i\rho_7}{\sqrt{2}} & \frac{\rho_0}{\sqrt{3}} - \sqrt{\frac{2}{3}} \rho_8 \end{pmatrix}$$

$$H(\xi) = \sum_{\mu=0}^8 h^\mu \lambda_\mu, \quad \lambda_\mu = \left\{ \frac{\mathbf{1}_{3 \times 3}}{\sqrt{3}}, \lambda_i \right\}, \quad \text{Tr}(\lambda^\mu \lambda^\nu) = \delta_{\mu\nu}$$

“Vectorizing” the time evolution

Defining

$$|\rho(t, \xi)\rangle = \begin{pmatrix} \rho_0 \\ \rho_1 \\ \vdots \\ \rho_8 \end{pmatrix}$$

**Expanding
both sides**

$$\partial_t \rho_\mu \lambda^\mu = -i h_\nu \rho_\theta [\lambda^\nu, \lambda^\theta]$$

**Collect
coefficients**

$$\partial_t |\rho(t, \xi)\rangle = \mathcal{H}(\xi) |\rho(t)\rangle$$

This can be treated analytically

$$\mathcal{U}(t, \xi) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos \Delta_{21}^{\text{eff}} & \sin \Delta_{21}^{\text{eff}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\sin \Delta_{21}^{\text{eff}} & \cos \Delta_{21}^{\text{eff}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos \Delta_{31}^{\text{eff}} & \sin \Delta_{31}^{\text{eff}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sin \Delta_{31}^{\text{eff}} & \cos \Delta_{31}^{\text{eff}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cos \Delta_{32}^{\text{eff}} & \sin \Delta_{32}^{\text{eff}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\sin \Delta_{32}^{\text{eff}} & \cos \Delta_{32}^{\text{eff}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Delta_{ij}^{\text{eff}} = \frac{\Delta m_{ij}^2}{2E_\nu} L + g_\phi \phi_0 \frac{(m_i - m_j)}{E_\nu} L \equiv \Delta_{ij} + \Delta_{ij}^\phi$$

The averaged evolution

$$\overline{\cos(\Delta_{ij}^{\text{eff}})} = J_0(\Delta_{ij}^{\phi}) \cos(\Delta_{ij})$$

Oscillation probability

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P^{\text{SM}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) + \frac{\cos^4 \theta_{13} \sin^2(2\theta_{12})}{2} \left(J_0(\Delta_{21}^{\phi}) - 1 \right) \cos(\Delta_{21}) \\ + \frac{\sin^2(2\theta_{13})}{2} \left(\cos^2 \theta_{12} \left(J_0(\Delta_{31}^{\phi}) - 1 \right) \cos(\Delta_{31}) + \sin^2 \theta_{12} \left(J_0(\Delta_{32}^{\phi}) - 1 \right) \cos(\Delta_{32}) \right)$$